Asymptotic entanglement of two atoms in a squeezed light field

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The dynamics of entanglement between two-level atoms interacting with a common squeezed reservoir is investigated. It is shown that for spatially separated atoms there is a unique asymptotic state depending on the distance between the atoms and the atom-photons detuning. In the regime of strong correlations there is a one-parameter family of asymptotic steady states depending on the initial conditions. In contrast to the thermal reservoir, both types of asymptotic states can be entangled. We calculate the amount of entanglement in the system in terms of concurrence.

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I. INTRODUCTION

Dynamical creation of entanglement by the indirect interaction between otherwise decoupled systems has been studied recently by many researchers, mainly in the case of two-level atoms interacting with the common vacuum. The idea that dissipation can create rather then destroy entanglement has been put forward in several publications [1–4]. In particular, the effect of spontaneous emission on the destruction and production of entanglement has been discussed [5–8]. When the two atoms are separated by a small distance compared to the radiation wavelength, there is a substantial probability that a photon emitted by one atom will be absorbed by the other, and the resulting process of photon exchange produces correlations between the atoms. Such correlations may cause initially separable states to become entangled.

The case of two atoms immersed in a common thermal reservoir was also investigated [9–12]. As shown in [12], similarly to the vacuum case, the collective properties of the atomic system can alter the decay process compared to the single atom. There are states with enhanced emission rates such that the emission rate is reduced. The important example of the latter is the antisymmetric superposition $|a\rangle$ constructed from energy levels of considered atoms. When the atoms are close to each other, this state is decoupled from the environment and therefore is stable. In that case the asymptotic states of the system are parametrized by the fidelity F of the initial state with respect to state $|a\rangle$ and the temperature T of the photon reservoir. Moreover, the asymptotic states can be identified with thermal generalization of Werner states, that is, mixtures of state $|a\rangle$ and the Gibbs equilibrium state at temperature T.

In the present paper, we consider the atoms interacting with a photon reservoir in a squeezed state [13]. In practice, squeezed light sources produce photon fields in multimode squeezed states, but here we assume a broadband approximation in which the parameters characterizing the photon field are constant over a sufficiently broad frequency range. The dynamics of atoms interacting with squeezed light has been studied by many authors (see, e.g., the review paper [14] and references therein). In the context of our studies, we mention the result of Palma and Knight [15] showing the existence of a highly correlated asymptotic state and the analysis of the cooperative behavior of atoms in broadband squeezed light in Ref. [16].

In this paper, we study the asymptotic entanglement of a system of atoms evolving according to the master equation considered by Tanaś and Ficek [18] but we allow nonzero detuning between the atomic transition frequency and the carrier frequency of the photon field. In the case of spatially separated atoms studied in detail in Ref. [18], there exists a unique asymptotic state, but in contrast to the vacuum or thermal reservoirs, this state can be entangled. However, the produced entanglement is maximal only when the atoms are in resonance with the squeezed photon field. Nonzero detuning significantly diminishes this production. The case where the atoms are separated by a small distance has not been studied previously by other researchers. In this experimental setup the dynamics of the system changes radically. Let us point out that in this case, in contrast to spatially separated atoms where the stationary asymptotic state is unique, there is a one-parameter family of steady states, which further implies that there are dynamically stable nontrivial observables in the system. The asymptotic states ρ_{as} depend on the initial fidelity F and parameters describing the reservoir, but nonzero detuning also modifies the matrix elements of ρ_{as}. We show that the asymptotic states can be expressed as a mixture of a separable Gibbs state and two pure entangled states: an antisymmetric state $|a\rangle$ and some symmetric superposition of ground and excited levels of the atoms. This realization of the asymptotic state simplifies for zero detuning and a minimum-uncertainty squeezed reservoir to a mixture of $|a\rangle$ and a two-atom squeezed state [15]. Thus, in that case, there are two linearly independent stable pure states so that the decoherence-free subspace is two-dimensional [17].

Depending on the initial fidelity, some of the asymptotic states are entangled. We calculate the amount of asymptotic entanglement using the concurrence as its measure. We also show that for initial fidelity greater than some threshold value (depending on the properties of the reservoir and detuning), the

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asymptotic concurrence is nonzero. This property is analogous to the thermal reservoir case. But when the reservoir is in a squeezed state, somehow an unexpected result occurs: initial states with low or even zero fidelity become asymptotically entangled. The possibility of production of entanglement starting from separable states with zero fidelity is very interesting. In this case the correlations present in a squeezed reservoir are transferred to the atomic system, entangling, for example two atoms, both in the ground state. But as before, large detuning between atoms and the photon field destroys this possibility.

II. MODEL DYNAMICS

Consider two-level atoms A and B with ground states $|0\rangle_j$ and excited states $|1\rangle_j$ ($j = A,B$), interacting with the radiation field in a broadband squeezed vacuum state with the carrier frequency ω_s. The parameters N and M characterizing the squeezing satisfy

$$M = |M|\, e^{i\vartheta} \quad \text{and} \quad |M| \leqslant \sqrt{N(N+1)},$$

where the equality holds for a minimum-uncertainty squeezed state. In the Markov approximation the influence of the reservoir on the system of atoms can be described by the dynamical semigroup with the Lindblad generator [18]

$$L = -i[H,\cdot] + L_{\mathrm{D}},$$

where

$$H = \frac{\omega_0}{2}\sum_{j=A,B}\sigma_3^j + \sum_{\substack{j,k=A,B\\ j\neq k}}\Omega_{jk}\sigma_+^j\sigma_-^k, \tag{II.1}$$

and

$$\begin{aligned} L_{\mathrm{D}}\rho = &\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}(1+N)\,(2\sigma_-^j\rho\sigma_+^k - \sigma_+^k\sigma_-^j\rho - \rho\sigma_+^k\sigma_-^j)\\ &+\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}\,N\,(2\sigma_+^j\rho\sigma_-^k - \sigma_-^k\sigma_+^j\rho - \rho\sigma_-^k\sigma_+^j)\\ &+\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}\,M\,(2\sigma_+^j\rho\,\sigma_+^k - \sigma_+^k\sigma_+^j\rho - \rho\sigma_+^k\sigma_+^j)e^{-2i\omega_s t}\\ &+\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}\overline{M}(2\sigma_-^j\rho\,\sigma_-^k - \sigma_-^k\sigma_-^j\rho - \rho\sigma_-^k\sigma_-^j)e^{2i\omega_s t}. \end{aligned} \tag{II.2}$$

Here

$$\sigma_\pm^A = \sigma_\pm \otimes \mathbb{1}, \quad \sigma_\pm^B = \mathbb{1}\otimes\sigma_\pm,$$
$$\sigma_3^A = \sigma_3 \otimes \mathbb{1}, \quad \sigma_3^B = \mathbb{1}\otimes\sigma_3.$$

In the Hamiltonian (II.1), ω_0 is the frequency of the transition $|0\rangle_j \to |1\rangle_j$ ($j = A, B$) and $\Omega_{AB} = \Omega_{BA} = \Omega$ describes interatomic coupling by the dipole-dipole interaction. In contrast, dissipative dynamics is given by the generator (II.2) with parameters γ_{AB} satisfying

$$\gamma_{AA} = \gamma_{BB} = \gamma_0, \quad \gamma_{AB} = \gamma_{BA} = \gamma. \tag{II.3}$$

In the above equalities, γ_0 is the single-atom spontaneous emission rate, and $\gamma = G(\vec{r}_{AB})\,\gamma_0$ is the collective damping constant. In the model considered, $G(\vec{r}_{AB})$ is the function of the interatomic distance $\vec{r}_{AB}$, and $G(\vec{r}_{AB})$ is small for large separation of atoms. In contrast, $G(\vec{r}_{AB}) \to 1$ when $\vec{r}_{AB}$ is small (for more details see, e.g., [19]).

The time evolution of the system of atoms is given by the master equation

$$\frac{d\rho}{dt} = L\,\rho. \tag{II.4}$$

In a frame rotating at frequency ω_s, the master equation (II.4) becomes an equation with time-independent coefficients, and it may be written as

$$\frac{d\rho_I}{dt} = \widetilde{L}\,\rho_I, \tag{II.5}$$

where

$$\widetilde{L} = -i[\widetilde{H},\cdot] + \widetilde{L}_{\mathrm{D}},$$

with

$$\widetilde{H} = \frac{\delta_0}{2}\sum_{j=A,B}\sigma_3^j + \sum_{\substack{j,k=A,B\\ j\neq k}}\Omega_{jk}\sigma_+^j\sigma_-^k, \quad \delta_0 = \omega_0 - \omega_s, \tag{II.6}$$

and

$$\begin{aligned} \widetilde{L}_{\mathrm{D}}\rho_I = &\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}(1+N)(2\sigma_-^j\rho_I\sigma_+^k - \sigma_+^k\sigma_-^j\rho_I - \rho_I\sigma_+^k\sigma_-^j)\\ &+\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}\,N(2\sigma_+^j\rho_I\sigma_-^k - \sigma_-^k\sigma_+^j\rho_I - \rho_I\sigma_-^k\sigma_+^j)\\ &+\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}\,M(2\sigma_+^j\rho_I\sigma_+^k - \sigma_+^k\sigma_+^j\rho_I - \rho_I\sigma_+^k\sigma_+^j)\\ &+\frac{1}{2}\sum_{j,k=A,B}\gamma_{jk}\,\overline{M}(2\sigma_-^j\rho_I\sigma_-^k - \sigma_-^k\sigma_-^j\rho_I - \rho_I\sigma_-^k\sigma_-^j). \end{aligned} \tag{II.7}$$

Notice that in the Hamiltonian (II.6), detuning δ_0 can be arbitrary. Only when the atoms are in resonance with the carrier frequency of the squeezed vacuum does $\delta_0 = 0$.

From now on we omit the subscript I. The master equation (II.5) can be used to obtain the equations for matrix elements of a state ρ of the system of two-level atoms with respect to some basis. To simplify the calculations one can work in the basis of collective states in the Hilbert space $\mathbb{C}^2\otimes\mathbb{C}^2$ [19], given by product vectors

$$|e\rangle = |1\rangle_A \otimes |1\rangle_B, \quad |g\rangle = |0\rangle_A \otimes |0\rangle_B, \tag{II.8}$$

symmetric superposition

$$|s\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A\otimes|1\rangle_B + |1\rangle_A\otimes|0\rangle_B), \tag{II.9}$$

and antisymmetric superposition

$$|a\rangle = \frac{1}{\sqrt{2}}(|1\rangle_A\otimes|0\rangle_B - |0\rangle_A\otimes|1\rangle_B). \tag{II.10}$$

In the basis of collective states, the two-atom system can be treated as a single four-level system with ground state $|g\rangle$, excited state $|e\rangle$, and two intermediate states, $|s\rangle$ and $|a\rangle$. From (II.5) it follows that the matrix elements of state ρ with respect to the basis $|e\rangle, |s\rangle, |a\rangle, |g\rangle$ satisfy the equations, which can be grouped into decoupled systems of differential equations. So the diagonal matrix elements and ρ_{eg} satisfy

$$\begin{aligned}\frac{d\rho_{ee}}{dt} &= (\gamma_0-\gamma)N\,\rho_{aa} + (\gamma_0+\gamma)N\,\rho_{ss} - 2\gamma_0 N\,\rho_{ee}\\ &\quad - \gamma\,(M\rho_{ge} + \overline{M}\,\rho_{eg}),\\ \frac{d\rho_{ss}}{dt} &= -(\gamma_0+\gamma)[(1+2N)\,\rho_{ss} - (1+n)\,\rho_{ee} - N\,\rho_{gg}\\ &\quad - M\rho_{ge} - \overline{M}\,\rho_{eg}],\\ \frac{d\rho_{aa}}{dt} &= -(\gamma_0-\gamma)\,[(1+2N)\,\rho_{aa} - (1+N)\,\rho_{ee} - N\,\rho_{gg}\\ &\quad + M\rho_{ge} + \overline{M}\,\rho_{eg}],\\ \frac{d\rho_{gg}}{dt} &= (\gamma_0-\gamma)(1+N)\,\rho_{aa} + (\gamma_0+\gamma)\,(1+N)\,\rho_{ss} - 2\gamma_0\,N\,\rho_{gg}\\ &\quad - \gamma\,(M\,\rho_{ge} + \overline{M}\,\rho_{eg}),\\ \frac{d\rho_{eg}}{dt} &= -(\gamma_0-\gamma)\rho_{aa} + (\gamma_0+\gamma)\,M\,\rho_{ss} - \gamma\,M\rho_{gg}\\ &\quad - (\gamma_0\,(1+2N) + 2i\,\delta_0)\,\rho_{eg}.\end{aligned} \tag{II.11}$$

In contrast, the elements $\rho_{ae},\rho_{ag},\rho_{se}$, and ρ_{sg} are connected by the following equations:

$$\begin{aligned}\frac{d\rho_{ae}}{dt} &= \left[\gamma\left(N+\frac{1}{2}\right) - \gamma_0\left(2N+\frac{1}{2}\right) + i\,(\delta_0+\Omega)\right]\rho_{ae}\\ &\quad - (\gamma_0-\gamma)\,\rho_{ga} + (\gamma_0-\gamma)\overline{M}\,\rho_{ea} - \gamma\overline{M}\,\rho_{ag},\\ \frac{d\rho_{ag}}{dt} &= \left[\gamma\left(N+\frac{1}{2}\right) - \gamma_0\left(2N+\frac{1}{2}\right) - i\,(\delta_0-\Omega)\right]\rho_{ag}\\ &\quad + (\gamma_0-\gamma)\,M\,\rho_{ga} - (\gamma_0-\gamma)(1+N)\rho_{ea} - \gamma\,M\rho_{ae},\\ \frac{d\rho_{se}}{dt} &= -\left[\gamma\left(N+\frac{1}{2}\right) + \gamma_0\left(2N+\frac{1}{2}\right) - i\,(\delta_0| - \Omega)\right]\rho_{se}\\ &\quad + (\gamma_0+\gamma)\,\overline{M}\rho_{es} + (\gamma_0+\gamma)N\rho_{gs} - \gamma\overline{M}\rho_{sg},\\ \frac{d\rho_{sg}}{dt} &= -\left[\gamma\left(N+\frac{1}{2}\right) + \gamma_0\left(2N+\frac{1}{2}\right) + i\,(\delta_0+\Omega)\right]\rho_{se}\\ &\quad + (\gamma_0+\gamma)(1+N)\,\rho_{es} + (\gamma_0+\gamma)\,M\,\rho_{gs} - \gamma\,M\,\rho_{se},\end{aligned} \tag{II.12}$$

and, finally,

$$\frac{d\rho_{as}}{dt} = -\,[\gamma_0\,(1+2N) - 2i\,\Omega\,]\,\rho_{as}. \tag{II.13}$$

The equations for the remaining matrix elements can be obtained by using the Hermiticity of ρ.

From Eqs. (II.11) it follows that, similarly to the case of a reservoir in the vacuum state (see, e.g., [19]) and thermal state [12], a system of atoms in symmetric state $|s\rangle$ decays at the enhanced rate $\gamma_0+\gamma$, whereas the antisymmetric initial state $|a\rangle$ leads to the reduced rate $\gamma_0-\gamma$. When the atoms are so close to each other that we can ignore the effects of their different spatial positions, we can put $\gamma=\gamma_0$. In this limiting case of strongly correlated atoms (Dicke model), state $|a\rangle$ is completely decoupled from the reservoir. It can also be checked that the master equation (II.5) describes two types of time evolution of the system of atoms, depending on the relation between γ and γ_0. When $\gamma<\gamma_0$, there is a unique asymptotic state. This state was found in Ref. [18] for the special case of zero detuning. In the general case, we compute it in the next section. In contrast, in the Dicke model case when $\gamma=\gamma_0$, we show that there is a one-parameter family of nontrivial asymptotic states depending on the initial states.

III. ASYMPTOTIC STATES

A. Spatially separated atoms

The case of spatially separated atoms when $\gamma<\gamma_0$ was studied in detail in Ref. [18], but for completeness of the exposition we also discuss this point briefly. In addition, we allow nonzero detuning, which makes the model more realistic. Direct calculations show that, in that case, there exists a unique stationary asymptotic state ρ_{u}, which in the canonical basis

$$|1\rangle_A\otimes|1\rangle_B,\quad |1\rangle_A\otimes|0\rangle_B,\quad |0\rangle_A\otimes|1\rangle_B,\quad |0\rangle_A\otimes|0\rangle_B$$

has nonvanishing matrix elements

$$\begin{aligned}\rho_{11} &= \frac{a_0}{u_0},\quad \rho_{22}=\rho_{33}=\frac{c_0}{u_0},\\ \rho_{23} &= \frac{b_0}{u_0},\quad \rho_{14}=\frac{z_0}{u_0},\quad \rho_{44}=\frac{d_0}{u_0},\end{aligned} \tag{III.1}$$

where for

$$\delta=\frac{\delta_0}{\gamma_0},\quad \widehat{\gamma}=\frac{\gamma}{\gamma_0},$$

we have

$$\begin{aligned}u_0 &= (1+2N)^2\,[\,(1+2N)^2+4\,\delta^2\,]\\ &\quad + 4|M|^2\,(\widehat{\gamma}^2-(1+2N)^2),\end{aligned} \tag{III.2}$$

and

$$\begin{aligned}a_0 &= N^2[\,(1+2N)^2 - 4|M|^2 + 4\delta^2] + |M|^2\widehat{\gamma}^2,\\ c_0 &= N(N+1)\left[(1+2N)^2 - 4|M|^2 + 4\delta^2\right] + |M|^2\widehat{\gamma}^2,\\ d_0 &= (1+N)^2\,[(1+2N)^2 - 4|M|^2 + 4\delta^2] + |M|^2\widehat{\gamma}^2.\end{aligned} \tag{III.3}$$

Moreover,

$$b_0 = -2\widehat{\gamma}\,|M|^2,\quad z_0 = -(1+2N-2i\delta)\,\widehat{\gamma}\,M. \tag{III.4}$$

State (III.1), in contrast to the analogous asymptotic state in the thermal reservoir, can be entangled and as we show later, its entanglement depends crucially on the value of the normalized detuning δ and the normalized damping constant $\widehat{\gamma}$.

B. Strongly correlated atoms

The main new results of the paper concern the atoms which are close to each other. Then we can put $\gamma=\gamma_0$ and equations (II.11)–(II.13) simplify. One can check that the solutions of (II.12) and (II.13) asymptotically vanish and the only contribution to the asymptotic states ρ_{as} comes from $\rho_{ee},\rho_{aa},\rho_{ss},\rho_{gg}$, and ρ_{eg}. Note that in this case

$$\frac{d\rho_{aa}}{dt}=0,\quad \text{so}\quad \rho_{aa}(t)=\rho_{aa}(0)=F,$$

where

$$F=\langle a|\rho|a\rangle$$

is the *fidelity* of the initial state ρ with respect to the antisymmetric state $|a\rangle$. Hence the fidelity of the asymptotic state ρ_{as} also equals F and one finds that in the canonical basis the matrix of ρ_{as} has the same "X" form as in the case of state (III.1), but with nonvanishing matrix elements given by

$$\rho_{11} = (1-F)\,\frac{a}{u}, \quad \rho_{22} = (1-F)\,\frac{c}{2u} + \frac{F}{2},$$
$$\rho_{23} = (1-F)\,\frac{c}{2u} - \frac{F}{2}, \quad \rho_{14} = (1-F)\,\frac{z}{u}, \qquad \text{(III.5)}$$
$$\rho_{44} = (1-F)\,\frac{d}{u},$$

and $\rho_{33} = \rho_{22}$. In Eqs. (III.5) we have

$$u = (1+2N)^2\,(1+3N+3N^2-3|M|^2) + 4\,(1+3N+3N^2)\,\delta^2, \qquad \text{(III.6)}$$

and

$$a = 4N^2[N(N+1)-|M|^2] + |M|^2 + N^2\,(1+4\delta^2),$$
$$c = (1+2N)^2[N(N+1)-|M|^2] + 2N(N+1)\delta^2,$$
$$d = (1+2N)[1+N+3(N(N+1)-|M|^2)] + 2N[N(N+1)-|M|^2] + 4(1+N)^2\delta^2$$
$$z = -(1+2N-2i\delta)\,M. \qquad \text{(III.7)}$$

The asymptotic states ρ_{as} defined by (III.5) exist for any initial state and, for fixed parameters characterizing the squeezing, depend on the initial fidelity and the normalized detuning $\delta = \delta_0/\gamma_0$ of the electromagnetic field. When $M = 0$, we recover the case of a standard thermal bath with N playing the role of the mean photon number [12].

To study the structure of the asymptotic states, we consider first the special case of minimum-uncertainty squeezing and zero detuning of the radiation field. One can check that, in that case, the matrix elements of ρ_{as} are given by

$$\rho_{11} = (1-F)\,\frac{N}{1+2N}, \quad \rho_{22} = (\rho_{as})_{33} = \frac{F}{2},$$
$$\rho_{23} = -\frac{F}{2}, \quad \rho_{44} = (1-F)\,\frac{1+N}{1+2N}, \qquad \text{(III.8)}$$
$$\rho_{14} = (1-F)\frac{\sqrt{N(N+1)}}{1+2N}\,e^{i\theta},$$

where $\theta = \vartheta + \pi$. The asymptotic state given by (III.8) has a remarkable structure: it is a mixture

$$\rho_{as} = (1-F)\,|N,\theta\rangle\langle N,\theta| + F\,|a\rangle\langle a| \qquad \text{(III.9)}$$

of the pure state

$$|N,\theta\rangle = \sqrt{\frac{N}{1+2N}}\,|0\rangle_A\otimes|0\rangle_B + e^{i\theta}\sqrt{\frac{1+N}{1+2N}}\,|1\rangle_A\otimes|1\rangle_B \qquad \text{(III.10)}$$

and the antisymmetric state $|a\rangle$. State $|N,\theta\rangle$ is known as a two-atom squeezed state and can be obtained from the ground state $|g\rangle = |0\rangle_A \otimes |0\rangle_B$ by applying the atomic squeezing transformation $S(\xi)$, given by

$$S(\xi) = \exp\,(\overline{\xi}\,\sigma_-^A\sigma_-^B - \xi\,\sigma_+^A\sigma_+^B), \qquad \text{(III.11)}$$

for the appropriate choice of the complex parameter ξ [15]. This state is entangled, and in the limit of maximal squeezing ($N \to \infty$), it becomes a maximally entangled generalized Bell state. Notice also that $|a\rangle$ and $|N,\theta\rangle$ span a decoherence-free subspace for this specific system, as recently established in Ref. [17].

In the general case the structure of ρ_{as} is much more involved. Define

$$F_{cr} = \frac{c}{c+u}. \qquad \text{(III.12)}$$

By a direct calculation we see that if $F \geqslant F_{cr}$, then

$$\rho_{as} = (1-p-q)\rho_\beta + p\,|a\rangle\langle a| + q\,|\psi\rangle\langle\psi|, \qquad \text{(III.13)}$$

where

$$p = \left(1+\frac{c}{u}\right)F - \frac{c}{u}, \quad q = \frac{|z|\,(a+d)}{u\,\sqrt{ad}}\,(1-F). \qquad \text{(III.14)}$$

State ρ_β is a Gibbs state,

$$\rho_\beta = \frac{e^{-\beta H_a}}{\mathrm{tr}\;e^{-\beta H_a}}, \qquad \text{(III.15)}$$

for the Hamiltonian $H_a = H_0 + H_1$, with

$$H_0 = \frac{\omega_0}{2}\sum_{j=A,B}\sigma_3^j, \quad H_1 = \frac{\omega_1}{2}\left(\mathbb{1}\otimes\mathbb{1} + \sigma_3^A\otimes\sigma_3^B\right),$$

the inverse temperature

$$\beta = \frac{1}{2\omega_0}\,\ln\frac{d}{a}, \qquad \text{(III.16)}$$

and the frequency

$$\omega_1 = \frac{2\omega_0}{\ln d/a}\,\ln\frac{c}{\sqrt{ad}-|z|}. \qquad \text{(III.17)}$$

Moreover, the pure state $|\psi\rangle$ is given by

$$|\psi\rangle = \sqrt{\frac{a}{a+d}}\,|0\rangle_A\otimes|0\rangle_B + e^{i\phi}\sqrt{\frac{d}{a+d}}\,|1\rangle_A\otimes|1\rangle_B, \qquad \text{(III.18)}$$

where $\phi = \arg\,z$.

Formula (III.13) is a generalization of Eq. (III.9) as well as the corresponding representation of ρ_{as} by the thermal generalization of Werner states in the case of a thermal reservoir [12]. Observe also that for $F < F_{cr}$, the asymptotic state cannot be expressed as the mixture (III.13), but in contrast to the purely thermal case, the states ρ_{as} can be entangled even if $F < F_{cr}$. We study this problem in the next section.

IV. ASYMPTOTIC ENTANGLEMENT

For characterization of the entanglement of the asymptotic state ρ_{as}, we use Wootters' concurrence [20], defined for any two-qubit state ρ as

$$C(\rho) = \max(\,0,\sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}), \qquad \text{(IV.1)}$$

where $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4$ are the eigenvalues of the matrix $\rho\widetilde{\rho}$, with $\widetilde{\rho}$ given by

$$\widetilde{\rho} = \sigma_2\otimes\sigma_2\,\overline{\rho}\,\sigma_2\otimes\sigma_2,$$

where $\overline{\rho}$ denotes complex conjugation of the matrix ρ. For states in the "X" form, concurrence is given by the function

$$C(\rho) = \max(\,0, C_1, C_2), \qquad \text{(IV.2)}$$

with

$$C_1 = 2(|\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}),$$
$$C_2 = 2(|\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}). \qquad \text{(IV.3)}$$

A. Entanglement of the asymptotic state ρ_u

Let us start with spatially separated atoms which have the unique asymptotic state ρ_u. Its concurrence is given by

$$C(\rho_u) = 2 \max\left(0, \frac{|z_0| - c_0}{u_0}, \frac{|b_0| - \sqrt{a_0 d_0}}{u_0}\right). \qquad \text{(IV.4)}$$

Analysis of this function in the general case of a broadband squeezed reservoir is involved, so we focus on the case of minimum-uncertainty squeezed states and consider (IV.4) as a function of the squeezed field intensity N, for fixed values of parameters $\widehat{\gamma}$ and δ. We plot this function in Fig. 1 for different values of detuning. It is evident that there is a range of values of mean photon number N for which the asymptotic concurrence is positive. Observe that the maximum of $C(\rho_u)$ appears for rather small values of N and the nonzero detuning diminishes the production of entanglement.

B. Entanglement of states ρ_{as}

The properties of the concurrence of ρ_{as} as a function of the initial fidelity can be studied in more detail. Notice that for these states we have

$$C_1 = \left(\frac{c - 2|z|}{u} - 1\right) F - \frac{c - 2|z|}{u}. \qquad \text{(IV.5)}$$

Define

$$F_1 = \max\left(0, \frac{c - 2|z|}{c - 2|z| - u}\right). \qquad \text{(IV.6)}$$

If $F_1 > 0$, then

$$C_1 > 0 \quad \text{for} \quad 0 \leqslant F < F_1.$$

In contrast,

$$C_2 = 2\left(\left|(1 - F)\frac{c}{2u} - \frac{F}{2}\right| - (1 - F)\frac{\sqrt{ad}}{u}\right). \qquad \text{(IV.7)}$$

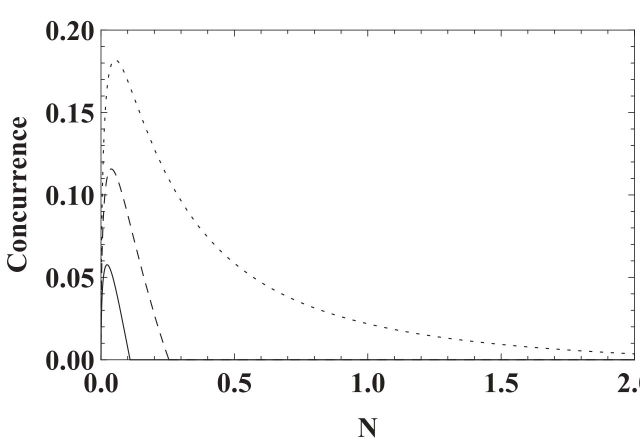

FIG. 1. Entanglement of state ρ_u as a function of N for $\widehat{\gamma} = 0.85$ and different values of detuning: $\delta = 0$ (dotted curve), $\delta = 0.5$ (dashed curve), and $\delta = 1$ (solid curve).

Notice that if $F < F_{cr}$, then

$$(1 - F)\frac{c}{2u} - \frac{F}{2} > 0,$$

and

$$C_2 = \left(\frac{2\sqrt{ad} - c}{u} - 1\right) F - \frac{2\sqrt{ad} - c}{u}. \qquad \text{(IV.8)}$$

Since

$$\frac{2\sqrt{ad} - c}{u} - 1 < 0,$$

so

$$C_2 < 0 \quad \text{when} \quad F < F_{cr}.$$

Let now $F \geqslant F_{cr}$, then

$$C_2 = \left(1 + \frac{c + 2\sqrt{ad}}{u}\right) F - \frac{c + 2\sqrt{ad}}{u}. \qquad \text{(IV.9)}$$

Define

$$F_2 = \frac{c + 2\sqrt{ad}}{c + 2\sqrt{ad} + u} \qquad \text{(IV.10)}$$

Form Eq. (IV.9) we see that

$$C_2 > 0 \quad \text{when} \quad F > F_2.$$

In contrast, direct calculations show that

$$F_2 \geqslant F_{cr} \quad \text{and} \quad F_1 \leqslant F_2.$$

Taking into account the above results, we arrive at the conclusion that, depending on the initial fidelity F, the asymptotic state ρ_{as} is entangled for all $F \in [0, F_1) \cup (F_2, 1]$ (provided $F_1 > 0$) and separable for $F \in [F_1, F_2]$ (see Fig. 2). The asymptotic concurrence reads

$$C(\rho_{as}) = \begin{cases} C_1, & 0 \leqslant F < F_1 \\ C_2, & F_2 < F \leqslant 1 \end{cases}, \qquad \text{(IV.11)}$$

with C_1 and C_2 given by Eqs. (IV.5) and (IV.9), respectively. This general result also covers the special cases of a vacuum reservoir where $F_2 = 0$, a thermal reservoir with $F_1 = 0$ and

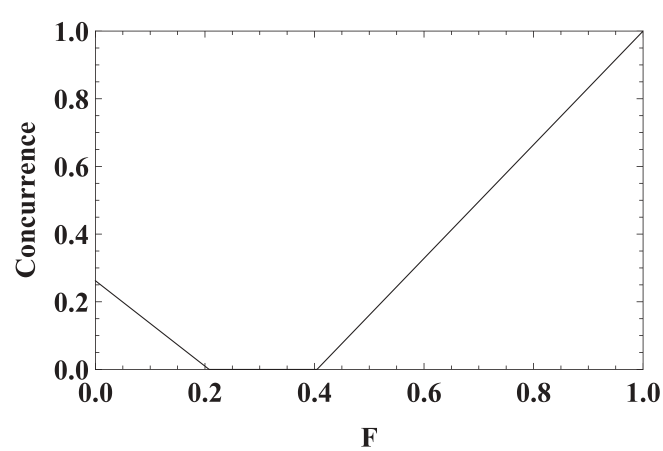

FIG. 2. Asymptotic entanglement versus fidelity for a minimum-uncertainty squeezed reservoir with $N = 1$ and detuning $\delta = 0.8$

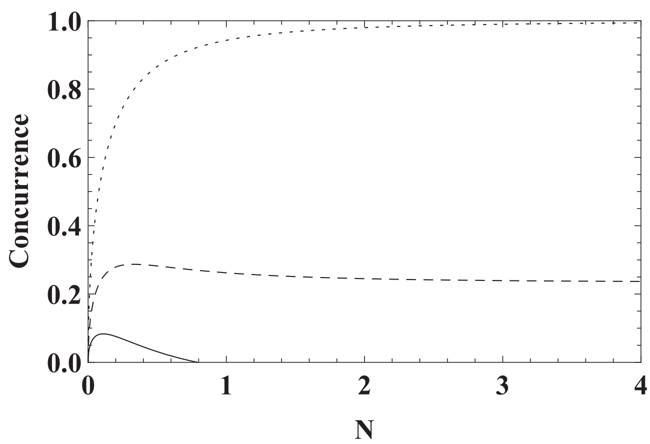

FIG. 3. Asymptotic entanglement of initial states with $F = 0$ as a function of N, for different values of detuning: $\delta = 0$ (dotted curve), $\delta = 0.8$ (dashed curve), and $\delta = 2$ (solid curve)

$F_2 > 0$, and a minimum-uncertainty squeezed reservoir where $F_1 = F_2$. It is worth stressing that the creation of asymptotic states with nonzero entanglement from initial states with low or even zero fidelity is only possible when the reservoir is in a squeezed state. Let us discuss this point in more detail in the special case of atoms which are in resonance with a minimum-uncertainty radiation field. In this case

$$F_1 = F_2 = \frac{2\sqrt{N(N+1)}}{2\sqrt{N(N+1)} + (1+2N)}, \qquad \text{(IV.12)}$$

and

$$C(\rho_{\text{as}}) = \begin{cases} -(1+C_0)F + C_0, & F < F_1 \\ (1+C_0)F - C_0, & F > F_1 \end{cases}, \qquad \text{(IV.13)}$$

with

$$C_0 = 2\frac{\sqrt{N(N+1)}}{1+2N}. \qquad \text{(IV.14)}$$

For all initial states with zero fidelity, we obtain a pure entangled state (III.10) with concurrence equal to C_0. Notice that in the limit of maximal squeezing, this state becomes maximally entangled. For pure product states

$$|\Psi\rangle = |\varphi\rangle \otimes |\psi\rangle, \qquad \text{(IV.15)}$$

the fidelity is given by the formula

$$F = \tfrac{1}{2}(1 - |\langle\varphi|\psi\rangle|^2), \qquad \text{(IV.16)}$$

so the zero fidelity corresponds, for example, to the case of two atoms prepared in the same initial states. This leads to a remarkable result: the interaction with a squeezed reservoir will entangle two atoms which are initially in the ground state $|g\rangle = |0\rangle_A \otimes |0\rangle_B$. The analogous phenomenon cannot occur when the photon field is in the vacuum or thermal state. Notice also that for nonzero detuning, the asymptotic state is no longer pure and the production of stationary entanglement is less effective (Fig. 3).

V. CONCLUSIONS

We have investigated the dynamics of two-level atoms interacting with a photon reservoir in a broadband squeezed vacuum state. The time evolution of the system depends crucially on the relative distance between the atoms. When the atoms are spatially separated, there is a unique asymptotic state, which can be entangled, in contrast to the analogous asymptotic state for a thermal reservoir. In the case of a small interatomic distance, there are nontrivial asymptotic states ρ_{as} which are parametrized by the fidelity F and the parameters N and M characterizing the squeezing. The states ρ_{as} also depend on the detuning between the atomic transition frequency and the carrier frequency of the photon field. For values of F above the threshold fidelity F_2, the states ρ_{as} are entangled. Nonzero entanglement can also occur for small values of F or even if $F = 0$, and this possibility is a unique feature of the squeezed reservoir. When the atoms are in resonance with the photon field and $|M| = \sqrt{N(N+1)}$, the asymptotic state corresponding to $F = 0$ is a pure entangled state known as the two-atom squeezed state.

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