Hyperfine structure interval of the 2s state of hydrogenlike atoms and a constraint on a pseudovector boson with mass below 1 keV/c^2

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A constraint on a spin-dependent interaction, induced by a pseudovector light boson, is presented. The interaction includes a Yukawa-type contribution $\alpha''(\mathbf{s}_1 \cdot \mathbf{s}_2)\, e^{-\lambda r}/r$ and a contact spin-spin term. To disentangle the long-range and contact terms we utilize experimental data on the 1s and 2s hyperfine intervals for light two-body atoms and construct a specific difference $8 \times E_{\rm hfs}(2s) - E_{\rm hfs}(1s)$. That allows one to constrain the spin-dependent coupling constant α'' of an electron-nucleus Yukawa-type interaction in hydrogen, deuterium, and the helium-3 ion at the level below a part in 10^{16}. The derived constraint is related to the range of masses below 4 keV/c^2. The combined constraint including the contact terms is also presented.

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I. INTRODUCTION

A strong constraint from atomic physics can be set on a spin-dependent long-range interaction induced by a light pseudovector particle. In principle, a constraint on a light particle with mass in the keV/c^2 range may be derived by various methods, involving cosmological estimation [1] and astrophysical phenomena [2]. (For a possible nature of such a particle see also [1,2], and references therein.) Such constraints involve a number of parameters, such as particle mass, its coupling to other particles, lifetime, etc. In contrast to that, a constraint based on limiting a possible deviation of the electron-nucleus interaction in the atomic distance range depends on two parameters only, namely, the particle mass λ and a strength of the interaction between an electron and a nucleus, mediated by the intermediate particle under consideration.

Certain atomic-physics constraints [3–5] are based on specific interactions, which can be induced by exchange of a light particle. The previous constraint of this kind on a spin-dependent Yukawa-type interaction was derived from data on the hyperfine structure (hfs) interval of the 1s state in light hydrogenlike atoms [3,5]. The result was for a particle substantially lighter than 4 keV/c^2, and the accuracy was limited either by the hfs experiment (for muonium and positronium) or by an uncertainty of the related contribution of nuclear effects (for hydrogen, deuterium, etc.). (It has also been extended there to heavier particles but with a reduced constraining strength.)

In the meantime, the pseudovector exchange produces not only a long-range spin-spin interaction of Yukawa type, but also a spin-spin contact term. Here we extend our previous result [3,5] in two directions. First, we separate the Yukawa-type contribution and the contact term. Elimination of the contact contributions should also improve accuracy of theoretical predictions, because leading contributions to the nuclear effects are also to be eliminated. That is possible by including additional experimental data into consideration. Next, we restore an appropriate contact term and reconsider the former study of the 1s hfs interval [3,5].

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Here, to disentangle the Yukawa-type and contact-term contributions and to avoid uncertainties due to nuclear effects, we consider a specific difference of the 1s and 2s hyperfine intervals

$$D_{21} = 8 \times E_{\rm hfs}(2s) - E_{\rm hfs}(1s), \qquad (1)$$

which is essentially free of such delta-function-like contributions [6,7]. Experimental data with appropriate accuracy are available for hydrogen [8,9], deuterium [10,11], and the helium-3 ion [12,13] for their 1s and 2s hyperfine intervals. The corresponding data are summarized in Appendix A. For theoretical results, which are summarized in Appendix B, we follow [14].

Theory suggests that there is a massive cancellation of various contributions, which are proportional to the squared value of the wave function at origin

$$|\Psi_{ns}(0)|^2 \propto n^{-3}.$$

Those include various uncertain nuclear-effect terms, and a theoretical prediction for the difference has a very safe grounds and has reached high accuracy (see [7] for details). The exotic contact spin-spin terms also vanish for the difference D_{21}. The nonleading terms are of reduced importance and can be neglected.

That is not the only theoretical advantage of using the difference. The cancellation also happens with the leading term (see below) and because of that the fractional uncertainty of measurements of the difference is relatively low. Even with such a fractional accuracy the difference remains very sensitive to many higher-order effects.

The theoretical accuracy in QED calculations for the hfs intervals is strongly affected by the accuracy of our knowledge of fundamental constants required for the calculations and, in particular, of the nuclear magnetic moments (see, e.g., [7]). In the case of the difference the leading contributions have a large theoretical uncertainty, however, they cancel out in the difference and, as a result, the theory of the difference is relatively immune to any problems in a determination of the magnetic moments and other fundamental constants, which is indeed quite advantageous for theoretical calculations.

Returning to the leading term, the cancellation happens for the leading term to the ns hfs interval, a so-called Fermi contribution,

$$\frac{E_F}{n^3} = C_s \frac{16\alpha}{3\pi n^3}\mu_{\rm B}\mu_{\rm nucl}R_\infty m_e^2, \tag{2}$$

where we apply relativistic units in which $\hbar = c = 1$, $e^2/(4\pi) = \alpha$ is the fine-structure constant, m_e is the electron mass, R_∞ is the Rydberg constant, $\mu_{\rm B}$ is the Bohr magneton, and $\mu_{\rm nucl}$ is the nuclear magnetic moment. The normalization constant C_s depends on the nuclear spin. In particular, $C_s = 1$ for the nuclear spin 1/2 (hydrogen, helium-3 ion), while for the spin 1 (deuterium) an additional factor $C_s = 3/2$ appears.

A pseudovector particle, which interacts both with an electron and a nucleus, induces various spin-dependent interactions (cf. the contributions of the Z boson [15] and a_1 meson [16] to the 1s hfs; see also [5]). Only the Yukawa-type one contributes to the D_{21} difference[1] and, if such an effect is present, the Coulomb exchange is modified at long distances by a spin-dependent term

$$-\frac{Z\alpha}{r} \rightarrow -\frac{Z[\alpha + \alpha''(\mathbf{s}_e\cdot\mathbf{s}_{\rm N})\,e^{-\lambda r}]}{r}, \tag{3}$$

where Z is the nuclear charge. Such a term is observable and may be used to produce a constraint on $\alpha''(\lambda)$ while comparing an actual value of D_{21} with theory.

In particular, in the limit

$$\lambda \ll Z\alpha m_e \sim Z \times\ 3.5\ \text{keV},$$

the energy of each hfs interval is shifted by

$$\Delta E_{\rm hfs}(ns) = -C_s\frac{2}{n^2}\frac{\alpha''}{\alpha}(Z^2R_\infty), \tag{4}$$

and the related contribution to the difference is

$$\Delta D_{21} = -2C_s\frac{\alpha''}{\alpha}(Z^2R_\infty) = -0.9\times 10^{18} C_s Z^2 \alpha''\ \text{Hz}, \tag{5}$$

which should be compared with the difference between the related experimental and theoretical values. The factor C_sZ^2 is unity for hydrogen, 3/2 for deuterium, and 4 for the helium-3 ion.

Consideration of the constraints based on Eq. (3) is present in Secs. II and III, while reconsideration in Sec. IV of the constraints from the 1s hfs interval (cf. [3,5]) involves the contact terms.

II. THE CONSTRAINT ON THE YUKAWA-TYPE COUPLING CONSTANT α''

The present situation with experiments and theory of the D_{21} difference is summarized in Table I, which covers all available data on the determination of D_{21} in light two-body atoms. We also present there a value of α'' for an asymptotic region $\lambda \ll 1$ keV. The result is indeed consistent with zero, since theory and experiment are in perfect agreement.

[1]Effects due to the presence of short distance contributions to the 1s hfs interval are considered in Sec. IV.

TABLE I. Comparison of experiment and theory for the D_{21} value in light hydrogenlike atoms. A negative sign for the hfs difference for the ^{3}He$^+$ ion reflects the fact that the nuclear magnetic moment is negative, i.e., in contrast to other nuclei in the table, its direction is antiparallel to the nuclear spin. The constraint on α'' is related to $\lambda \ll 1$ keV. The confidence level of the constraint corresponds to one standard deviation.

Atom	Experiment (kHz)	Theory (kHz)	α''
H	48.923(54)	48.953(3)	$(3.3\pm5.9)\times10^{-17}$
D	11.280(56)	11.3125(5)	$(2.4\pm4.1)\times10^{-17}$
^{3}He$^+$	−1189.979(71)	−1190.08(15)	$(-2.8\pm4.6)\times10^{-17}$

If we consider α'' as a certain universal constant, an average value over the constraints in Table I is found as

$$\alpha''_{\rm av} = (0.7\pm 2.7)\times 10^{-17}. \tag{6}$$

To consider a constraint on a heavier intermediate particle, we have to calculate the contribution of the Yukawa correction in Eq. (3) to the D_{21} difference. As a result, the correction (5) should include an additional factor $\mathcal{F}_{12}(\lambda/(Z\alpha m_e))$ and the constraint takes the form

$$\alpha''(\lambda) = \frac{\alpha''_0}{\mathcal{F}_{12}(\lambda/(Z\alpha m_e))}, \tag{7}$$

where α''_0 is a constraint for $\lambda/(Z\alpha m_e) \ll 1$, listed in Table I, and the profile function

$$\mathcal{F}_{12}(x) = 4\left[\left(\frac{1}{1+x}\right)^2 - 2\left(\frac{1}{1+x}\right)^3 + \frac{3}{2}\left(\frac{1}{1+x}\right)^4\right] - \left(\frac{2}{2+x}\right)^2$$

satisfies the condition $\mathcal{F}_{12}(x\rightarrow 0)\rightarrow 1$.

The related constraints extended to higher λ are presented in Fig. 1 [3], however, they have sharp λ dependence and are not efficient above a few-keV level.

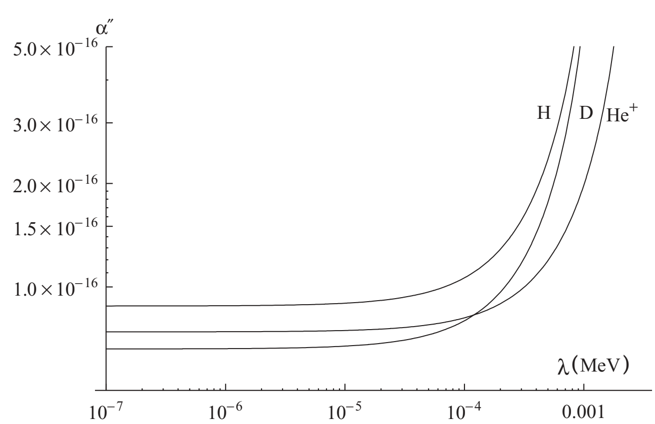

FIG. 1. Constraints on a pseudovector intermediate boson from D_{21} in hydrogen, deuterium, and helium-3 ion. The lines present an upper bound for $|\alpha''|$. The confidence level corresponds to one standard deviation.

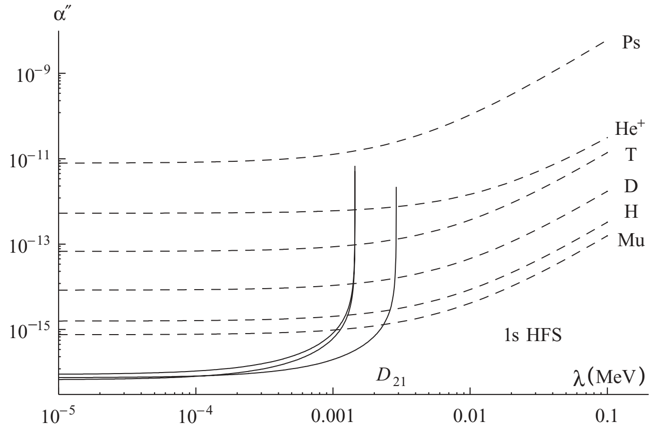

FIG. 2. Constraints on a pseudovector intermediate boson from the hfs study. The lines present the upper bound for $|\alpha''|$ from data on D_{21} (solid lines; see Fig. 1 for details) and the $1s$ hfs interval (dashed lines) in various two-body atoms. The $1s$ results are from [5]. The confidence level corresponds to one standard deviation.

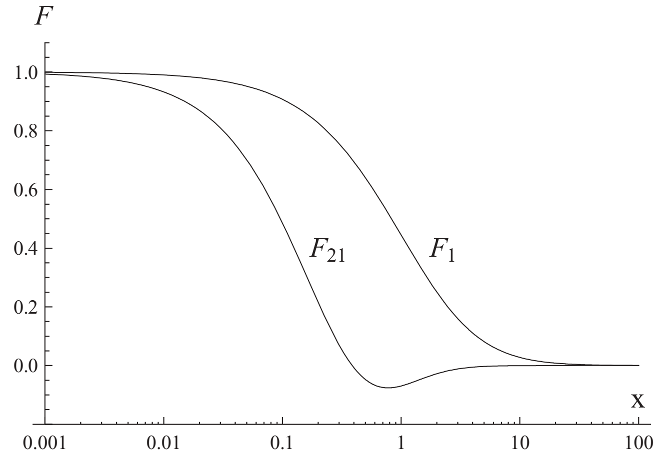

FIG. 3. The profile functions giving the upper bound for $|\alpha''|$ from data on D_{21} and the $1s$ hfs interval (in various two-body atoms. The $1s$ results are from [5].

III. COMPARISON TO OTHER HFS CONSTRAINTS ON THE YUKAWA-TYPE COUPLING CONSTANT α''

Because of the low efficiency of the constraints in Eq. (7) above the keV region, we have to combine the results of this paper with the constraint derived previously [5] from the data on the $1s$ hfs interval. Those constraints are weaker in the keV range but they are more suitable for extension to higher masses.

The overall constraint [3] from a study of the hyperfine intervals is summarized in Fig. 2. Three low lines are from D_{21} (cf. Fig. 1) and the related constraints are much stronger in the 1 keV region and below. However, the lines related to the $1s$ hfs interval [5] produce stronger constraints for above a few keV.

That is expectable. In the case of the Yukawa radius larger than atomic distances, the D_{21} constraints gain in accuracy because of the cancellation of the nuclear contributions which have large uncertainties. (The same mechanism turns the D_{21} difference into a powerful tool to test QED bound states [7].) However, once the radius is shorter than atomic distances, the Yukawa contribution becomes proportional to $|\Psi_{ns}(0)|^2$ and it is canceled out almost completely. Technically, that shows up as a special behavior of the function $\mathcal{F}_{12}(x) \propto x^{-4}$ at $x \to \infty$, while the related behavior for the $1s$ contribution [5]

$$\mathcal{F}_1(x) = \left(\frac{2}{2+x}\right)^2, \qquad (8)$$

which, in particular, determines λ dependence of the $1s$ constraints in Fig. 2, is $\propto x^{-2}$. That makes the D_{21} difference insensitive to shorter-distance Yukawa spin-spin interactions.

For illustration, we present both profile functions in Fig. 3. Both are equal to unity for low λ and that is the area where the constraints are the strongest. At large λ, both functions decrease to zero, which means that the Yukawa correction vanishes. However, as we mentioned, the behavior at high λ is different, which produces a different sensitivity for the high λ region. The results are obtained within a nonrelativistic approximation. Taking into account relativistic effects does not change the sharp-edge behavior of $\mathcal{F}_{12}$.

Thus, it is really fruitful to combine hfs constraints obtained by both methods: the D_{21} study for a longer wing of λ and the $1s$ hfs tests for the shorter one as summarized in Fig. 2. The constraints derived are complementary to various high-energy physics constraints reviewed in [17].

To conclude our consideration of the Yukawa-type contributions, we remind one that the vertex for an interaction of a vector particle with a fermion is $-ig_V\gamma_\mu$, while for the pseudovector it is $-ig_A\gamma_5\gamma_\mu$. That means that the long-range interaction for particles x and y mediated by a pseudovector boson is of the form

$$\frac{\alpha_A(xy)(\boldsymbol{\sigma}_x \cdot \boldsymbol{\sigma}_y)}{r},$$

where $\alpha_A(xy) = g_A(x)g_A(y)/(4\pi)$. Comparing with substitution (3), where the spin-dependent coupling constant α'' is introduced, we note that $\alpha_A = \alpha''/4$ (since $\mathbf{s}_x = \boldsymbol{\sigma}_x/2$). That is, rather, the constant α_A that is the properly normalized coupling constant.

We summarize in Fig. 4 the constraints on $\alpha_A(xe)$ for proton, neutron, and muon (i.e., for $x = p,n,\mu$), where we have taken into account all results derived in [5] and in this paper. To separate the proton and neutron contributions, we assume that nuclear binding effects can be neglected, and thus for the deuteron we find

$$\alpha_A(de) = \frac{\alpha_A(pe) + \alpha_A(ne)}{2},$$

while the helion constant is assumed to be equal to a free neutron value $[\alpha_A(he) = \alpha_A(ne)]$. Indeed, the binding effect could add some additional uncertainty, which is to be estimated. We do not think that would change the general situation.

IV. LONG-RANGE INTERACTIONS AND CONTACT TERMS

Above we have constrained the long-range spin-dependent interaction, which takes the form of Eq. (3) in the coordinate

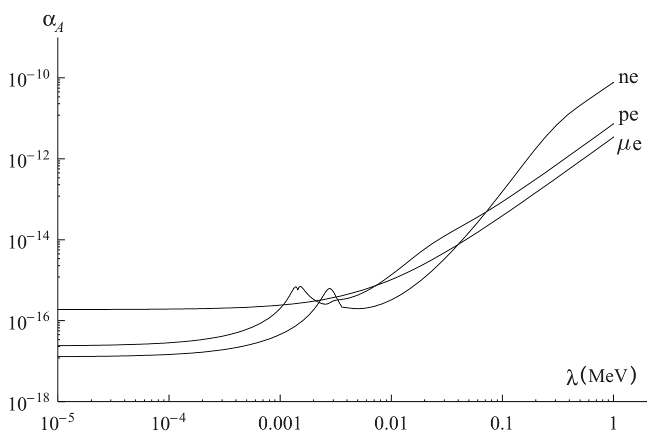

FIG. 4. Constraints on a pseudovector intermediate boson. The lines present the upper bound for the coupling constant $|\alpha_A(xy)|$ for $xy = pe, ne, \mu e$ from data on the hfs intervals in various two-body atoms. The confidence level corresponds to one standard deviation.

space. The results are also interpreted in terms of an effective coupling of an electron and a nucleus by exchange of an intermediate pseudovector particle (see Fig. 4).

Indeed, an exchange by a massive particle is to induce a certain Yukawa-type potential, but that is not a complete result. The propagator of a massive pseudovector in the momentum space has the form

$$-\frac{i}{q^2-\lambda^2}\left[g_{\mu\nu}-\frac{q_\mu q_\nu}{\lambda^2}\right]. \quad (9)$$

While converting it into the interaction in the coordinate space, not one kind of an effective interaction, but two should appear. One is a long-range spin-dependent interaction (3) studied in [5] and in this paper, while the other is a contact interaction induced by the term

$$\frac{i}{q^2-\lambda^2}\frac{q_\mu q_\nu}{\lambda^2}$$

in Eq. (9).

The related coupling is similar to the coupling by an axion exchange with the axion-matter coupling constant $\propto \sqrt{\alpha''}(m_i/\lambda)$, where m_i is the mass of the matter particle (see, e.g., [18,19]).

If the mass is below the characteristic atomic momentum (that is an area of $\lambda \leqslant 1$ keV for the light two-body atoms under consideration), the additional term mostly acts as a nonrelativistic contact term, effectively proportional to the delta function in the coordinate space (see Appendix C for details). That leads to a contribution to the ns hfs interval, which has relative order $(Z\alpha)^2(m_e/\lambda)^2$ compared to the long-term contribution. In contrast to the long-range contribution, which is proportional to n^{-2} [see Eq. (4)], the contact-term contribution is proportional to n^{-3}.

Meanwhile, for hydrogen, deuterium, and the helium-3 ion the results on two hfs intervals, namely, on the $1s$ and $2s$ states, are available (see Appendix A for details). Because of that, in the area of $\lambda \leqslant 1$ keV one can separate the long-range and contact-term contributions since they depend on n differently. Actually, our treatment of the D_{21} difference in this paper is such a procedure. Since the uncertainty caused by the nuclear-structure effects scales in the same way as the contact contributions, i.e., as n^{-3}, the procedure not only removes the contact contributions, but also reduces the uncertainty dramatically.

The area of $\lambda \leqslant 1$ keV can be considered as free of contact contributions for the D_{21} constraints. That is not only because of the cancellation of the leading contact-term contributions, but also because of the theoretical approach to higher-order nuclear-structure corrections developed in [6]. The approach considers various nuclear-structure effects as induced by certain contact terms and, after determination of these terms by a comparison of the $1s$ hfs theory with experiment, the remaining state-dependent tail of the nuclear-structure effects, due to the higher-order in $(Z\alpha)$, is obtained within an effective theory (see Appendix B for details and references). That means that once a certain contact term is present in the $1s$ hfs interval, its leading contribution is removed from the D_{21} difference and its next-to-leading contribution to D_{21} is effectively taken into account.

In principle, we could still apply the results from the $1s$ and $2s$ HFS intervals to constrain the coupling constant α^A_{eX} by taking into account the contact axionlike interaction. In particular, evaluating experimental and theoretical data on the $1s$ and $2s$ hfs intervals, we find the constraint for the coupling constant related to the contact term for $\lambda < 1$ keV,

$$\alpha_A(\text{contact}) = \frac{\alpha_A(1s)}{Z^2}\left(\frac{\lambda}{4.3\ \text{keV}}\right)^2,$$

where $\alpha_A(1s)$ is a value of a plateau part of the related $1s$ hfs constraint in Fig. 2, which is found [3,5] as $\alpha_A^{pe}(1s) = \pm 0.4\times 10^{-15}$, $\alpha_A^{ne}(1s) = \pm 4\times 10^{-15}$, and $\alpha_A^{\mu e}(1s) = (0.4 \pm 1.5)\times 10^{-16}$. The constraints on α_A(contact) are stronger than those on α_A from the D_{21} difference for λ in the eV-keV range.

For the higher mass range above 4 keV, the situation is more complex and the contact term is even more singular and has an additional suppression compared to lighter masses λ. The long-range contribution also shrinks to a contact term but this is only $(Z\alpha)^2(m_e/\lambda)^2$ in respect to the long-range non-Yukawa Coulomb-like contribution [i.e., a contribution where we put unity instead of $\exp(-\lambda r)$]. One can see that the contribution of the contact term is not competitive and may be neglected.

The typical constraints for $\alpha_A(\mu e)$ and $\alpha_A(pe)$, which follow from taking into account the longitudinal terms in the complete propagator (9) of the pseudovector boson, are presented in Fig. 5. The muonium case is for a constraint, where only data on the $1s$ hfs interval are available, while the hydrogen constraints are for an atom with two hfs intervals measured. The other constraints, which can be derived from the hyperfine structure in deuterium and the helium-3 ion (from the $1s$ and $2s$ hfs intervals) and from positronium and tritium (from the $1s$ hfs), are similar to those from hydrogen ($1s$ and $2s$) and muonium ($1s$), while the latter are the most accurate.

Nevertheless, in this paper we do not intend to separate the contact-term and long-range contributions. A reason for that is that, generally speaking, the contact term could appear from different sources. It may produce certain theoretical problems and because of that in certain theories it may be canceled out. Meanwhile we do not expect such a cancellation for the long-range force (3), e.g., due to an exchange by a

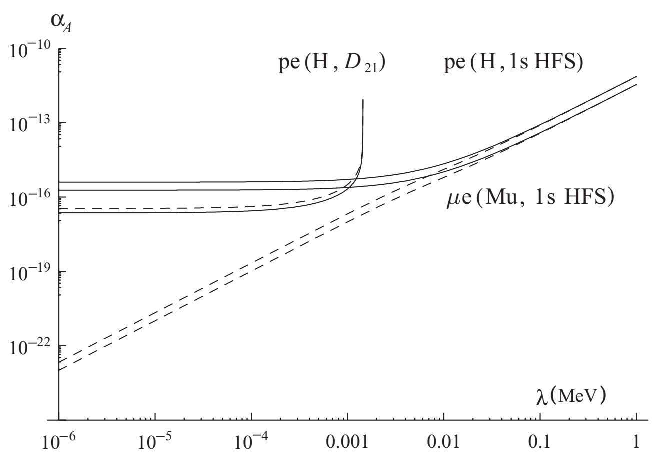

FIG. 5. Constraints on a pseudovector intermediate boson from data on the hfs structure of muonium and hydrogen. The solid lines represent the upper bound for the coupling constant $|\alpha_A(xy)|$ for $xy = pe$ and μe ignoring the longitudinal term, while the dashed lines are for the constraints taking those terms into account. The confidence level corresponds to one standard deviation.

pseudovector. The relation between the eventual contact term and the long-range interaction is not necessarily completely determined by the propagator (9). Because of that we prefer the interpretation of our results in Fig. 4 as a conservative constraint on a pseudovector particle from the long-range interaction (3) with the understanding that it may happen that a constraint due to an axionlike contact interaction could produce an even stronger limitation (as seen in Fig. 5).

V. CONCLUSIONS

In conclusion, we have improved a model-independent limitation on spin-spin Yukawa-type interactions as compared with our previous paper [5] (see Fig. 2). Such an interaction can be mediated by a light pseudovector boson. The improvement relates to the mass range below 1 keV. The constraint concerns electron-proton, electron-neutron, and electron-muon interactions (see Fig. 4).

The constraints for the intermediate boson are derived ignoring the longitude term in its propagator, however, various consequences of taking this term into account are considered in detail (see, e.g., Fig. 5).

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APPENDIX A: SUMMARY ON EXPERIMENTAL DATA ON THE 1*s* AND 2*s* HFS INTERVALS IN LIGHT TWO-BODY ATOMS

The experimental results on the metastable 2*s* state are available for only three hydrogenlike atoms, namely, for hydrogen, deuterium, and the helium-3 ion. Only a few measurements have been performed for over 50 years since the 1950s when the first results on the 2*s* hfs interval in hydrogen [20] and deuterium [21] atoms and the helium-3 ion [22] were obtained. We summarize in Table II all obtained results.

TABLE II. All results on the 2*s* hfs interval in light hydrogenlike atoms obtained up until now. A negative sign for the $^3He^+$ ion reflects the fact that the nuclear magnetic moment is negative and thus its direction is antiparallel to the nuclear spin.

Atom	E_{HFS} (expt.) (kHz)	Ref.
Hydrogen	177 556.8343(67)	[9]
	177 556.860(16)	[23]
	177 556.785(29)	[24]
	177 556.860(50)	[20]
Deuterium	40 924.454(7)	[11]
	40 924.439(20)	[21]
$^3He^+$ ion	−1083 354.980 7(88)	[13]
	−1083 354.99(20)	[22]

Since only these three atoms are important for calculations of a specific difference of the hfs intervals in the 1*s* and 2*s* states, in Table III we collect the experimental results on the 1*s* hfs interval for involved atoms.

The results on the difference D_{21}, based on the most accurate experimental data, are presented in Table I of the paper.

APPENDIX B: SUMMARY ON THEORY OF THE D_{21} DIFFERENCE IN LIGHT TWO-BODY ATOMS

A detailed review on theory of the D_{21} difference in hydrogen, deuterium, and the helium-3 ion can be found in [6,7]. The results are summarized in Table IV. “QED3” and “QED4” stand for pure QED corrections in units of the Fermi energy E_F, defined in Eq. (2).

There are three small parameters in QED theory: α stands for QED loops and is for the QED perturbation effects, $Z\alpha$ is for the Coulomb strength and describes binding effects, while the mass ratio m/M (electron-to-nucleus) is for the recoil effects in two-body atoms. Theoretical evaluations have a certain history, which started in [25–27], shortly after the first results on the 2*s* hfs interval were achieved [20–22].

The QED3 term involves various combinations of these three parameters up to the third order, which were mainly calculated a long time ago. A more recent development was due to the fourth-order contributions (QED4) and due to higher-order nuclear effects.

TABLE III. The most accurate results for the 1*s* hfs interval in those light hydrogenlike atoms, for which the results on the 2*s* hfs interval are available.

Atom	E_{hfs} (expt.) (kHz)	Ref.
Hydrogen	1 420 405.751 768(1)	[8]
Deuterium	327 384.352 522(2)	[10]
$^3He^+$ ion	−8 665 649.867(10)	[12]

TABLE IV. Theory of the specific difference D_{21} in light hydrogenlike atoms [14]. The numerical results are presented for the related frequency D_{21}/h. QED3 and QED4 stand for the third- and fourth-order QED corrections in units of the Fermi energy E_F (see [6,7] for details).

Contribution to D_{21}	Hydrogen (kHz)	Deuterium (kHz)	$^3He^+$ ion (kHz)
D_{21}(QED3)	48.937	11.305 6	−1 189.253
D_{21}(QED4)	0.018(5)	0.004 4(10)	−1.13(14)
D_{21}(nucl.)	−0.002	0.002 6(2)	0.307(35)
D_{21}(total)	48.953(5)	11.312 5(10)	−1 190.08(15)

As we mentioned above, there is a substantial cancellation of the nuclear-structure contribution in difference D_{21}. The leading term, which takes into account the nuclear charge and magnetic moment distribution, cancels completely. However, certain higher-order nuclear-effect contributions survive the cancellations and they are denoted as "nucl". Those higher-order terms were found in [14].

For QED3 terms and for higher-order nuclear effects we follow [7], while for the QED4 terms we apply the results of [14], a recent correction in which follows a reexamination of the former QED4 calculation in [6] and numerical medium-Z calculation of one-loop effects in [28] (cf. [29]).

After the result was already published [3], I learned about new results for the one-loop self-energy contribution [30], which shifts the helium ion theory to −1190.14(5) kHz and marginally affects our constraint for the related upper bound in Fig. 1.

APPENDIX C: CONTRIBUTION OF THE LONGITUDINAL TERM OF THE PSEUDOVECTOR PROPAGATOR

The longitudinal term

$$\frac{i}{q^2-\lambda^2}\,\frac{q_\mu q_\nu}{\lambda^2}$$

of the propagator of the pseudoscalar boson in Eq. (9) induces a certain contact potential. The related coupling is proportional to

$$\frac{\alpha''}{4}\,\gamma_5^{(e)}\gamma_\mu^{(e)}\,\gamma_5^{(\mathrm{N})}\gamma_\nu^{(\mathrm{N})}\frac{1}{q^2-\lambda^2}\,\frac{q_\mu q_\nu}{\lambda^2},$$

where we consider the case of the nuclear spin $I=1/2$.

The dominant spin-spin interaction appears from the term with $\mu=\nu=0$. Taking into account only large components of the spinors, we arrive at an effective spin-spin potential, which can be presented in terms of an effective substitution in the momentum space

$$\frac{(\mathbf{s}_e\cdot\mathbf{s}_\mathrm{N})}{\mathbf{q}^2+\lambda^2}\to\frac{(\mathbf{s}_e\cdot\mathbf{s}_\mathrm{N})}{\mathbf{q}^2+\lambda^2}\left(1+\frac{1}{3}\,\frac{\mathbf{q}^2}{\lambda^2}\right),$$

where $\mathbf{q}$ is momentum transfer. In the coordinate space instead of Eq. (3) we arrive at

$$-\frac{\alpha}{r}\to-\left[\frac{\alpha}{r}+\frac{2}{3}\frac{\alpha''(\mathbf{s}_e\cdot\mathbf{s}_\mathrm{N})e^{-\lambda r}}{r}+\frac{4\pi}{3\lambda^2}\alpha''(\mathbf{s}_e\cdot\mathbf{s}_\mathrm{N})\delta(\mathbf{r})\right],$$

where $\alpha''=4\alpha_A$.

For λ below atomic momenta, the potential, as a nonrelativistic potential, produces corrections of the order of $(Z\alpha)^2(m_e/\lambda)^2$ in units of the long-range term, which scales as n^{-3} for the ns states. A calculation of relativistic effects produces various corrections which are of the order of $(Z\alpha)^3(m_e/\lambda)^2$ and $(Z\alpha)^4(m_e/\lambda)^2$. Only the latter has n dependence different from n^{-3} and can thus contribute to D_{21}.

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