Four-photon indistinguishability transition

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We demonstrate the formation of many-particle interferences of different degrees to determine the transmission of four photons of tunable indistinguishability through a four-port beam splitter array. The probability of certain output events depends *nonmonotonically* on the degree of distinguishability, due to distinct multiparticle interference contributions to the transmission signal.

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I. INTRODUCTION

The symmetrization postulate imposed by the indistinguishability of particles is a fundamental quantum concept with no classical counterpart, which strongly influences the behavior of matter at any energy scale. In quantum optics, the most prominent manifestation thereof is the Hong-Ou-Mandel (HOM) effect [1]: two indistinguishable photons falling on the opposite input ports of a balanced beam splitter always leave the setup together, i.e., all amplitudes with two photons in different output modes interfere destructively, while events with both photons in the same mode are enhanced due to the photons' *bosonic nature*. When tuning the transition from distinguishability to indistinguishability of the photons, which can be described by a *single* parameter [2], the visibility of the HOM dip in the probability to detect one photon per mode increases monotonically.

Signatures for the *full* indistinguishability of *more than two* particles can be observed, e.g., when many indistinguishable photons bunch at one output mode of a beam splitter [3–5]. In addition, when several particles enter a multiport beam splitter [6] simultaneously, *many-particle interferences* strongly influence the probability of individual counting events, such that many distinct events with a given number of particles per output port are totally suppressed [7–9]. The *transition* between many fully distinguishable and fully indistinguishable particles has not received theoretical or experimental attention beyond its impact on bosonic bunching [3–5]. In the latter case, the distinguishability of the photons is reflected in a monotonic way by the event probabilities, since all amplitudes interfere constructively, and the strength of the enhancement therefore always grows with the indistinguishability of the particles. The role of partial indistinguishability for the behavior of other events, e.g., the ones that are suppressed for indistinguishable particles since the contributing many-particle amplitudes interfere *destructively* [8,9], is widely open. A thorough understanding of partial indistinguishability is, however, mandatory for the experimental characterization of the degree and nature of many-particle interferences.

In the present paper, we consider four photons that propagate through a four-port beam splitter array. In contrast to the intuitive extrapolation of the features of the well-known two-photon case [1], which is often used to understand the bunching behavior of many photons [4], we show that (i) the degree of distinguishability manifests itself in *nonmonotonic*

event probabilities and that (ii) events with large occupation numbers are not necessarily more likely with increasing indistinguishability, despite the bosonic nature of photons. These results are established by quantitative predictions on experimentally directly accessible quantities.

II. EXPERIMENTAL SETUP

We study many-particle interference within a setup with four ports [10] [see Fig. 1(a)]. In this arrangement, no single-particle (Mach-Zehnder-like) interference can occur, what makes it perfect to study the genuine manifestation of many-particle interference.

Single-particle evolution is here described by a unitary matrix which relates particle creation operators of input and output ports, $\hat{a}^{\dagger}_{i,\omega}$ and $\hat{b}^{\dagger}_{k,\omega}$, respectively, via [11]

$$\hat{a}^{\dagger}_{i,\omega} \rightarrow \sum_{k=1}^{4} U_{ik}(\alpha,\phi)\hat{b}^{\dagger}_{k,\omega}, \qquad (1)$$

with the photon frequency ω unchanged, and $U(\alpha,\phi)$ given by

$$U(\alpha,\phi) = \frac{1}{2} \begin{pmatrix} e^{i\phi} & e^{i\phi} & e^{i(\phi+\alpha)} & e^{i(\phi+\alpha)} \\ 1 & 1 & -e^{i\alpha} & -e^{i\alpha} \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}. \qquad (2)$$

For any value of α and ϕ, the matrix $U(\alpha,\phi)$ is a complex unitary matrix with $|U_{jk}| = 1/2$, i.e., a Hadamard matrix [12]. The phases α, ϕ have two rather distinct physical interpretations: ϕ absorbs all relative phases between input modes and is well known from the two-mode interference of many particles, e.g., in N00N-state interferometry [13]. α is of distinct origin: it corresponds to the phase enclosed by the interfering modes (similar to a Sagnac interferometer [14]), emerges only in the four-mode case [12], and effectively controls the relative phases of the *output* components, conditioned on the input modes.

Four photon states are created by the double passage of a laser pulse through a nonlinear crystal, through spontaneous parametric down conversion [see Fig. 1(b)]. Such states consist of the coherent superposition of a *quadruplet* part, with one photon in each mode, created when the laser pulse induces one pair of photons at each time it passes the crystal, with two *double-twin* parts, where four photons are distributed among two modes, resulting from events where the pulse generates
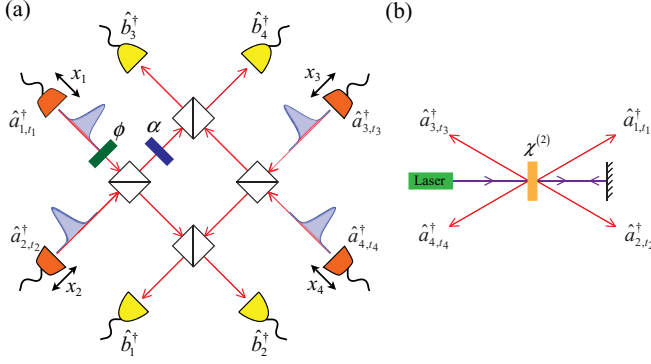
FIG. 1. (Color online) (a) Diamond-shaped multiport (balanced) beam splitter array. Four ingoing modes $\hat{a}_i^\dagger$ are redistributed onto the output modes $\hat{b}_j^\dagger$. The path length x_j of each incoming mode controls the mutual distinguishability of the particles in the setup. (b) Creation of four photons by spontaneous parametric down conversion, by double passage of a laser pulse through a nonlinear crystal.

two pairs in the one or in the other passing direction.[1] Since all three processes occur with the same probability, the initial state reads

$$|\Psi\rangle = \frac{1}{\sqrt{3}} \left(\prod_{j=1}^{4} \hat{a}_{j,t_j}^\dagger + \frac{\hat{a}_{1,t_1}^{\dagger 2} \hat{a}_{2,t_2}^{\dagger 2}}{2} + \frac{\hat{a}_{3,t_3}^{\dagger 2} \hat{a}_{4,t_4}^{\dagger 2}}{2} \right) |0\rangle. \quad (3)$$

The operator

$$\hat{a}_{j,t_j}^\dagger = \int_{-\infty}^{\infty} d\omega \frac{1}{\sqrt{\pi}\,\Delta\omega} e^{-[(\omega-\omega_0)^2/(2\Delta\omega^2)]} e^{i\omega t_j} \hat{a}_{j,\omega}^\dagger \quad (4)$$

creates a single photon with central frequency ω_0 and spectral width $\Delta\omega$ at the input port j, at time t_j. The arrival times t_j of the photons can be tuned through variable path lengths $x_j = ct_j$ depicted in Fig. 1(a), and hence their overlap, or indistinguishability, by virtue of

$$\left| \langle 0 | \hat{a}_{i,t_j} \hat{a}_{i,t_k}^\dagger | 0\rangle \right|^2 = \exp\left[-\tfrac{1}{2}\Delta\omega^2(t_j - t_k)^2\right]. \quad (5)$$

A change of relative path lengths induces a phase shift between the input modes, which can be accounted for in ϕ:

$$\phi \to \phi + \omega(t_1 + t_2 - t_3 - t_4). \quad (6)$$

All possible final states that emerge from the multiport can be characterized in terms of the photon number detected in each port. For four photons, simple combinatorics yields 35 distinct events which can be labeled with vectors $\vec{s} = (s_1, s_2, s_3, s_4)$, where s_j is the number of particles in port j. Their order is hereafter given by their relative abundance in the fully distinguishable case, Eq. (9) below, and such that vectors $\vec{s}$ with large s_1 come first, i.e., $\vec{s}_1 = (4,0,0,0)$, $\vec{s}_2 = (0,4,0,0), \ldots, \vec{s}_5 = (3,1,0,0)$, and finally, $\vec{s}_{35} = (1,1,1,1)$.

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[1]The relative phase between double-twin and quadruplet contribution is absorbed in the phase ϕ, Eq. (2).

III. FULL (IN)DISTINGUISHABILITY

For the interpretation of the interference effects that we will discuss hereafter, it is useful to decompose the initial state (3) into a superposition of states in which the photons in different modes are all either fully indistinguishable or fully distinguishable. In the two-photon case, only two contributions (distinguishable and indistinguishable) arise. For four particles, we perform a Gram-Schmidt orthonormalization of the spatiotemporal degrees of freedom of the four single-particle states, i.e., of $\hat{a}_{j,t_j}^\dagger |0\rangle$ ($j = 1, \ldots, 4$), such that

$$\hat{a}_{j,t_j}^\dagger = \sum_{k=1}^{j} c_{j,k} \hat{a}_{j,\tilde{t}_k}^\dagger, \quad (7)$$

with $\sum_{k=1}^{j} |c_{j,k}|^2 = 1$, and where the $\hat{a}_{j,\tilde{t}_k}^\dagger |0\rangle$ are orthonormalized single-particle states which satisfy

$$\langle 0 | \hat{a}_{j,\tilde{t}_k} \hat{a}_{l,\tilde{t}_m}^\dagger | 0\rangle = \delta_{j,l}\delta_{k,m}. \quad (8)$$

In other words, the description of a particle in the jth mode needs up to j nonvanishing orthogonal components. For our calculation of event probabilities, we expand the initial state (3) in products of the orthonormalized single-particle states $\hat{a}_{j,\tilde{t}_k}^\dagger |0\rangle$, and thereby obtain a superposition of many mutually orthogonal terms. For the quadruplet part in Eq. (3), this procedure yields $4! = 24$ different terms. Each term corresponds to a certain distinguishability *setting* that we will denote by $\{i_1, i_2, i_3, i_4\}$, where photons in port k and l are fully indistinguishable if $i_k = i_l$. Consequently, the weight of the distinguishability setting $\{i_1, i_2, i_3, i_4\}$ is given by the product of the corresponding coefficients, i.e., $|\prod_{j=1}^{4} c_{j,i_j}|^2$. Fully indistinguishable (distinguishable) particles correspond to $\{1,1,1,1\}$ ($\{1,2,3,4\}$), and only indistinguishable particles interfere. Hence, the distinguishability setting determines the degree of many-particle interference. In general, a setup with given arrival times for all photons corresponds to a situation in which several distinguishability settings contribute to the initial state, with different degrees of multiparticle interference occurring simultaneously.

The extreme case of full distinguishability $\{1,2,3,4\}$ is realized when all photons have pairwise delays $|t_i - t_j| \gg 1/\Delta\omega$. In this case, we can safely neglect the exponentially suppressed components that still exhibit many-particle interference, and simple combinatorics can be applied to yield the output event probabilities (remember that no single-particle interference can occur in our setting):

$$P_{\text{dist}}(\vec{s}) = \frac{4!}{4^4 \prod_j s_j!}. \quad (9)$$

The opposite, fully indistinguishable limit $\{1,1,1,1\}$ is realized for $t_1 = t_2 = t_3 = t_4$, when all photons can interfere perfectly. The resulting probabilities for five representative events are compared in Table I, for the fully distinguishable and indistinguishable case, respectively. The event $\vec{s}_{14} = (0,1,0,3)$ is fully suppressed for indistinguishable photons, for any choice of the phases α, ϕ, while $\vec{s}_{21}$, $\vec{s}_{22}$, and $\vec{s}_{35}$ exhibit an intricate α, ϕ

TABLE I. Event probabilities for fully distinguishable (P_{dist}) and indistinguishable particles (P_{id}).

Event	P_{dist}	$P_{\text{id}}(\vec{s})$
$\vec{s}_1 = (4,0,0,0)$	1/256	$\cos^4(\phi/2)/8$
$\vec{s}_{14} = (0,1,0,3)$	1/64	0
$\vec{s}_{21} = (0,2,0,2)$	3/128	$[\cos(\alpha) + \cos(\alpha + \phi)]^2/48$
$\vec{s}_{22} = (0,0,2,2)$	3/128	$[1 - 3\cos(2\alpha + \phi)]^2/48$
$\vec{s}_{35} = (1,1,1,1)$	3/32	$[\cos(\alpha) - \cos(\alpha + \phi)]^2/12$

dependence. Most importantly, there is no unambiguous correlation between the event probability and the photon distribution on the output modes: as anticipated above, $s_1 = (4,0,0,0)$ may be enhanced (as expected for bosonic bunching) as well as strictly suppressed (for the experimentalist's choice $\phi = \pi$). Unexpectedly, $\vec{s}_{35} = (1,1,1,1)$ may be enhanced up to a weight 32/9. In stark contrast to the two-photon HOM effect, the manifestation of perfect indistinguishability is not unique in the present, multiparticle interference scenario, and has no intuitive interpretation in terms of the occupation of modes. This is highlighted by Fig. 2, which displays all event probabilities, for fully distinguishable and indistinguishable particles, and three different choices of α and ϕ, and thus demonstrates the loss of any indistinguishability-induced hierarchy in the event probabilities. Furthermore, not only are the event probabilities no unambiguous witnesses of (in)distinguishability anymore, but they even evolve *nonmonotonically* with decreasing distinguishability of the particles, as can be demonstrated in our setting, by subsequently rendering different pairs of photons indistinguishable, and thus adding interference terms between an increasing number of photons.

IV. TRANSITIONS

Let us focus on the effects of partial interference on the three events $\vec{s}_{35} = (1,1,1,1)$, $\vec{s}_{14} = (0,1,0,3)$, and $\vec{s}_{21} = (0,2,0,2)$ (all listed in Table I), within two exemplary distinguishability transitions: for specificity, we assume that the single photons
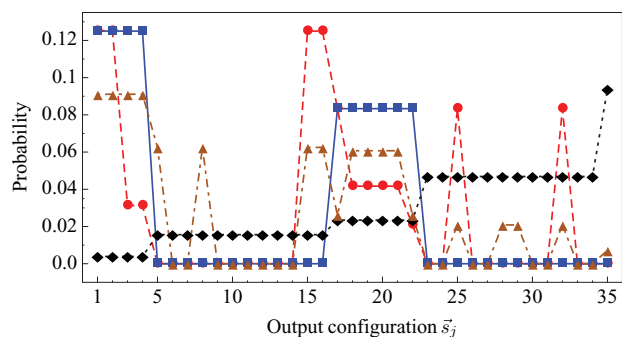

FIG. 2. (Color online) Event probabilities of all 35 possible output configurations $\vec{s}_j$, for distinguishable (black diamonds) and fully indistinguishable particles, with $\phi = \alpha = 0$ (blue squares), $\phi = 0$, $\alpha = \pi/4$ (red circles), and $\phi = \pi/4$, $\alpha = 0$ (brown triangles).

have a central wavelength of $\lambda_0 = 780$ nm and a full width at half maximum (FWHM) of $\Delta_\lambda = 5$ nm, which corresponds to a coherence length of $l_c \approx 122\ \mu$m.

(1) We first adjust the path lengths in a continuous way by parametrizing $x_1 = 0$, $x_2 = y$, $x_3 = -y$, and $x_4 = 2y$. Hence, for $y = -180\ \mu$m, the particles are fully distinguishable $\{1,2,3,4\}$, while for $y = 0\ \mu$m, their wave functions fully overlap; the particles are fully indistinguishable $\{1,1,1,1\}$. During the transition, several distinct contributions to the output signal arise simultaneously [see Fig. 3(a)]. We initially choose $\alpha = 0$ and $\phi = 0$. The latter parameter remains constant for all values of y with our chosen parametrization [cf. Eq. (6)]. Consequently, only variations on length scales of the order of the coherence length l_c appear.

The signal evolution during the (controlled) indistinguishability transition is shown on the bottom left of Fig. 3: The event $\vec{s}_{35} = (1,1,1,1)$ is progressively suppressed as higher-order interference contributions, from two to over three to four photons, kick in. This contrasts with the clearly nonmonotonic event probability of $\vec{s}_{14} = (0,1,0,3)$. While two- and three-photon interference increase its probability, it is eventually strictly suppressed for fully indistinguishable particles, resulting in a maximum for partial distinguishability. For $\vec{s}_{21} = (0,2,0,2)$, a steplike behavior results from the competition of destructive three-particle and constructive four-particle interference.

(2) Another remarkable impact of the distinguishability transition is borne out when we adjust the path lengths step by step: starting from fully distinguishable photons, with $x_1 = 0$, $x_2 = 220\ \mu$m, $x_3 = 440\ \mu$m, and $x_4 = 660\ \mu$m, we first tune down x_2 and subsequently, x_3 and x_4, as indicated in Fig. 3(c), with $\alpha = \pi$. The thus chosen, dominant distinguishability settings are $\{1,2,3,4\} \rightarrow \{1,1,3,4\} \rightarrow \{1,1,1,4\} \rightarrow \{1,1,1,1\}$, and this stepwise transition is reflected by the event probabilities in Fig. 3(d). In particular, the probability of $\vec{s}_{14} = (0,1,0,3)$ is again nonmonotonic, due to a *destructive* two-photon contribution, followed by a *constructive* three-photon contribution for the $\{1,1,1,4\}$ setting.

An additional effect manifests when four-photon interference sets in: the event probabilities start to depend on the phase ϕ. This leads to fast oscillations on the scale of the photon wavelength λ, an intrinsic feature of the interference of four or more photons, shown in the plots as shaded areas between minima and maxima. The dependence of the event probability on ϕ for fully indistinguishable photons (Table I) explains the onset of fast oscillations of the event probabilities for $\vec{s}_{21}$ and $\vec{s}_{35}$, and the growth of their amplitudes with increasing $\{1,1,1,1\}$ contribution. Unexpectedly, also $\vec{s}_{14} = (0,1,0,3)$ exhibits such oscillations which, however, vanish when four-photon interference fully dominates, since this event is then strictly suppressed (Table I). It is the interplay of the $\{1,1,1,4\}$ and $\{1,1,1,1\}$ settings that leads to such dependence on ϕ: the double-twin part of the wave function in the setting $\{1,1,1,4\}$ interferes with the quadruplet part in the $\{1,1,1,1\}$ setting, resulting in four-photon interference that depends on ϕ, and a local maximum of the event probability at the point where the contributions of $\{1,1,1,4\}$ and $\{1,1,1,1\}$ are equal. This feature is specific to the *partial* distinguishability of the photons.
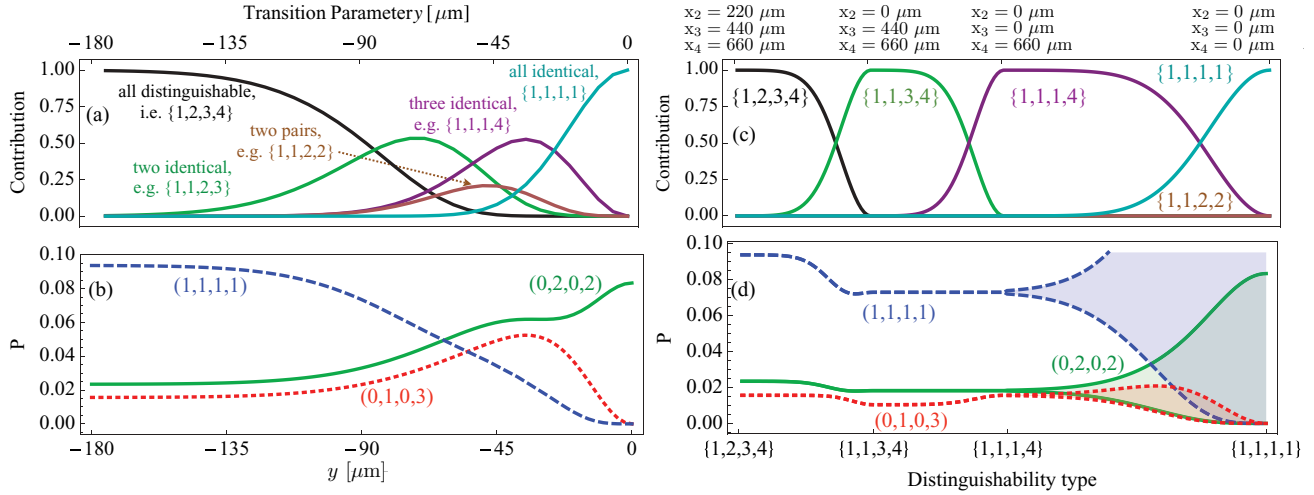
FIG. 3. (Color online) Evolution of event probabilities during the transition from distinguishable to indistinguishable particles (bottom row). Left: Continuous transition, parametrized by y, with $\alpha = 0$. Right: Stepwise transition, with corresponding interferometer path lengths, Fig. 1, on top of panel (c), with $x_1 = 0\,\mu$m, $\alpha = \pi$. The path lengths x_2, x_3, and x_4 are tuned from $220\,\mu$m to $0\,\mu$m, one after the other. [(a),(c)] Distinguishability-type contributions to the output signal: While for the stepwise transition, at most two distinguishability settings are relevant at a time, several may contribute in the continuous transition. [(b),(d)] Event probabilities of $\vec{s}_{35} = (1,1,1,1)$ (blue, dashed), $\vec{s}_{14} = (0,1,0,3)$ (red, dotted), and $\vec{s}_{21} = (0,2,0,2)$ (green, solid). The shaded areas in (d) represent four-photon interference fringes oscillating as a function of ϕ.

V. CONCLUSIONS

The discussed phenomena constitute showcase examples for the intricate effects that manifest when quantifying or controlling the particles' degree of indistinguishability in an experiment. Despite the mere doubling of the number of particles with respect to the HOM setup, the (in)distinguishability transition cannot be explained anymore by extrapolation of the two-photon effect: interference dominates over bosonic bunching, and events with large occupation numbers are not necessarily enhanced when approaching indistinguishability. The phase α which quantifies the phase enclosed by the setup constitutes an input- and output-mode dependent parameter with measurable impact, as a physical consequence emerging from the formal definition of four-dimensional complex Hadamard matrices [12]. Finally, not only the degree, but also the quality of interference changes with the number of interfering particles.

Our results impressively demonstrate that we need to fully abandon the idea that many-particle interference manifests itself in a unique fashion that can be predicted from the bosonic nature of particles alone, as usually stated in the two-particle case [16]. Constructive and destructive interference effects occur for *all* final events, and they can be turned into one another by variation of the phases. Our decomposition of the initial state into distinguishability settings not only offers a powerful tool for the computation of event probabilities, necessary for the experimental implementation of many-particle interference, but also allows us to understand and classify the occurring phenomena. Due to the combinatorial explosion of possible many-particle paths, a description of the indistinguishability transition becomes prohibitive for many more than four particles, and it remains to be studied whether one can find other, coarse-grained observables [15] with a monotonic dependence on a single parameter, to quantify many-particle distinguishability.

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