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Optical rogue waves in telecommunication data streams

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Large broadening of short optical pulses due to fiber dispersion leads to a strong overlap in information data streams resulting in statistical deviations of the local power from its average. We present a theoretical analysis of rare events of high-intensity fluctuations—optical freak waves—that occur in fiber communication links using bit-overlapping transmission. Although the nature of the large fluctuations examined here is completely linear, as compared to commonly studied freak waves generated by nonlinear effects, the considered deviations inherit from rogue waves the key features of practical interest—random appearance of localized high-intensity pulses. We use the term "rogue wave" in an unusual context mostly to attract attention to both the possibility of purely linear statistical generation of huge amplitude waves and to the fact that in optics the occurrence of such pulses might be observable even with the standard Gaussian or even rarer-than-Gaussian statistics, without imposing the condition of an increased probability of extreme value events.

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The increase of channel rates of optical communication systems imposes a use of shorter time slots allocated for each transmitted symbol and, consequently, shorter optical carrier pulses. For instance, in fiber-optic systems operating at the 40 Gbit/s channel rate the corresponding temporal interval between neighboring carrier pulses is 25 ps, and for 100 and 160 Gbit/s links being a subject of intensive laboratory research the time slots are just 10 and 6.25 ps, respectively. The width of a carrier pulse has to be even shorter, typically, by factor of 2 or more. Propagation of ultrashort pulses is strongly affected by the fiber dispersion resulting in a substantial temporal broadening. This leads to overlapping of a large number of transmitted information bits making the properties of the propagating field envelope dependent on the statistical characteristics of a data stream. Transmission of information in fiber links using bit overlapping and ultrashort optical pulses is a new and largely unexplored research field.

The transmission regime with large broadening of carrier pulses is often called in the literature either the quasilinear [1] to stress that dispersion dominates over nonlinear effects or the bit-overlapping transmission [2]. The bit-overlapping effect could be even more pronounced in the modern coherent optical communication systems where dispersion is not compensated over thousands of kilometers [3,4]. There are two important physical consequences of large pulse broadening and bit overlapping. The first-order linear effect is that such a massive overlapping of uncorrelated bits effectively creates a new type of randomness in the transmitted field due to the random nature of the information content in a bit stream. As a result of the bit-overlapping effect, signal power experiences statistical fluctuations that might lead to nondesirable events when local peak power pulsations in some time slots greatly exceed the average level leading to high local nonlinearity and as a result to signal degradation. The second effect occurs due to accumulation of nonlinear interactions of many overlapping bits. Despite the fact that for a single pulse the dispersion is a dominant effect, the nonlinearity does affect the signal as the local power in every time slot during transmission is determined by a large number of overlapping bits. Such nonlinear interactions of many randomly (due to information content of the bit stream) distributed bits lead to strong patterning effects and degrades the quality of a transmitted signal (see, e.g., [1,2,5,6]). In this Rapid Communication we consider the first problem, focusing on description of statistical properties of optical power in the quasilinear (bit-overlapping) regime. There is an interesting link between the considered effect of occurrence of large amplitude optical fluctuations in the telecommunication data stream and the so-called optical rogues waves, actively studied recently [7–10]. Though the generation of the optical freak (rogue) waves studied in [7–10] has a completely different nonlinear nature compared to the pure linear statistical problem considered here, the manifestation has many similarities, a rare random appearance of a large-amplitude localized optical wave packet on a lower power background. In both cases the theoretical challenge is to predict the probability of generation of a wave with a certain large amplitude {rogue waves were first studied in the context of hydrodynamics (see, e.g., [11] and references therein)}. We use the term "optical rogue wave" without imposing a requirement that occurrence of such waves should be statistically more probable than in the Gaussian distribution. The importance of such high-amplitude wave is determined both by the impact of a single event of high-intensity fluctuation and the probability of its occurrence. Knowledge to what extent occurrence of high-amplitude fluctuations is suppressed compared to the Gaussian distribution could be essential for system designers. Therefore, although we focus on the theoretical aspects of the general problem and our approach can be used in many other fields, signal statistics are also important for the practical design of transmission systems providing information when peak power fluctuations become too high and dispersion compensation has to be applied in-line. In this Rapid Communication we derive analytical expressions for the probability density functions of the optical peak power in systems using ultrashort optical pulses.

First, recall the very well known analytical presentation for the linear evolution of the data stream along the fiber span. The optical signal $A(t)$ at the beginning of the fiber span $z = 0$ has the standard form: $A(t,z=0)=\sum_n c_n f_0(t-nT)$. Here the input carrier pulse shape $f_0(t)$ could be any localized function in the case of the so-called return-to-zero (having zero level of power in between two consecutive ones) data format. To simplify expressions, in what follows we assume Gaussian pulse shape $f_0(t)=\sqrt{P_0}\exp[-t^2/(2\tau_0^2)]$. However, any other pulse shape can be considered in a similar manner. It is convenient to characterize the initial pulse width in units of the bit slot time T using dimensionless parameter $\eta=\tau_0/T$. Our approach is general and can be modified in a straightforward manner and applied to a variety of modulation formats [12]. Recall that in the modern coherent optical communication systems dispersion of the transmission line can be compensated electronically at the receiver allowing for a large accumulated broadening even for relatively wide pulses [3,4]. Therefore, the results presented below can be applied to a number of optical communication applications including coherent transmission. Here, without loss of generality, we consider an important data format: the so-called return-to-zero M-level differential-phase-shift-keying (M-DPSK) modulation formats that have many advantages such as high spectral efficiency and high tolerance to linear or nonlinear impairments. In the RZ M-DPSK signal logical ones and zeros are determined by the phase change between the consecutive time slots with $c_k=\exp[i2\pi(k-1)/M]$, where $k=1,2,\ldots,M$. Two currently most popular formats are binary 2-DPSK (or DBPSK) and quadrature (4-DPSK or DQPSK) formats with $c_k=\pm1$ and $c_k=1,i,-1,-i$, respectively. We assume that information bits are statistically independent. We consider here regimes of quasilinear propagation [1,2] when very short pulses are used to carry information data. In this regime dispersion dominates over nonlinearity and in the first approximation we can consider simple dispersion broadening of the pulse train leading to large bit overlapping. Evolution of the Gaussian pulses in a fiber span with dispersion β_2 and length L is a classical textbook material and is given by $f(t,L)=\tau_0 P_0^{1/2}\tau^{-1}\exp[-t^2/(2\tau^2)]$, $\tau^2=\tau_0^2-2iL\beta_2$. We assume in what follows that the distance L is large enough (or to be more precise the accumulated dispersion $L\beta_2$ is large) and all initially separated pulses are spread over many time slots. Mathematically it is convenient to introduce the parameter that characterizes a level of broadening and bit overlapping: $\varphi=2L\beta_2/(\tau_0 T)$. It is also useful to decompose amplitude and phase factors $f(t,L)=|f(t,L)|\exp(i\beta)$ that at large φ are $|f(t,L)|=\sqrt{P_0\eta/\varphi}\exp[-\varphi^{-2}(t/T)^2]$ and $\beta=t^2/(2\eta\varphi)-\pi/4$.

It is critically important for transmission that since at each time slot a large number (order of φ) of pulses are overlapped, the properties of the field envelope are *not anymore deterministic and are defined by the statistical properties of a bit pattern*. Thus we assume that averaging of signal power over the same position in many time slots can be replaced by averaging of the power over bits statistics.

The main goal of our work is to describe statistical properties of optical power fluctuations due to large bit overlapping. Therefore, in what follows we consider local power of the signal $\mathcal{P}(t,L)=|A(t,L)|^2$ that will be averaged over the bit pattern $\langle\ \rangle$. In mathematical terms we study a one-point probability density function (PDF) for a local power PDF($\mathcal{P}$).

Under the assumption that bits in an information stream are statistically independent, it is easy to derive the statistical properties for values c_k. Moments that are particularly important for the following analysis are

$$\langle c_k\rangle=0,\quad \langle c_i c_k^*\rangle=\delta_{ik}. \tag{1}$$

Note that, in general, one cannot apply the Wick theorem [13] and reduce higher order correlation functions of c_i to the first two moments, since some irreducible corrections arise. However, in the limit $\varphi\gg1$ higher order terms make negligible contributions to the core of the probability density function describing most probable fluctuations of the power. It will be shown below that high-order moments of c_i and difference between cases $M=2$ and $M>2$ are relevant only for very rare events.

Let us establish general statistical properties of the signal in the bit-overlapping regime $\varphi\gg1$. The general expression for an optical field reads

$$A(t,L)=\sum_k c_k\exp(i\beta_k)\,|f(t-kT,L)|, \tag{2}$$

where $\beta_k=\arg\{f(t-kT,L)\}$ and the argument function "arg" takes values within the interval $[0,2\pi]$. Technically we need to calculate the sum (over k) of contributions from bits at the time slot "k" to the field at some chosen time slot, for instance the site "0". Let us start from the estimates which are fundamental for the following analysis. A phase β_k as a function of k changes on the scale $\sim\sqrt{\varphi}$, whereas the amplitude of impulse $|f(t-kT,L)|$ as a function of k changes on the scale $\sim\varphi\gg\sqrt{\varphi}$. Thus, considering the signal $A(t,L)$ a phase of the factor $c_k e^{i\beta_k}$ can be treated as a random function with plane probability distribution on the interval $[0,2\pi]$, independently of specific values of M. As a result, there are $\sim\varphi$ summands with almost equal amplitude $|f(t-kT,L)|\sim1/\sqrt{\varphi}$ and uncorrelated, effectively random realizations of phase. Hence, sum (2) can be treated as a sum of $\sim\varphi$ almost equally distributed independent random quantities. This fact allows us to apply the central limit theorem and obtain Gaussian distribution for the most probable values of power. The central limit theorem, however, does not cover the structure of the far tails of the PDF. Therefore rare fluctuations of power given by far tails of the PDF will be analyzed separately using the *generalized limit theorem* (see, e.g., [14]).

The mean value of A is zero for considered RZ M-DPSK data format with all bit patterns having equal probabilities. Note that the PDF of the $A(t,L)$ is isotropic on the complex plane, as the resulting phase of the field $A(t,L)$ is random, because most of the summands in (2) have effectively random phases. The randomness of phases simplifies derivation of the standard deviation of $A(t,L)$. Only $\langle\mathcal{P}\rangle=\langle|A(t,L)|^2\rangle$ contributes to the standard deviation, whereas $\langle A^2(t,L)\rangle$ tends to zero as $\varphi\to\infty$. Averaging $\langle|A(t,L)|^2\rangle$ over the signal statistics (1) leads to

$$\langle\mathcal{P}\rangle\left(\frac{t}{T},\varphi;\frac{\tau_0}{T}\right)=\sum_k|f(t-kT,L)|^2.$$

We keep only the sum of powers for each pulse, because phases of overlapping pulses are effectively incoherent due to statistical independence of transmitted information signal.

The remaining sum is not sensitive to the phase of broadened pulses and it contains $\sim\varphi$ summands with an amplitude of each on the scale of $\propto 1/\varphi$. In the limit $\varphi \gg 1$ the sum can be replaced by an integral leading the obvious result for the average power $\bar{P} = \sqrt{\pi}\eta P_0$. Using the central limit theorem we arrive at $\mathrm{PDF}(A) \propto \exp(-|A^2|/\bar{P})$, that reads in terms of power

$$\mathrm{PDF}(\mathcal{P}) = \bar{P}^{-1}\exp(-\mathcal{P}/\bar{P}). \quad (3)$$

This formula provides a compact analytical expression for the probability density function of local optical power in the quasilinear transmission regime. The standard deviation $\sigma = \sqrt{\langle\mathcal{P}^2\rangle - \langle\mathcal{P}\rangle^2} = \bar{P}$ characterizes fluctuations of power. Our derivation indicates in particular that there is no difference between RZ DBPSK and RZ M-DPSK in the statistical analysis of the expression above. The derived probability density function is valid for relatively small fluctuations $\mathcal{P} \ll \varphi\bar{P}$ (see discussion below).

The more nontrivial problem however is the probability of giant fluctuations of local optical power (freak optical waves) that create extremely high-local nonlinearity in the fiber line. Such fluctuations happen due to very specific random arrangements in the bit pattern c_k and require special analysis.

Let us first find the bit pattern c_k which leads to maximum amplitude of the signal at some point t (we denote in what follows field phase as β). Mathematically one has to find the solution of the maximum problem $\max_{c_k}\{\mathrm{Re}[A(t,L)\exp(-i\beta)]\}$. Using the effective randomness of phase β_k it is straightforward to derive from (2) that the maximum amplitude of the optical field $A(t,L)$ produced by the solution of the maximum problem, extremal realization of c_k, in the limit of large φ can be presented as

$$A(t,L)_{\max} = e^{i\beta}\sqrt{q\,\varphi\,\bar{P}}, \quad (4)$$

where $q = 2M^2\pi^{-3/2}\sin^2(\pi/M)$ is of the order of unity. Expression (4) has a rather transparent physical meaning: the maximum is proportional to $\sqrt{\varphi}$ because on the optimal realization there are about φ in-phase summands with amplitudes of the order of $1/\sqrt{\varphi}$.

Next we derive the full PDF of the power in the whole interval. This problem is technically similar to derivation of PDF for $A(t,L)$ due to isotropy of the PDF in the complex plane. Generalization of the central limit theorem that includes the far tails of the PDF is based on applying the Cramér function S (see for details, e.g., [14]). The outline of the procedure described in [14] is that we first construct a generating function $Q(u) = \langle\exp[i(u'\,\mathrm{Re}\,A + u''\,\mathrm{Im}\,A)]\rangle_{c_k}$ for the random A defined in (2), where $u = u' + iu''$. Since A is the algebraic sum of random quantities, the generation function can be represented in the form $Q = \exp(\sum_k q_k)$, where $\exp[q_k(u)]$ presents generation function for each of the summand k in (2). Effective randomness of the phase factor β_k and weak dependence of the pulse amplitude on k at large φ (discussed above) allows us to treat the sum over k as an averaging over the phase and replace the sum by an integral that leads to the expression $Q = \exp(\int dk\langle q_k\rangle_{\beta_k})$. Note that averaged $\langle q_k\rangle_{\beta_k}$ still depends on M and is a function of the absolute value $|u|$ only. It can be shown that the

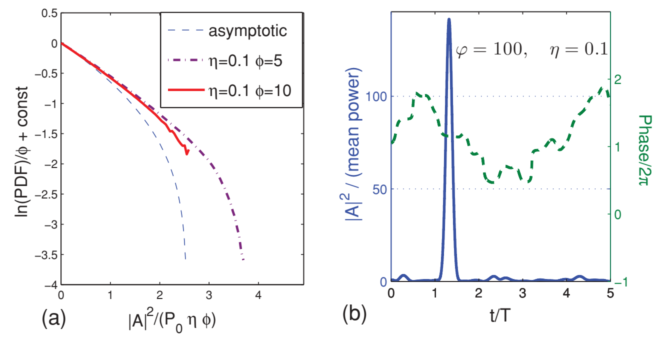

FIG. 1. (Color online) (a) Probability density function for normalized power for $\eta = 0.1$ and three values of $\varphi = 5$ (dashed-dotted line); 10 (solid line) at $M = 2$. Dashed line shows the asymptotic curve for PDF in the limit of very large φ. (b) Giant statistical fluctuation (optical rogue wave) that occurs in the signal power distribution. Power level in such freak optical wave is 138 times higher [$q\varphi \approx 145$, see (4)] than the average signal power. Dashed line is the phase of the signal.

integral saturates at $k \sim \varphi$. Omitting the technical details of calculations, the derived PDF reads

$$\mathrm{PDF}(\mathcal{P}) \;\propto\; \exp[-\varphi\, S_M(\mathcal{P}/\varphi\bar{P})], \quad (5)$$

where the nonimportant preexponential factor is ignored. This is the main result of the paper. The Cramér function S_M [14] has minimum at $\mathcal{P} = 0$, is concave, and tends to infinity as power $\mathcal{P}$ approaches its maximum possible value related to special fluctuation (4). Using tabulated (or easily computed, as shown in Fig. 1) Cramér functions S_M for each type of RZ M-DPSK format it is possible now to present the power PDF in the whole interval of powers including rare high-power fluctuations. The PDF(P) in the limit of very large φ is shown in Fig. 1(b) by a dashed line. In the limit of small fluctuations $\mathcal{P} \ll \varphi\bar{P}$ expression (5) is reduced to (3). Thus non-Gaussian statistics [and higher order correlators in (1)] of c_k and different M affect only the far tails of the PDF for power, while small fluctuations are described by the generic formula (3).

Figure 1(a) depicts the computed probability density function for several values of the parameters φ and η. The dashed line shows PDF in the limit of very large φ given by the Cramér function (5).

Figure 1(b) shows a particular realization of the statistics in the bit pattern leading to a very large excursion (peak-to-average ratio) of a local power. It is seen from Fig. 1(b) that the power level in rare variations can be much higher than the average power of the signal. Such a rare fluctuation of the optical field, an optical rogue (freak) wave, occurs due to statistical properties of the signal in the data stream propagating along fiber links. The freak optical wave shown in Fig. 1(b) corresponds to the fluctuations described by the tail of the probability density function shown in Fig. 1(a) and given by (5).

The peak of the optical rogue wave shown in Fig. 1(b) dramatically exceeds an average signal power. This leads to detrimental nonlinear effects causing overall degradation of the optical signal. Therefore, in long communication fiber links using ultrashort optical pulses in-line dispersion compensation might be required even in the quasilossless propagation regime and coherent transmission. The derived PDF (5) can be used to estimate the required amount and spatial periodicity of in-line dispersion compensation in such systems. For example, one can estimate the length N (measured in bit time T) of a signal bit stream which will contain a peak power fluctuation of a given intensity $\mathcal{P}$: $N \sim \eta/\mathrm{PDF}(\mathcal{P})$. This value practically does not depend on L when the dimensionless parameter $\varphi \gg \mathcal{P}/\bar{\mathcal{P}}$ according to (3). The developed theory can be used to describe both a rare event of high-power fluctuation and also the conditions leading to increase in an average frequency of occurrence of rogue optical waves in the information data stream.

In conclusion, we have presented theoretical analysis of optical signal statistics in the quasilinear transmission regime when locally dispersion is dominating over nonlinear effects. Large bit overlapping combined with the random nature of the information pattern leads to a statistical possibility of high-peak power fluctuations that can be damaging for signal transmission. Such freak optical waves in telecommunication data stream increase local nonlinearity and overall degrade transmitted signal quality. We have derived a analytical expression for the probability density functions describing such rare events of high-peak power fluctuations.

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