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Quantum teleportation in the spin-orbit variables of photon pairs

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We propose a polarization to orbital angular momentum teleportation scheme using entangled photon pairs generated by spontaneous parametric down-conversion. By making a joint detection of the polarization and angular momentum parity of a single photon, we are able to detect all the Bell states and perform, in principle, perfect teleportation from a discrete to a continuous system using minimal resources. The proposed protocol implementation demands experimental resources that are currently available in quantum optics laboratories.

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Teleportation [1] is probably one of the most amazing quantum phenomena relying on the existence of entanglement, also presenting direct applications to quantum-state transmission over long distances. Briefly, the teleportation protocol can be described as follows: Alice (A) and Bob (B) share an entangled state of two qubits, so that each one of them receives one qubit, henceforth called qubit A and qubit B. In addition to this qubit, A has another qubit prepared in a given (unknown to both parts) state that she wants to teleport to B. This qubit is henceforth called qubit S. To achieve teleportation, A makes a joint Bell measurement of qubit S and qubit A. As a result, she finds one of the four possible Bell states and transmits this result to B through a classical channel. Depending on the measurement result, B applies to his qubit one of the three Pauli matrices or simply the identity. After this operation, the quantum state of qubit S (which has been destroyed by A's measurement) is reconstructed on qubit B.

Different experimental schemes for quantum-state teleportation have been reported in the literature, using photons [2,3], trapped ions [4], or cavity QED systems [5]. Most experimental realizations employ three or more particles or subsystems, as is the case in [3] and [5]. However, by using different degrees of freedom of the same particle, one can reduce the number of particles involved [2]. Combination of the photon polarization with its spatial degrees of freedom has recently led to interesting results such as the demonstration of a topological phase for entangled qubits with spin-orbit modes [6], proposals of hyperentanglement schemes in parametric oscillators [7], and investigation of a spin-orbit Bell inequality [8,9]. Also, spin-orbit photonic devices useful for quantum information protocols have been proposed, such as cryptography schemes [10], controlled-not (CNOT) gates [11], and so-called qplates [12–15]. In the present work, we propose a teleportation scheme using two photons produced by spontaneous parametric down-conversion, which are entangled in orbital angular momentum (OAM). While the proposed scheme benefits from the advantages offered by photonic implementations, it also allows for complete Bell-state measurement of the spin-orbit degrees of freedom. Another interesting aspect of our proposal is that it does not depend on the specific entangled OAM state that is shared between A and B, relying only on its parity. Different entangled angular momentum states with the same parity properties can be used to implement the protocol. In Refs. [16] and [17] teleportation protocols for OAM states have been proposed. They present, nevertheless, several differences from the one we describe here, as we show below.

The proposed setup consists of a nonlinear crystal, cut for type I phase match so that parametric down-converted photons are produced in the same polarization state. The nonlinear crystal is then pumped by a vertically polarized beam, prepared in a Laguerre-Gaussian mode with topological charge l. Assuming that the phase match condition is satisfied, the down-converted photon pairs are produced with horizontal polarization. In terms of OAM conservation, phase match imposes that the sum of the down-converted charges equals the pump charge [18–21]. In [22], it was shown that the photon pairs are entangled in OAM, and their quantum state is

$$|\chi_0\rangle = \sum_{m=-\infty}^{+\infty} c_m\, |m,H\rangle_A |l-m,H\rangle_B, \qquad (1)$$

with $c_m = c_{l-m}$. This symmetry is due to the conservation of the total angular momentum l. Each photon carries some angular momentum and the sum of both must correspond to the pump beam's OAM. Under type I phase match, frequency degeneracy, and noncollinear geometry, the correlated photons are distinguishable from each other by their wave vectors $\mathbf{k}_s$ and $\mathbf{k}_i$, symmetrically disposed with respect to the pump wave vector. The symmetry $c_m = c_{l-m}$ then means that the probablity amplitude for generating one photon with wave vector $\mathbf{k}_s$ and angular momentum m and another photon with wave vector $\mathbf{k}_i$ and angular momentum $l-m$ has to be the same as the probability amplitude for generating one photon with wave vector $\mathbf{k}_s$ and angular momentum $l-m$ and another photon with wave vector $\mathbf{k}_i$ and angular momentum m. Of course, a complete description of the spatial quantum correlations between the twin photons should also involve entanglement in the radial indexes of the down-converted Laguerre-Gaussian modes. However, the usual measurement setups have finite apertures, so that only the lowest radial order contributes to the coincidence counts, while higher orders can be neglected.

Now, let us suppose that the pump beam is prepared in a Laguerre-Gaussian mode with $l = 1$. In this case, we can rewrite the entangled state above, separating its parity components in the following way:

$$\begin{aligned}|\chi_0\rangle &= \sum_{-\infty}^{+\infty} c_{2m}\,|2m,H\rangle_A|1-2m,H\rangle_B \\ &\quad + \sum_{-\infty}^{+\infty} c_{2m+1}\,|2m+1,H\rangle_A|-2m,H\rangle_B \\ &= \sum_{-\infty}^{+\infty} c_{2m}\,(|2m,H\rangle_A|1-2m,H\rangle_B \\ &\quad + |1-2m,H\rangle_A|2m,H\rangle_B)\,, \end{aligned} \tag{2}$$

where in the second equality we reordered the summation in the odd-even component by making $m \to -m$ and used $c_{1-q} = c_q$. To describe the protocol, it will be useful to define single-photon OAM parity states:

$$\begin{aligned}|E\rangle &= \sqrt{2}\sum_{-\infty}^{+\infty} c_{2m}\,|2m\rangle, \\ |O\rangle &= \sqrt{2}\sum_{-\infty}^{+\infty} c_{2m}\,|1-2m\rangle = \sqrt{2}\sum_{-\infty}^{+\infty} c_{2m+1}\,|2m+1\rangle. \end{aligned} \tag{3}$$

The principle of our proposal is the following: an arbitrary quantum state is first encoded on the polarization of photon A and then teleported to the OAM of photon B by a complete spin-orbit Bell measurement realized on photon A only. The polarization quantum state of photon A can be prepared by a sequence of wave plates capable of implementing a general unitary transformation and producing an arbitrary polarization state $|\varphi\rangle \equiv \alpha|H\rangle + \beta|V\rangle$, where α and β are arbitrary complex coefficients satisfying the normalization condition [23,24]. After the state preparation scheme, we have a total state of the type

$$\begin{aligned}|\chi\rangle &= \sum_{-\infty}^{+\infty} c_{2m}\,(|2m,\varphi\rangle_A|1-2m,H\rangle_B \\ &\quad + |1-2m,\varphi\rangle_A|2m,H\rangle_B). \end{aligned} \tag{4}$$

It is now useful to define a spin-orbit Bell basis as follows:

$$\begin{aligned}|\phi_\pm^q\rangle &= \frac{1}{\sqrt{2}}(|q,H\rangle \pm |1-q,V\rangle) \\ |\psi_\pm^q\rangle &= \frac{1}{\sqrt{2}}(|1-q,H\rangle \pm |q,V\rangle). \end{aligned} \tag{5}$$

State (4), rewritten in the basis (5), gives

$$\begin{aligned}|\chi\rangle = \sum_{m=-\infty}^{+\infty} \frac{c_{2m}}{\sqrt{2}}\,[\,&|\phi_+^{2m}\rangle_A\,(\,\alpha|1-2m\rangle_B + \beta|2m\rangle_B\,) \\ &+ |\phi_-^{2m}\rangle_A(\alpha|1-2m\rangle_B - \beta|2m\rangle_B) \\ &+ |\psi_+^{2m}\rangle_A(\alpha|2m\rangle_B + \beta|1-2m\rangle_B) \\ &+ |\psi_-^{2m}\rangle_A(\alpha|2m\rangle_B - \beta|1-2m\rangle_B)]|H\rangle_B. \end{aligned} \tag{6}$$

Alice can now follow the prescription of [1], as described above, and perform a complete Bell measurement on state (6). Alice's Bell measurement corresponds to detecting one of the four maximally entangled (ME) states of two different degrees of freedom (polarization and OAM) of the same photon. This basis can be completely measured, providing a deterministic teleportation protocol, using the setup sketched in Fig. 1: first, Alice's photons are sent through an OAM sorter like the one described in Ref. [25], where the even (E) and odd (O) OAM modes are discriminated in the two outputs. Since the $2m$ and $1-2m$ components of entangled state (1) have opposite parities, they will exit the OAM sorter through different outputs. We thus have the state

$$\begin{aligned}|\chi'\rangle &= \sum_{-\infty}^{+\infty} c_{2m}[|(2m,\varphi);0\rangle_A|1-2m\rangle_B \\ &\quad + |0;(1-2m,\varphi)\rangle_A|2m\rangle_B], \end{aligned} \tag{7}$$

where we have added extra slots to Alice's photon state and grouped together orbital and polarization labels to stress that the photon takes a parity-dependent path. For example, $|(2m,\varphi);0\rangle_A$ refers to a single photon with polarization $|\varphi\rangle$ and OAM $|2m\rangle$ on the even output of the OAM sorter and vacuum on the odd output. We have omitted the common $|H\rangle_B$ state, which multiplies all the states, to make notation more clear. Labels in parentheses thus refer to the spin-orbit state in a given path, while the other one is empty. Each output then passes through a 50:50 polarizing beam splitter (PBS) where the H and V components are discriminated. We now have four possible paths for the photon, a situation that can be described by the state

$$\begin{aligned}|\chi''\rangle &= \sum_{-\infty}^{+\infty} c_{2m}\,[\alpha|(2m,H);0;0;0\rangle_A|1-2m\rangle_B \\ &\quad + \beta|0;(2m,V);0;0\rangle|1-2m\rangle_B \\ &\quad + \alpha|0;0;(1-2m,H);0\rangle|2m\rangle_B \\ &\quad + \beta|0;0;0;(1-2m,V)\rangle_A|2m\rangle_B]\,. \end{aligned} \tag{8}$$

Projection onto the spin-orbit Bell basis of Alice's photon is then achieved by recombining the PBS outputs in regular beam splitters (BSs), forming two nested Mach-Zehnder interferometers whose phases have to be stable. In a regular BS, the photon, arriving from whichever path, exits in a

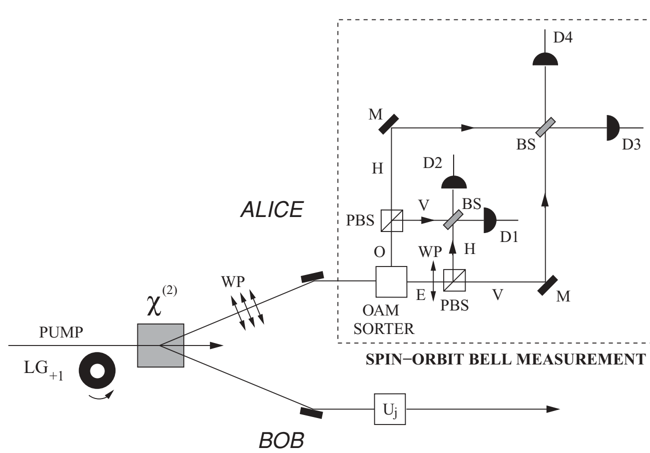

FIG. 1. Proposed experimental setup. M, mirror; WP, wave plate; BS, beam splitter; PBS, polarizing beam splitter; D1–D4, detectors.

superposition of two paths, irrespective of its polarization or OAM, and the phase in this superposition depends on its entering port. We thus have a transformation between modes of type $|1,0\rangle \rightarrow 1/\sqrt{2}(|1,0\rangle + |0,1\rangle)$ and type $|0,1\rangle \rightarrow 1/\sqrt{2}(|1,0\rangle - |0,1\rangle)$. By carefully keeping track of how modes combine, it is easy to check that the states arriving at each detector, after recombination in the BSs, are as follows.

$$\begin{aligned}
&\mathrm{D1}: \sum_{-\infty}^{+\infty} c_{2m}(\alpha|2m,H\rangle_A|1-2m\rangle_B + \beta|1-2m,V\rangle_A|2m\rangle_B).\\
&\mathrm{D2}: \sum_{-\infty}^{+\infty} c_{2m}(\alpha|2m,H\rangle_A|1-2m\rangle_B - \beta|1-2m,V\rangle_A|2m\rangle_B).\\
&\mathrm{D3}: \sum_{-\infty}^{+\infty} c_{2m}(\alpha|1-2m,H\rangle_A|2m\rangle_B - \beta|2m,V\rangle_A|1-2m\rangle_B).\\
&\mathrm{D4}: \sum_{-\infty}^{+\infty} c_{2m}(\alpha|1-2m,H\rangle_A|2m\rangle_B + \beta|2m,V\rangle_A|1-2m\rangle_B).
\end{aligned} \tag{9}$$

Photodetection at detectors D1–D4 corresponds to applying an annihilation operator destroying the corresponding photon irrespective of its state. We thus have that, after detection of Alice's photon, Bob's photon is left in one of the four states, depending on which detector has clicked.

$$\begin{aligned}
&\mathrm{D1}: (\alpha|O\rangle + \beta|E\rangle)|H\rangle,\\
&\mathrm{D2}: (\alpha|O\rangle - \beta|E\rangle)|H\rangle,\\
&\mathrm{D3}: (\alpha|E\rangle - \beta|O\rangle)|H\rangle,\\
&\mathrm{D4}: (\alpha|E\rangle + \beta|O\rangle)|H\rangle,
\end{aligned} \tag{10}$$

Here we have reincorporated into the notation the polarization state of Bob's photon. Comparing Eq. (10) to Eq. (6), we see that each detector acts on the quantum state $|\chi\rangle$ as one of the Bell-state projectors

$$\begin{aligned}
P_{\phi_\pm} &= \sum_{m=-\infty}^{\infty} |\phi_\pm^{2m}\rangle\langle\phi_\pm^{2m}|,\\
P_{\psi_\pm} &= \sum_{m=-\infty}^{\infty} |\psi_\pm^{2m}\rangle\langle\psi_\pm^{2m}|.
\end{aligned} \tag{11}$$

To complete the teleportation protocol, we allow for the exchange of two classical bits of information between A and B. A then tells B which one of the Bell states she detected or, equivalently, which one of the detectors clicked. Using the information A provides, B applies a unitary transformation U_j (one of the three Pauli matrices, or the identity) to photon B to reconstruct state $|\varphi\rangle$, photon A's initial state, in photon B, and resume teleportation. In this case, Alice's polarization state is teleported to Bob's OAM parity state. The required unitary transformations in the $\{|E\rangle,|O\rangle\}$ subspace can be achieved by means of simple optical setups. A Dove prism performs an image reflection making $|m\rangle \rightarrow |-m\rangle$. A spiral phase hologram (SPH) adds one unit of OAM to an incoming beam, so that $|m\rangle \rightarrow |m+1\rangle$ [26–28]. Since $c_m = c_{1-m}$ one can easily see that a Dove prism followed by a spiral phase hologram makes the transformations $|E\rangle \rightarrow |O\rangle$ and $|O\rangle \rightarrow |E\rangle$. Also, any relative phase can be introduced between the even and the odd components with an OAM sorter followed by a delay line. These resources allow Bob to implement the unitary transformation needed to resume the protocol.

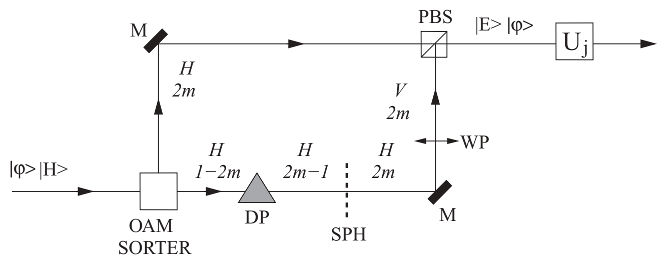

FIG. 2. Scheme for OAM parity-polarization swap. M, mirror; DP, Dove prism; SPH, spiral phase hologram; WP, wave plate; PBS, polarizing beam splitter.

One interesting remark about the protocol is that the only condition imposed on the OAM entangled states is that $c_m = c_{1-m}$. Therefore, the whole protocol does not depend on details of the state created by parametric down-conversion, relying only on its symmetry properties. Starting from any one of states (10), Bob can reconstruct $|\varphi\rangle$ with the help of the aforementioned simple optical devices. As a result, one teleports a discrete polarization state to the OAM parity of the single-photon wavefront. Parity is an usual dichotomization of continuous variables [29]. In our protocol it allows for the quantum-state teleportation from a discrete degree of freedom to a continuous one. Another possibility is to swap the OAM parity state to the polarization (see Fig. 2) and then make the necessary unitary transformation with polarization devices only. This procedure would simplify the tomography of the teleported quantum state.

It is important to notice the crucial role played by the OAM pump in the teleportation scheme. For any value of $l \neq 0$, the dominant term in the expansion in Eq. (1) is the ME component $|l\rangle_A|0\rangle_B + |0\rangle_A|l\rangle_B$. Moreover, all the rest of the expansion can be cast in the form of a superposition of ME components as in Eq. (2). Our teleportation scheme works simultaneously in each ME state's subspace. However, in the absence of the OAM pump ($l = 0$), the dominant term in (1) will be the product component $|0\rangle_A|0\rangle_B$, and the teleportation protocol would not work.

We now briefly discuss the main difference between the present teleportation scheme and those in [16] and [17], where the same goal is pursued: teleporting an OAM state. In both [16] and [17], the nonlinear crystal responsible for the photon pair generation is pumped by a beam with zero angular momentum. As a consequence, the highest probability is to create photon pairs with both null OAMs. In this case, the main component of the OAM quantum state is not entangled and must be ruled out by the setup. In the present proposal, we increase the proportion of parity entangled photon pairs by pumping the nonlinear crystal with a beam carrying $m = 1$. All components of the OAM quantum state are entangled and the protocol acts in parallel in all of them.

In conclusion, we have proposed a scheme to teleport the quantum state of a two-dimensional variable (polarization) to another one belonging to an infinite-dimensional space (OAM) using two photons only. Our proposal is possible by dichotomizing the infinite-dimensional OAM state space and making a Bell measurement on the 2 degrees of freedom of the same photon. It demands experimental resources already available in laboratories and can be realized in a short delay.

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