Coherent perfect absorption in a homogeneously broadened two-level medium

Stefano Longhi
Dipartimento di Fisica, Politecnico di Milano, Piazza L. da Vinci 32, IT-20133 Milano, Italy
(Received 30 March 2011; published 24 May 2011)

In recent works, it has been shown, rather generally, that the time-reversed process of lasing at threshold realizes a coherent perfect absorber (CPA). In a CPA, a lossy medium in an optical cavity with a specific degree of dissipation, equal in modulus to the gain of the lasing medium, can perfectly absorb coherent optical waves that are the time-reversed counterpart of the lasing field. Here, the time-reversed process of lasing is considered in detail for a homogeneously broadened two-level medium in an optical cavity and the conditions for CPA are derived. It is shown that, owing to the dispersive properties of the two-level medium, exact time-reversal symmetry is broken and the frequency of the field at which CPA occurs is generally different than the one of the lasing mode. Moreover, at a large cooperation parameter, the observation of CPA in the presence of bistability requires one to operate in the upper branch of the hysteresis cycle.

DOI: 10.1103/PhysRevA.83.055804 PACS number(s): 42.55.Ah, 42.65.Pc, 42.65.Sf

Introduction. It is well-known that the absorption properties of a lossy medium can be conveniently controlled by optical feedback. Several early studies have investigated coherent absorption effects in optical cavities [1–4] with application in the realization of special modulators and cavity-enhanced detectors [1–3]. The transmission properties of an optical cavity filled by an absorbing medium have also attracted great attention since long time in connection to optical bistability (see, for instance, Refs. [5–7] and the references therein). In recent works, the idea of controlling the properties of an absorption of a lossy medium in the presence of feedback has been revisited [8,9], and the unusual scattering properties of gain (absorber) systems with certain symmetries have been highlighted [10,11]. In particular, in Ref. [8] it was proven rather generally that a coherent perfect absorber (CPA) realizes the time-reversed process of lasing at threshold [8], whereas in Ref. [9] an experimental demonstration of interferometric control of the absorption was reported using a thin slice of silicon illuminated by two beams [9]. The main idea underlying CPA is that, since in a steady-state process time reversal corresponds to interchanging incoming and outgoing fields, the time-reversed process of lasing at threshold corresponds to the perfect absorption of certain incoming coherent-light fields. In the reverse process of lasing at threshold, the gain medium in the resonator is replaced by a lossy medium, corresponding to a positive imaginary refractive index equal in absolute value to that at the lasing threshold. Then, whenever the system is illuminated coherently and monochromatically by the time-reverse of the output of a lasing mode, the incident radiation is perfectly absorbed [8]. Such a simple picture of CPA strictly holds provided that phenomena like saturation of the absorption and absorption-induced dispersion in the medium (responsible for the well-known frequency pulling effect in the time-reversed process of lasing) are negligible. While the former condition can be satisfied for low-power fields, the latter phenomenon (being a linear one) occurs also for low input powers and can be non-negligible when absorption exploits a narrow resonance frequency of the atomic medium. In this Brief Report, we consider the time-reversed process of lasing for a homogeneously broadened two-level medium in an optical cavity and discuss in details the conditions for CPA when the saturation of absorption and loss-induced dispersion are properly considered in the model. Owing to the dispersive properties of the two-level medium, it is shown that the frequency of the field at which CPA occurs is generally different than the one of the lasing mode and that a somewhat singular case appears when the cavity photon and polarization decay rates are equal. Moreover, it is shown that, for a strong cooperation parameter leading to optical bistability, the observation of CPA requires one to operate in the upper branch of the bistable cycle.

The model. Let us consider a standard model describing light absorption (or amplification) in a homogeneously broadened two-level medium embedded in an optical cavity similar to the one encountered in the theory of optical bistability [7,12] or in the semiclassical theory of homogeneously broadened lasers with an injected signal [13]. Specifically, we consider a two-level medium with a resonance frequency, ω_0, of length, l, placed in a ring cavity of total length, $\mathfrak{L}$, which is coupled to the outside by a single mirror of power transmittance, T [see Fig. 1(a)]. An input field of frequency, ω, close to ω_0, and amplitude, $\mathcal{E}_0$, can be injected into the cavity, as shown in Fig. 1(a). As compared to the most common configurations considered in the theory of optical bistability [7,12], the optical cavity considered here is coupled to the outside by a single coupler [see mirror 1 in Fig. 1(a)], whereas all other mirrors are assumed to have 100% reflectivity. For such a cavity, the lasing and its time-reversed counterpart, i.e., CPA, are schematically shown in Figs. 1(b) and 1(c). To study the operation of laser and CPA, let us indicate by $\mathcal{E}(z,t) = (1/2)[(\sqrt{\gamma_\parallel \gamma_\perp}\hbar/\mu)A(z,t)\exp(i\omega t - ikz) + \text{c.c.}]$, $\mathcal{P}(z,t) = (1/2)[(i\mu N_e\sqrt{\gamma_\parallel/\gamma_\perp})\Lambda(z,t)\exp(i\omega t - ikz) + \text{c.c.}]$, and $N(z,t) = N_e n(z,t)$ the electric field, macroscopic polarization, and population difference in the medium, respectively, where z is the longitudinal spatial coordinate along the ring, $\gamma_\parallel$ and $\gamma_\perp$ are the population and dipole decay rates, respectively, μ is the modulus of the electric dipole moment of the atoms, and N_e is the population difference at the equilibrium between the lower and upper atomic levels. For an absorber, N_e is positive and equal to the total atomic population, N_t, whereas for an amplifying medium, N_e is negative and its value is determined by the pumping rate. The evolution of the slowly varying amplitudes, A and Λ, of the electric field and polarization in the medium are governed by the Maxwell-Bloch equations [7,12,13]:

$$\partial_t A = -c\partial_z A + c\alpha\Lambda, \tag{1}$$
$$\partial_t \Lambda = -\gamma_\perp[(1+i\Delta)\Lambda + nA], \tag{2}$$
$$\partial_t n = -\gamma_\parallel\left[n - 1 - \tfrac{1}{2}(A^*\Lambda + A\Lambda^*)\right], \tag{3}$$

where $\alpha \equiv \mu^2\omega_0 N_e/(2\hbar\gamma_\perp\epsilon_0 c)$ is the unsaturated absorption ($N_e > 0$) or amplification ($N_e < 0$) coefficient and $\Delta \equiv (\omega - \omega_0)/\gamma_\perp$ is the detuning between the frequency, ω, of the injected field and the resonance frequency, ω_0, of the atoms, normalized to the polarization decay rate, $\gamma_\perp$ [i.e., to the gain (absorption) linewidth]. Equations (1)–(3) should be supplemented by the boundary conditions imposed by the coupling mirror 1 at $z = 0$, which relates the incident and scattered fields [see Fig. 1(a)]. For a lossless mirror, these are given by

$$A(0,t) = i\sqrt{T}A_0 + \sqrt{1-T}A(\mathcal{L},t)\exp(-ik\mathcal{L}) \tag{4}$$

and

$$A_{\rm out} = \sqrt{1-T}A_0 + i\sqrt{T}A(\mathcal{L},t)\exp(-ik\mathcal{L}), \tag{5}$$

where $A_0 \equiv \mu\mathcal{E}_0/(\hbar\sqrt{\gamma_\parallel\gamma_\perp})$ and $A_{\rm out} \equiv \mu\mathcal{E}_{\rm out}/(\hbar\sqrt{\gamma_\parallel\gamma_\perp})$ are the normalized amplitudes of the injected and reflected electric fields at the mirror 1, respectively [see Fig. 1(a)]. Note that the laser ($\alpha < 0$) and CPA ($\alpha > 0$) systems in Figs. 1(b) and 1(c) correspond to $\mathcal{E}_0 = 0$, $\mathcal{E}_{\rm out} \neq 0$ and $\mathcal{E}_0 \neq 0$, $\mathcal{E}_{\rm out} = 0$, respectively. To simplify the analysis, here, we consider the mean-field limit of the Maxwell-Bloch equations [7,12,13], in which the fields are almost uniform along z. This limit requires a small transmissivity, $T \to 0$, a small single-pass absorption (or amplification) coefficient, $\alpha l \to 0$ (of order $\sim T$), a small amplitude of the injected signal, $A_0 \to 0$ (of order $\sim\sqrt{T}$), and a small detuning, $k\mathcal{L} - 2n\pi \to 0$ (of order $\sim T$), where n is an integer defining the resonance frequency of the cold cavity closest to ω. In this limit, the Maxwell-Bloch equations read

$$\partial_t A = \kappa[2C\Lambda - (1+i\theta)A + Y], \tag{6}$$
$$\partial_t \Lambda = -\gamma_\perp[(1+i\Delta)\Lambda + nA], \tag{7}$$
$$\partial_t n = -\gamma_\parallel\left[n - 1 - \tfrac{1}{2}(A^*\Lambda + A\Lambda^*)\right], \tag{8}$$

where $Y \equiv 2A_0 i/\sqrt{T}$, $C \equiv \alpha l/T$ is the cooperation parameter, $\kappa \equiv cT/(2\mathcal{L})$ is the photon decay rate in the cold cavity, and $\theta \equiv (\omega - \omega_c)/\kappa$ is the detuning between the frequency, ω, of the injected field and the cavity resonance frequency, $\omega_c = 2n\pi c/\mathcal{L}$, closest to ω, normalized to the photon decay rate, κ. Moreover, in the mean-field limit the normalized amplitude, $A_{\rm out}$, of the field leaving the cavity, as obtained by Eq. (5), simply reads

$$A_{\rm out} \simeq i\sqrt{T}(A - Y/2). \tag{9}$$

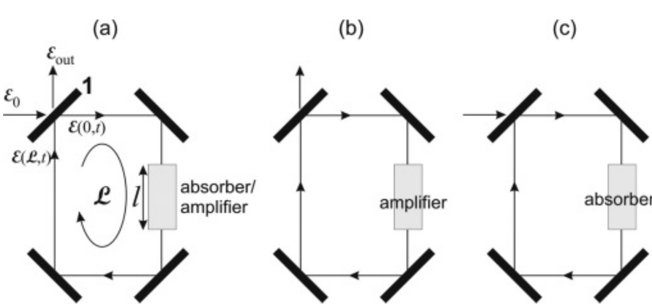

FIG. 1. (a) Schematic representation of a ring cavity with a homogeneously broadened two-level medium and an injected signal; (b) and (c) show schematically the operation of a laser and of a CPA system.

Let us now discuss separately the cases of lasing [Fig. 1(b)] and CPA [Fig. 1(c)], highlighting some distinct features not considered in Refs. [8,9] and arising from dispersive and saturation effects.

The laser system. The laser configuration in Fig. 1(b) corresponds to the absence of an injected field, $Y = 0$, and to an amplifying medium, $C < 0$. In this case, as is well-known laser emission occurs above threshold, for $|C| > |C_{\rm th}| \equiv (1+\Delta^2)/2$. The frequency, $\omega = \omega_{\rm las}$, of the emitted radiation is obtained from the condition $\Delta = -\theta$, which yields the well-known frequency-pulling relation:

$$\omega_{\rm las} = \frac{\gamma_\perp\omega_c + \kappa\omega_0}{\gamma_\perp + \kappa}. \tag{10}$$

Above threshold, the steady-state solution of the normalized intracavity power, $X = |A|^2$, is given by

$$X = 2|C| - 1 - \Delta^2 \tag{11}$$

and it is stable within the mean-field model. The normalized output power is simply given by $|A_{\rm out}|^2 = TX$.

The CPA system. CPA is realized for an absorbing medium ($C > 0$) with an injected signal ($Y \neq 0$) of appropriate amplitude and frequency such that the output field, $A_{\rm out}$, vanishes. Contrary to the predictions of Ref. [8], we show here that the absorption-induced dispersion of the two-level atoms breaks exact time-reversal symmetry and the frequency at which CPA occurs is generally different than the lasing frequency, $\omega_{\rm las}$, given by Eq. (10). Moreover, as compared to Ref. [8] our model properly accounts for saturation of the absorber, and thus can predict CPA beyond the condition of lasing at threshold. To determine the conditions that realize a CPA, let us first notice that the steady-state solution to Eqs. (6)–(8) gives the following implicit relation between the normalized powers, $|Y|^2$ and $X = |A|^2$, of incoming and intracavity fields, respectively,

$$|Y|^2 = X\left[\left(1+\frac{2C}{1+\Delta^2+X}\right)^2 + \left(-\theta+\frac{2C\Delta}{1+\Delta^2+X}\right)^2\right]. \tag{12}$$

The normalized power of the field leaving the cavity can be then obtained using Eq. (9). The steady-state solution of Eq. (12), and its stability have been extensively studied in the theory of optical bistability and the results can be briefly summarized as follows [7]: (i) for $2C > \Delta\theta - 1$ and $(2C - \Delta\theta + 1)^2(C + 4\Delta\theta - 4) > 27C(\Delta + \theta)^2$, the curve $X = X(|Y|^2)$ is S-shaped. Within the mean-field approximation the solution is stable in the upper and lower branches and unstable in the intermediate (negative-slope) branch. (ii) If either one of the two previous inequalities is not satisfied, the curve $X = X(|Y|^2)$ turns out to be single-valued and the solution is always stable. In particular, for $C < 4$ the curve is always single-valued. As an example, in Figs. 2(a) and 2(b) the

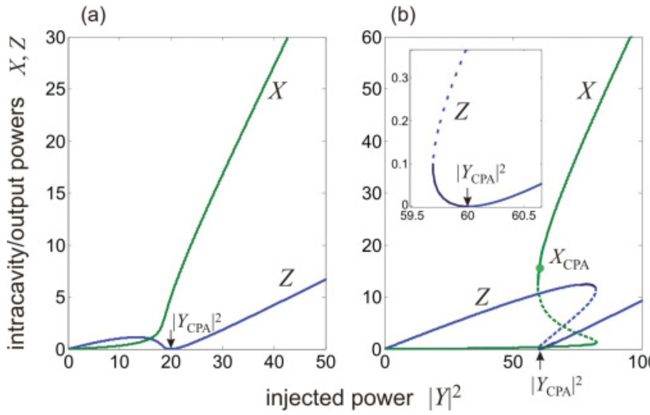

FIG. 2. (Color online) Behavior of the normalized intracavity power, $X = |A|^2$, and output power, $Z = |A_{\rm out}|^2/T$, versus the normalized intensity, $|Y|^2$, of the incident beam for $\Delta = \theta = 0$ and for (a) $C = 3$ (monostable regime) and (b) $C = 8$ (bistable regime). In (b) the dashed curves denote the unstable branches. The value $Y = Y_{\rm CPA}$ corresponds to the input field amplitude that realizes CPA. In (b) the amplitude $Y_{\rm CPA}$ is close to the boundary of the hysteresis cycle, as shown in the enlargement depicted in the inset of (b).

behaviors of the normalized intracavity power, X, and output power, $Z = |A_{\rm out}|^2/T$, versus the normalized intensity, $|Y|^2$, of the incident beam are depicted for typical monostable [Fig. 2(a)] and bistable [Fig. 2(b)] cases.

Let us now focus our attention to the conditions that realize a CPA. According to Eq. (9), CPA corresponds to a steady-state solution of Eqs. (6)–(8) with $A = Y/2$. This leads to the following two requirements for CPA: (i) the intracavity power, X, should have the value $X = X_{\rm CPA} = 2C - 1 - \Delta^2$, which is precisely the value given by Eq. (11); correspondingly, the normalized power, $|Y|^2$, of the injected field that realizes CPA reads $|Y_{\rm CPA}|^2 = 4X_{\rm CPA} = 4(2C - 1 - \Delta^2)$. In the case of optical bistability, it can be easily proven that $X_{\rm CPA}$ is always located on the upper stable branch of the S-shaped curve [see, for instance, Fig. 2(b)]. (ii) The frequency, ω, of the injected field should satisfy the condition $\theta = \Delta$.

Let us first discuss the condition (i). This condition shows that to observe CPA, a necessary requirement is that the cooperation parameter, $C = \alpha l/T$, must be *larger* than a minimum value, which is precisely the threshold value, $|C_{\rm th}|$, of the lasing mode once the absorber is replaced by the amplifier. If the injected power is small enough, in such a way that saturation of absorption is negligible, i.e., $X_{\rm CPA} \ll 1$, according to Ref. [8], CPA is obtained for a lossy medium corresponding to a positive imaginary refractive index equal in absolute value to that at the lasing threshold. For a cooperation parameter larger than the threshold value, CPA is attained at the intracavity power, $X_{\rm CPA}$, that makes the *saturated* positive imaginary refractive index of the absorber equal in absolute value to that at the lasing threshold. Thus perfect absorption occurs solely at a precise power level of the injected field given by $|Y|^2 = |Y_{\rm CPA}|^2 = 4X_{\rm CPA} = 4(2C - 1 - \Delta^2)$. Let us now assume that the injected signal is adiabatically increased from $Y = 0$ to the final value, $Y_{\rm CPA}$, that realizes CPA. If the input-output curve [Eq. (12)] is not S-shaped, CPA is obviously realized once Y reaches $Y_{\rm CPA}$. This is shown, as an example in Fig. 2(a). However, if the input-output curve [Eq. (12)] is S-shaped and $X_{\rm CPA}$ lies inside the bistable interval, CPA *is not* attained when Y reaches $Y_{\rm CPA}$ because $X_{\rm CPA}$ lies on the upper branch of the hysteresis cycle, as shown in Fig. 2(b). To realize CPA, one should therefore follow the bistable cycle: first, by increasing the power level, $|Y|^2$, above the hysteresis threshold to switch the system from the lower to the upper stable branch, and then, by decreasing the input power level down to $|Y_{\rm CPA}|^2$. Thus at large values of the cooperation parameter such that the optical bistability appears, CPA could be prevented by the appearance of the hysteresis, and its observation requires one to switch the operation into the upper branch of the bistable curve.

Let us now discuss the condition (ii), $\Delta = \theta$, required to observe CPA. Such a condition, basically, determines the frequency, $\omega = \omega_{\rm CPA}$, of the injected field for which CPA is observable. Here, the main result is that $\omega_{\rm CPA} \neq \omega_{\rm las}$, where $\omega_{\rm las}$ is given by the frequency-pulling equation, Eq. (10). Such a difference is basically related to the physical circumstance that, if the two-level amplifier is replaced by a two-level absorber, the resonant contribution to the *real* part of the refractive index changes sign as well and the condition $\epsilon(z) \to \epsilon^*(z)$, invoked to explain the CPA process as the time-reversed process of lasing at threshold [8], is not strictly satisfied. In other words, time-reversal symmetry is broken for the Maxwell-Bloch equations. The frequency, $\omega = \omega_{\rm CPA}$, of the injected field that yields a CPA is simply calculated by the requirement $\Delta = \theta$. For $\kappa \neq \gamma_\perp$, such a condition gives [compare with Eq. (10)]

$$\omega_{\rm CPA} = \frac{\gamma_\perp \omega_c - \kappa\omega_0}{\gamma_\perp - \kappa}. \tag{13}$$

Note that, for $\gamma_\perp \gg \kappa$, i.e., in the good-cavity limit and for a sufficiently broadened absorption line, one has $\omega_{\rm CPA} \simeq \omega_{\rm las} \simeq \omega_c$. More interesting is the case of $\gamma_\perp \to \kappa$, i.e., the case where the bandwidth of the absorption line equals to the decay rate of photons in the cold cavity. In this case, the condition $\Delta = \theta$ is never satisfied for $\omega_c \neq \omega_0$ and it is satisfied for any frequency, ω, for $\omega_c = \omega_0$. Therefore, if $\gamma_\perp = \kappa$ and a resonance frequency, ω_c, of the cavity is exactly tuned in resonance with the two-level atoms, the observation of CPA *does not require* a precise frequency tuning of the injected field.

Conclusions. In this Brief Report, the time-reversed process of lasing and coherent perfect absorption proposed in recent works [8,9] have been investigated in the framework of the semiclassical (Maxwell-Bloch) laser equations for a homogeneously broadened two-level medium. It has been shown that, owing to the dispersive properties of the two-level medium, exact time-reversal symmetry is broken and the frequency of the field at which CPA occurs is generally different than the one of the lasing mode. Moreover, at a large cooperation parameter the observation of CPA in the presence of bistability requires to operate the system in the upper branch of the hysteresis cycle.

Acknowledgment. This work was supported by the Italian MIUR (Grant No. PRIN-20082YCAAK, “Analogie ottico-quantistiche in strutture fotoniche a guida d’onda”).

[1] R. H. Yan, R. J. Simes, and L. A. Coldren, IEEE Photonics Technol. Lett. **1**, 273 (1989); K.-K. Law, R. H. Yan, J. L. Merz, and L. A. Coldren, Appl. Phys. Lett. **56**, 1886 (1990); K.-K. Law, R. H. Yan, L. A. Coldren, and J. L. Merz, *ibid.* **57**, 1345 (1990).
[2] J. F. Heffernan, M. H. Moloney, J. Hegarty, J. S. Roberts, and M. Whitehead, Appl. Phys. Lett. **58**, 2877 (1991).
[3] M. S. Ünlü, K. Kishino, H. J. Liaw, and H. Morkoç, J. Appl. Phys. **71**, 4049 (1992).
[4] M. Cai, O. Painter, and K. J. Vahala, Phys. Rev. Lett. **85**, 74 (2000); A. Yariv, IEEE Photonics Technol. Lett. **14**, 483 (2002); J. R. Tischler, M. S. Bradley, and V. Bulovic, Opt. Lett. **31**, 2045 (2006).
[5] E. Abraham and S. D. Smith, Rep. Prog. Phys. **45**, 815 (1982).
[6] H. M. Gibbs, *Optical Bistability: Controlling Light with Light* (Academic, Orlando, FL, 1985).
[7] L. A. Lugiato, Prog. Opt. **21**, 69 (1984).
[8] Y. D. Chong, Li Ge, Hui Cao, and A. D. Stone, Phys. Rev. Lett. **105**, 053901 (2010); S. Longhi, Physics **3**, 61 (2010).
[9] W. Wan, Y. Chong, Li Ge, H. Noh, A. D. Stone, and Hui Cao, Science **331**, 889 (2011).
[10] S. Longhi, Phys. Rev. A **82**, 031801(R) (2010).
[11] Y. D. Chong, Li Ge, and A. D. Stone, Phys. Rev. Lett. **106**, 093902 (2011).
[12] R. Bonifacio and L. A. Lugiato, Lett. Nuovo Cimento **21**, 505 (1972); Phys. Rev. A **18**, 1129 (1978); M. Gronchi, V. Benza, L. A. Lugiato, P. Meystre, and M. Sargent III, *ibid.* **24**, 1419 (1981).
[13] N. B. Abrahm, P. Mandel, and L. M. Narducci, Prog. Opt. **25**, 1 (1988).