Two peaks in the momentum distribution of bosons in a weakly frustrated two-leg optical ladder

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The ground-state properties of neutral hard-core bosons trapped in an optical two-leg ladder in the presence of an artificial magnetic field are studied. For a weak field, two separated peaks appear in the momentum distribution as a signature of the Meissner state in which bosons, carrying persistent currents on each leg, condense into finite-momentum states, while for a strong field, a central peak and tiny bumps associated with the vortex lattice structure indicate that the ground state is the vortex state.

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I. INTRODUCTION

Recent advances in creating an artificial magnetic field [1–5] for neutral bosons trapped in an optical lattice [6,7] provide a laboratory to explore macroscopic phase coherence in the presence of phase frustration. The condensate state, analogous to the superconductivity in a real magnetic field, either expels a weak field to form the Meissner state or generates quantized vortices for a strong field in the vortex state. These two states have been explored in type-II superconductors mostly by measuring macroscopic quantities, such as magnetizations, or by detecting vortices. However, the state of bosons confined in typical condensed-matter systems, such as Cooper pairs in Josephson junction arrays or superconductors in the presence of a magnetic field, cannot be directly probed. Ballistic expansion measurements of cold atoms in an optical lattice provide a unique opportunity to explore the macroscopically coherent state of the frustrated bosonic matter waves.

Parameters, such as interaction strengths and hopping amplitudes, of cold bosonic atoms in an optical lattice as well as geometries of the lattice can be manipulated to realize in a variety of strongly correlated systems [8–11]. Setting up a three-dimensional lattice and then increasing the trapping potential along the transverse directions to confine bosons in columns, we can realize one-dimensional chains [12,13]. Interacting bosonic quantum gases in a chain is a Luttinger liquid [14–16] which has a central peak at zero momentum in the momentum distribution as a hallmark of superfluidity. Superimposing a lattice with twice the period by controlling the polarization of the laser beams [17,18], one can create a two-leg ladder [19–21].

Since a ladder, still a one-dimensional system, permits orbital motions around the planar loops, it can implement some two-dimensional effects such as in-plane phase frustration. How the ground-state properties will be changed due to the phase frustration is a fundamental question on the behavior of frustrated conducting bosons [22,23]. Innovative approaches to create phase frustration [1–5] for neutral atoms in an optical lattice are proposed and some of them are implemented by using a spatially dependent optical coupling, freed from the limitations of rotating systems. Inhomogeneous phase shifts build an artificial magnetic field for neutral atoms bringing phase frustration into the system. These artificial phase shifts can create rather easily even strong frustration, seldom available in condensed-matter systems where subnanosize of the elementary plaquette needs a huge magnetic field to obtain one fluxon per plaquette.

In this work, we show that the momentum distribution provides tools to probe not only the vortex state, in which the existence of vortices is often regarded as an evidence of phase frustration due to a strong field, but also the Meissner state when the system is subject to a weak field. Specifically, hard-core bosons in a two-leg ladder under weak phase frustration for $f < f_c$ with $f_c \lesssim 0.3$ have two peaks in the momentum distribution as a signature of the Meissner state, whereas one central peak at zero momentum and tiny bumps at finite momenta, indicating the vortex lattice structure, appear in the vortex state for $f > f_c$. Furthermore, from the correlation functions, we find that, in the Meissner state, the phase angles of bosons are aligned across the rungs at the cost of being twisted along the legs, driving persistent currents in opposite directions on each leg. On the other hand, in the vortex state, the phase angles are locally twisted to adjust to strong frustration, generating currents that alternate directions on the rungs with a period characterized by the frustration.

II. MODEL

The system we are considering is that of cold bosonic atoms confined in a frustrated two-leg optical ladder (Fig. 1) described by the Hamiltonian

$$\begin{aligned} H = & -t \sum_{\alpha,x} (b^{\dagger}_{\alpha,x} b_{\alpha,x+1} + \text{H.c.}) \\ & - t \sum_{x} \left(e^{i A_x} b^{\dagger}_{2,x} b_{1,x} + \text{H.c.} \right), \end{aligned} \tag{1}$$

where $b^{\dagger}_{\alpha,x}$ ($b_{\alpha,x}$) denotes boson creation (destruction) operator at the xth site on the leg indexed by $\alpha = 1,2$, t is the hopping matrix element, and A_x is the phase shift on the xth rung. Here, focusing on the frustration effect, we choose the hard-core limit and assume that the inter- and intraleg hopping strengths are the same. We take a gauge choice $A_x = 2\pi f x$ to have a frustration

$$f = \frac{1}{2\pi} \sum_x A_x, \tag{2}$$

where the sum runs over an elementary plaquette. A different gauge choice, possibly taken in experiments, does not alter the results.

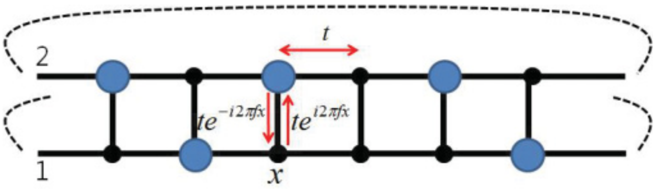

FIG. 1. (Color online) Schematic illustration of hard-core bosons (large dots) in a two-leg ladder under the influence of an artificial magnetic field due to nonvanishing phase shifts associated with hopping.

Momentum distributions observed in ballistic expansion experiments of cold atoms in an optical lattice are proportional to one-body correlators within the factor of the Wannier function in k space [8]. We describe them in terms of the correlator along a leg defined by the formula

$$n_\alpha(k) = \frac{1}{L}\sum_{x,x'=0}^{L-1}\langle b^\dagger_{\alpha,x} b_{\alpha,x'}\rangle e^{ik(x-x')} \tag{3}$$

for momentum $k = (2\pi/L)l$ $(l = 0,1,\ldots,L-1)$ where L is the length of the ladder and the average $\langle\cdots\rangle$ is taken over the ground-state wave function. Here, we set the lattice constant $a = 1$. For frustrated cases ($f \neq 0$), the total momentum distributions are $n(k_x,k_y) \sim \frac{1}{2}\{n_1(k_x) + n_2(k_x)\}$ for $k_y = 0$ and $k_y = \pi$ since the values of the interleg correlators are very small, but, for the unfrustrated case, $n(k_x,0) \sim \{n_1(k_x) + n_2(k_x)\}$ while $n(k_x,\pi)$ are negligible.

Correlation functions are obtained by taking the Fourier transform of the momentum distributions:

$$C_\alpha(x) = \frac{1}{L}\sum_k n_\alpha(k)e^{-ikx}. \tag{4}$$

We also consider the currents on the rungs

$$J_r(x) = i(e^{iA_x}b^\dagger_{2,x}b_{1,x} - e^{-iA_x}b^\dagger_{1,x}b_{2,x}) \tag{5}$$

and on the legs

$$J_{l\alpha}(x) = i(b^\dagger_{\alpha,x+1}b_{\alpha,x} - b^\dagger_{\alpha,x}b_{\alpha,x+1}) \tag{6}$$

and their correlations.

The ground-state wave functions and then their momentum distributions are calculated by the modified Lanczos method [24] for N bosons under the frustration $f = p/q$ for some small integers p and q with the periodic boundary condition. Note that we need a size $L = 2sq$ (s is an integer) to avoid any twisted boundary effects, especially in the Meissner state.

III. RESULTS

Figure 2 shows the momentum distributions $n_\alpha(k)$ at different f for $L = 20$ (40 sites in total) and $N = 8$. Apparently they show distinct features for different f. For $f = 0$, $n_\alpha(k)$ shows the Luttinger liquid behavior: it has peaks at zero momentum for both legs and its size dependence [25] at a fixed filling [$n_\alpha(0) \sim C_0 + C_1 L^{1-1/2K_s}$ with fitting constants C_0 and C_1] is roughly characterized by the Luttinger liquid parameter $K_s \approx 1$. Unfortunately, because of the limited sizes, further elaboration of K_s is not feasible.

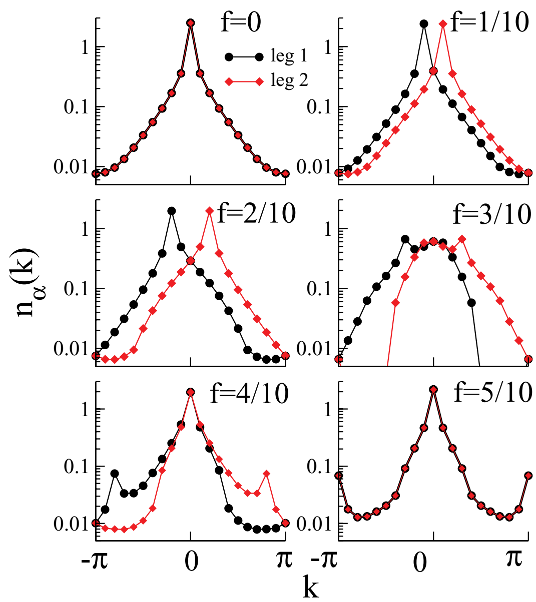

FIG. 2. (Color online) The momentum distributions of hard-core bosons, denoted by black (circles) and red (squares) lines for each leg, on an optical ladder with phase frustration f. They show distinct features below and above $f_c \lesssim 0.3$: there are two separate peaks for $f < f_c$, characterized as the Meissner state, and one central peak and tiny bumps at $k = \pm 2\pi f$ for $f > f_c$ in the vortex state.

In the presence of frustration, for $0 < f < f_c$ with $f_c \lesssim 0.3$, however, an interesting feature appears: $n_1(k)$ and $n_2(k)$ have peaks at finite $k = -\pi f$ and $+\pi f$, respectively. This is a one-dimensional quantum liquid of bosons condensed at finite k, in which the lowest energy states are not the zero-momentum state, but the states with finite velocity (or momentum) in opposite directions at each leg. This state can be viewed as the Meissner state with persistent currents at the edges: the currents exist on the legs, $\langle J_{l1}(x)\rangle = -\langle J_{l2}(x)\rangle \neq 0$, but not on the rungs, $\langle J_r(x)\rangle = 0$.

For $f > f_c$, vortices appear. The existence of vortices is reflected in the current-current correlation functions of currents on the rungs (Fig. 3): they oscillate with the period characterized by f in the vortex state and agree fairly well with the theoretically predicted form [22],

$$\langle J_r(x)J_r(0)\rangle \sim \frac{(2\pi/L)^{2/K}\cos(2\pi f x)}{[2 - 2\cos(2\pi x/L)]^{1/K}}, \tag{7}$$

for a finite system with the periodic boundary condition [26], where K is a Luttinger parameter. By fitting data to this form, we find that $1/K = 0.27$ for $f = 4/10$ and $1/K = 0.29$ for $f = 5/10$. However, the thermodynamic properties of the strongly frustrated system with vortices are not readily discriminated from those of the unfrustated system. Both of them are mainly governed by the sharp central peak of $n_\alpha(k)$. In the vortex states, however, $n_\alpha(k)$ has bumps at $k = \pm 2\pi f$ as remnants of the vortex lattice structure. They will, then, serve as useful markers in identifying the vortex state by measuring $n_\alpha(k)$.

Between the Meissner and the vortex states, a transition occurs at $f = f_c$ [22,23]. In a finite-size system, however, the transition is smeared and, near f_c, the central peak and the

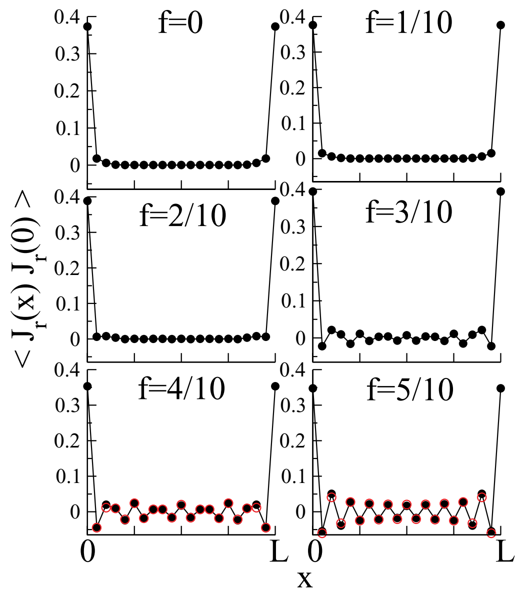

FIG. 3. (Color online) The current-current correlation functions (denoted by filled circles) for the currents on the rungs. In the vortex states for $f > f_c$, a periodic oscillation of currents associated with the vortex lattice structure appears, which is absent in the Meissner states for $f < f_c$, fitted to the theoretical predictions (denoted by open circles).

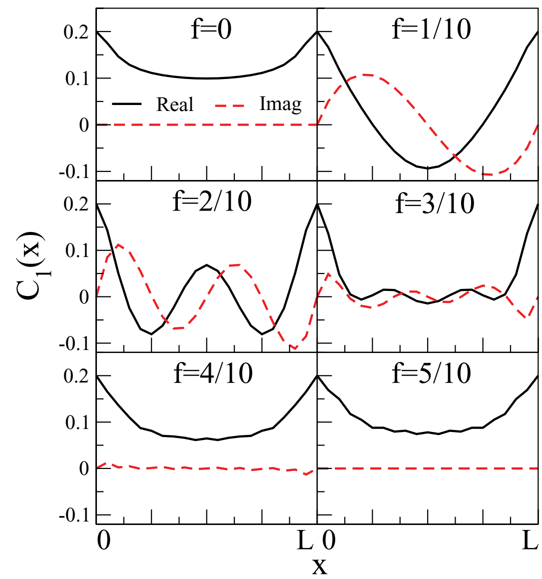

FIG. 4. (Color online) The correlation functions for different f on leg 1.

peaks at finite momenta coexist as shown in Fig. 2 at $f = 3/10$. Therefore we can estimate $f_c \lesssim 0.3$, nearly consistent with the value obtained in the Josephson-junction ladder at commensurate filling [23]. However, it is inappropriate to make a direct comparison of our results to the case at commensurate filling where the coupling to the periodic lattice potential leads to a Kosterlitz-Thouless (KT)-like transition. As shown in Ref. [22], our flux driven transition is a commensurate-incommensurate transition and the exact value of f_c will depend upon the details of the system such as the ratio between the inter- and intraleg hopping amplitudes or the filling fractions.

We may consider the system with $f = p/q$ as the one with q-fold degeneracy in the ground states. Then, as f decreases, for example, by increasing q with fixed $p = 1$, the number of degeneracy increases. This picture fails for a small f: at $f = 0$, the system has one ground state in the limit where q goes infinity. It turns out that the picture of q-fold degeneracy is valid only in the vortex state. As f decreases, there is a transition to the Meissner state which has one ground state with persistent currents at edges, manifested by two peaks in the momentum distribution. The peaks become closer and eventually merge into one central peak as f approaches zero.

The properties of the momentum distribution discussed above show how conducting bosons adjust to phase frustration. In the Meissner state for weak frustration, the phase angles of bosons are twisted gradually along the legs over long distance, as can be described by the correlation functions, $C_\alpha(x)$, in Fig. 4, which have roughly a sinusoidal oscillation with the wavelength $\lambda \sim 2/f$. This results in persistent currents on legs, $\langle J_{l\alpha}(x)\rangle \neq 0$, in directions opposite each other but little twist across the rungs, which may cause relatively larger energy cost. In the vortex state, however, the phase angles are locally twisted to adjust to strong frustration, generating vortices with currents alternating in directions on legs and rungs in the period characterized by f. This makes the phase angles of the matter wave aligned along the legs with tiny tilts so that the real part of the correlation function is always positive while the imaginary part is nearly vanishing. Figure 5 shows a schematic plot of angles and currents of the Meissner and the vortex states which represent two different ways how conducting bosons in a two-leg ladder cope with external orbital frustration.

IV. SUMMARY

In this work, we have studied the ground-state properties of hard-core bosons in a phase frustrated two-leg ladder through a Lanczos calculation of the momentum distributions and the current-current correlation functions. We have found that, in the weakly frustrated case, the ground state is the Meissner state with persistent currents on the legs, which is marked in the momentum distribution by two peaks at finite momenta $k = \pm\pi f$. In the strongly frustrated case, on the other hand, the vortex state is formed, in which vortex lattice structure characterizes the oscillating behavior in the correlation functions of currents on the rungs. In the momentum distribution

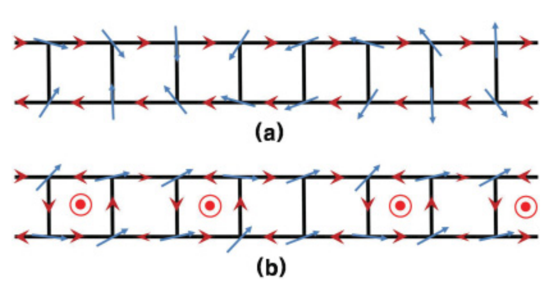

FIG. 5. (Color online) A schematic plot of the currents (red arrows) and the phase angles (blue arrows) in (a) the Meissner state for $f = 2/10$ and (b) the vortex state for $f = 4/10$ with vortices (circles).

of this state, a central peak at zero momentum and tiny bumps at $k = \pm 2\pi f$ appear. Experimentally, these features can be detected and used in a ladder of coupled optical chains in an artificial magnetic field to measure the phase frustration realized on the system and to clarify the properties of the corresponding ground state.

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