Generation of optical crystals and quasicrystal beams: Kaleidoscopic patterns and phase singularity

Y. F. Chen,* H. C. Liang, Y. C. Lin, Y. S. Tzeng, K. W. Su, and K. F. Huang
Department of Electrophysics, National Chiao Tung University, Hsinchu 30050, Taiwan
(Received 21 January 2011; published 10 May 2011)

We explore the feasibility of the generation of pseudonondiffracting optical beams related to crystal and quasicrystal structures. It is experimentally confirmed that optical crystal and quasicrystal beams can be remarkably generated with a collimated light to illuminate a high-precision mask with multiple apertures regularly distributed on a ring. We also found that exotic kaleidoscopic patterns can be exhibited with the high-order quasicrystal beams. More importantly, the structures of phase singularities in optical quasicrystal beams are manifested.

DOI: 10.1103/PhysRevA.83.053813 PACS number(s): 42.65.Jx, 42.15.−i, 42.65.Tg

I. INTRODUCTION

Durnin, Miceli, and Eberly [1] first realized a nondiffracting Bessel beam that is intriguing because it has a comparatively longer invariance length than a conventional Gaussian beam. Since then, pseudonondiffracting optical beams have been extensively studied [2] and applied in diverse fields, such as in optical manipulation [3], biophotonics [4], optical interconnects [5], and optical coherence tomography [6]. Despite a great number of works devoted to the investigation of pseudonondiffracting beams [7], generation of pseudonondiffracting crystal and quasicrystal beams has not been investigated in detail so far. It has been demonstrated that optical quasicrystal beams can be used to create the quasicrystalline substrate potential for developing new materials and devices [8]. Optical quasicrystal patterns have been also employed in the optical induction technique [9,10] for exploring solitons in two-dimensional nonlinear optical lattices [11,12] and Anderson localization phenomena [13]. Furthermore, the patterns with quasicrystalline and kaleidoscopic structures have fascinated scientists, mathematicians, and artists in both ancient and modern cultures [14,15]. Therefore, it is of general interest to realize optical beams with quasicrystalline and kaleidoscopic structures.

Multibeam interference technique is often used to generate desired two-dimensional (2D) optical patterns for fabricating different quasiperiodic structures [16]. However, splitting a laser beam into several components of equal intensity usually requires a complicated experimental setup and suffers from low stability. Alternatively, the experimental technique based on Fourier transform of the amplitude mask has been demonstrated to be an extremely stable method for generating the 2D square photonic lattice [12,17,18]. In this paper, we will employ this approach to generate the desired quasicrystal patterns. We first verify that the pseudonondiffracting optical beams related to crystal and quasicrystal structures can be formed by using a collimated light to illuminate a mask with multiple apertures regularly distributed on a ring. We experimentally fabricate stencil masks with high precision and set up an optical configuration to realize the pseudonondiffracting crystal and quasicrystal beams. Optical pseudonondiffracting beams from low-order crystalline to high-order kaleidoscopic structures are generated excellently. Also, experimental results reveal that various quasicrystal beams with different phase factors can be generated by slightly tilting the mask relative to the optical axis. We finally manifest the structures of phase singularities for some experimental quasicrystal beams. Optical fields with phase singularities, also called optical vortices, have generated a lot of interest in recent years [19,20]. Although the singularities can be created by random scattering, the singular structures carried by quasicrystal beams are believed to play an important role in many applications.

II. THEORETICAL ANALYSIS

The optical fields with two-dimensional crystal or quasicrystal structures in polar coordinates (ρ,ϕ) can be expressed as a sum of plane waves [16]

$$\psi_q(\rho,\phi) = \sum_{s=0}^{q-1} A_s e^{i\varphi_s} e^{i\mathbf{K}_s\cdot\boldsymbol{\rho}}, \tag{1}$$

where $\mathbf{K}_s = (K\cos(2\pi s/q), K\sin(2\pi s/q))$, $\boldsymbol{\rho} = (\rho\cos\phi, \rho\sin\phi)$, q is an integer number, and A_s, $\mathbf{K}_s$, and φ_s are the amplitude, the transverse wave vector, and the initial phase of the sth plane wave, respectively. The fields $\psi_q(\rho,\phi)$ with $q = 2$, 3, 4, and 6 yield periodic lattices, which correspond to the standard 2D crystal structures. All other values of q correspond to quasicrystals. Unlike periodic crystals, the quasicrystals are not invariant under spatial translation. For the convenience of description, the fields $\psi_q(\rho,\phi)$ with the most representative parameters of $A_s = 1/q$ and $\varphi_s = 0$ are expressed as

$$\Psi_q(\rho,\phi;K) = (1/q)\sum_{s=0}^{q-1} e^{iK\rho\cos[\phi-(2\pi s/q)]}. \tag{2}$$

Figure 1 depicts the calculated patterns for the intensity $|\Psi_q(\rho,\phi;K)|^2$ with $q = 3$, 4, 5, 6, 7, and 8. It can be seen that the patterns with $q = 3$, 4, and 6 display translational symmetry, whereas the patterns with $q = 5$ and 7 exhibit quasicrystal structures.

The field of a general pseudonondiffracting beam can be generated with a specific input field transformed by a lens. Consider a coherent light with field distribution expressed by polar coordinates $u_o(\rho',\phi')$ to be produced in the focal plane in front of the lens with the focal length f. By Fresnel synthesis

*yfchen@cc.nctu.edu.tw

 ©2011 American Physical Society

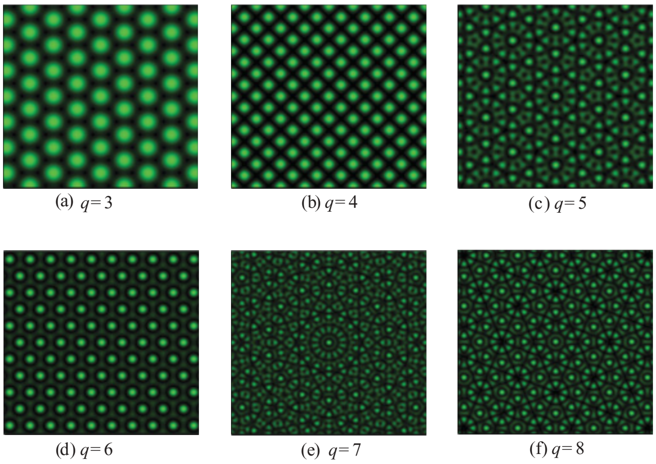

FIG. 1. (Color online) Numerical patterns for the crystal and quasicrystal intensity $|\Psi_q(\rho,\phi;K)|^2$ with $q=3$, 4, 5, 6, 7, and 8.

based on Fourier optics, the complex field $u(\rho,\phi,z)$ behind the lens at a distance z is given by

$$u(\rho,\phi,z)=\frac{-i}{\lambda f}e^{ik(f+z)}\int\int u_o(\rho',\phi')e^{i\frac{k\rho'^2}{2f}\left(1-\frac{z}{f}\right)} \times e^{-i\frac{2\pi\rho\rho'}{\lambda f}\cos(\phi-\phi')}\rho'\,d\phi'\,d\rho', \quad (3)$$

where $k=2\pi/\lambda$ and λ is the wavelength of the coherent light. Durnin, Miceli, and Eberly [1] obtained a circular hollow beam $u_o(\rho',\phi')=\delta(\rho'-a)$ by directly illuminating a thin circular slit with collimated light. Substituting this hollow beam into Eq. (3), the output field can be found to be an approximate Bessel beam propagating in the z direction, i.e.,

$$u(\rho,\phi,z)=\frac{-i2\pi a}{\lambda f}e^{ik(f+z)}e^{i\frac{ka^2}{2f}\left(1-\frac{z}{f}\right)}J_0\left(\frac{2\pi a}{\lambda f}\rho\right), \quad (4)$$

where $J_0(\cdot)$ is the Bessel function of zero order of the first kind.

When a mask with multiple apertures (small circular holes) regularly distributed on a ring is illuminated with collimated light, the field just after the mask can be approximately expressed as $u_o(\rho',\varphi')=(1/q)\delta(\rho'-a)\sum_{s=0}^{q-1}\delta(\varphi'-\varphi_s)$. Substituting this field distribution into Eq. (3), the output field can be derived to be given by

$$u(\rho,\phi,z)=\frac{-ia}{\lambda f}e^{ik(f+z)}e^{i\frac{ka^2}{2f}\left(1-\frac{z}{f}\right)}\Psi_q^*\left(\rho,\phi;\frac{2\pi a}{\lambda f}\right). \quad (5)$$

Equation (5) indicates that the output field represents a pseudonondiffracting beam propagating in the z direction with the transverse pattern revealing a crystal or quasicrystal structure.

III. EXPERIMENTAL RESULTS AND DISCUSSION

To realize the pseudonondiffracting crystal and quasicrystal beams, we set up an optical system that is essentially similar to Durnin's approach, as depicted in Fig. 2. The light source was a linearly polarized 20-mW He-Ne laser with a wavelength of 632.8 nm. A beam expander was employed to reduce the beam divergence to less than 0.1 mrad. Metal masks of different forms were fabricated with a laser stencil-cutting

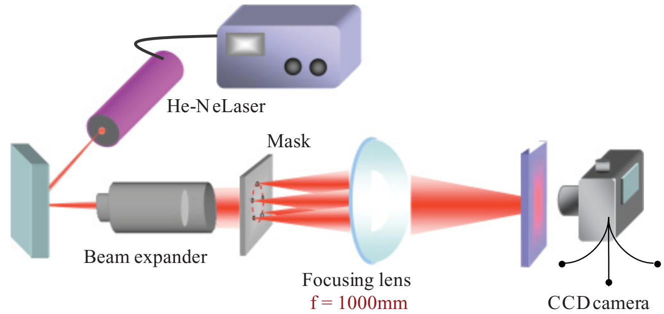

FIG. 2. (Color online) Experimental setup for generating optical crystal and quasicrystal beams.

machine. The radii of the aperture and the ring are 0.1 and 5.0 mm, respectively. The focal length of the lens is 1000 mm. Interference patterns formed in the region behind the focal lens were imaged by a CCD camera.

Figure 3 depicts the interference patterns for crystal and quasicrystal structures observed in the experiment under the condition of the optimal alignment. It can be seen that the experimental observations agree very well with the numerical patterns shown in Fig. 1. The good agreement between the experimental and theoretical patterns confirms that the pseudonondiffracting optical beams related to crystal and quasicrystal structures can be manifestly generated with multiple tiny apertures regularly distributed on a ring. Since the optical configuration is essentially similar to Durnin's approach, the overall properties of diffraction lengths are nearly the same as a pseudonondiffracting Bessel beam. Figure 4 shows the experimental generated quasicrystal patterns for $q=5$ at different propagation distances along z. The patterns can be seen clearly to be diffraction free over a finite distance, $z_{\max}$, the so-called propagation-invariant region, behind which the center of the pattern rapidly turns vague. The finite distance of the nondiffracting region comes from the finite size of the beam. The maximum distance $z_{\max}$ can be derived geometrically to be given by $z_{\max}=R/\tan\theta$, where R is the

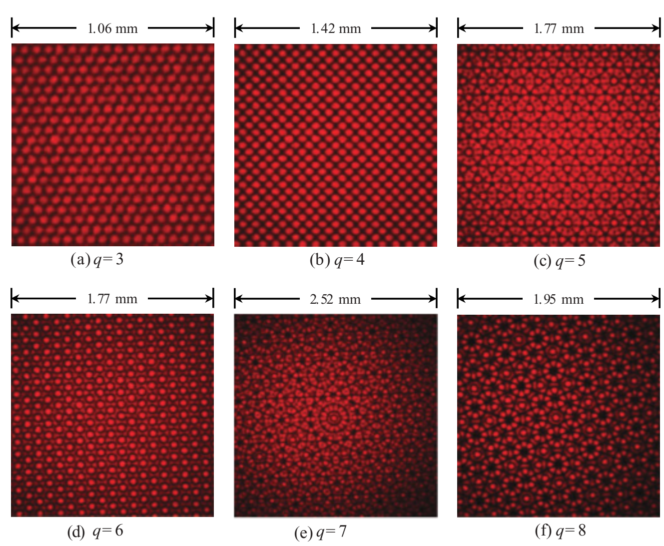

FIG. 3. (Color online) Experimental patterns for crystal and quasicrystal structures observed under the optimal alignment.

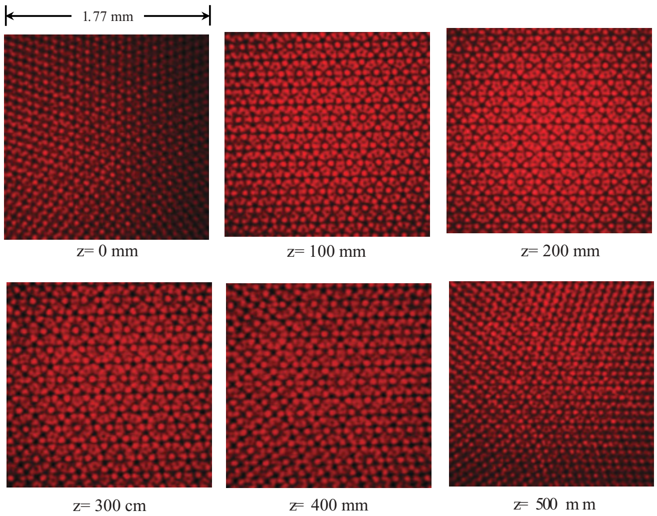

FIG. 4. (Color online) Experimental generated quasicrystal patterns for $q = 5$ at different propagation distances along z.

radius of each interference beam in the plane of the focal lens and θ is the inclination angle of each interference beam with respect to the optical axis. In our experiment, R and θ are approximately 2.0 mm and 5 mrad, respectively. As a result, $z_{\max}$ can be estimated to be approximately 400 mm, which agrees very well with the experimental observation, as shown in Fig. 4.

It is intriguing to explore the characteristics of experimental quasicrystal patterns for the large value of the index q. Figures 5(a)–5(c) and 5(a′)–5(c′) show the experimental and numerical quasicrystal patterns for $q = 16$, 21, and 30. The high-order quasicrystal beams display exotic kaleidoscopic patterns. The central part of the high-order patterns can be seen to have some resemblance to a Bessel beam. This feature comes from the limiting behavior $\lim_{q\to\infty} \Psi_q(\rho,\varphi;K) = J_0(K\rho)$.

By slightly tilting the mask relative to the optical axis, rich patterns related to various quaiscrystal structures can be observed. The experimental patterns for different tiny tilt angles can be reconstructed with the fields $\psi_q(\rho,\phi)$ with some specific phase parameters φ_s, although the relationship between the experimental tilt angle and the phase parameters φ_s cannot be analytically expressed. Figures 6(a) and 6(a′) show two experimental octagonal patterns obtained for two

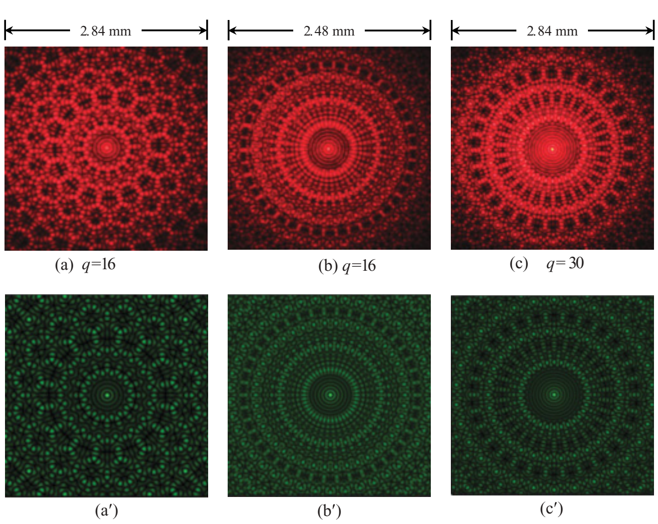

FIG. 5. (Color online) Experimental [(a)–(c)] and numerical [(a′)–(c′)] quasicrystal patterns for $q = 16$, 21, and 30.

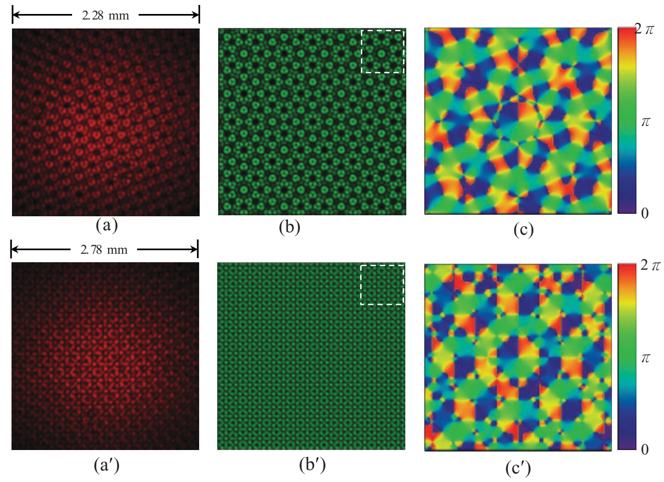

FIG. 6. (Color online) (a), (a′) Two experimental octagonal patterns obtained for two tilt angles. (b), (b′) Reconstructed patterns with the fields $\psi_8(\rho,\phi)$ with $\varphi_s = s\pi/4$ and $\varphi_s = s\pi/2$, respectively. (c), (c′) Contour plots of phase field $\Theta(\rho,\phi)$ for the boxed regions shown in (b) and (b′), respectively.

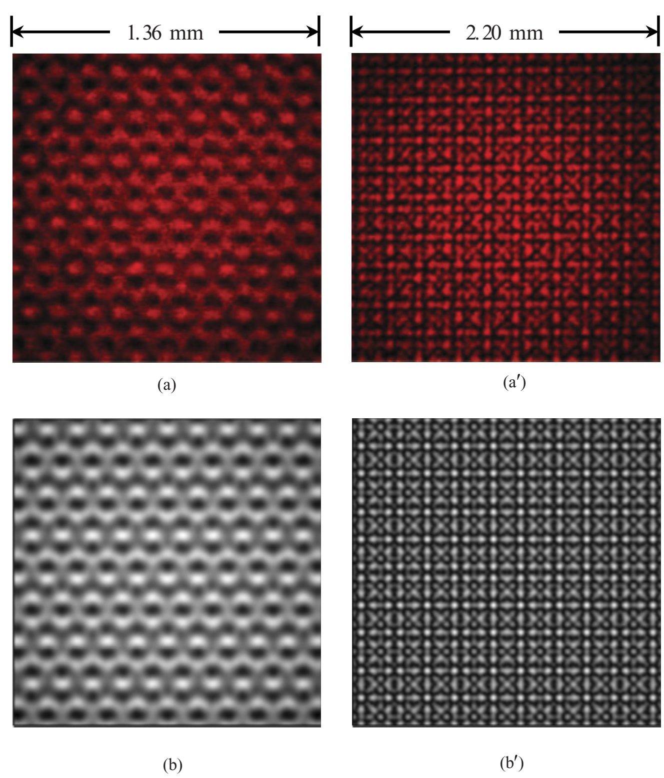

FIG. 7. (Color online) (a), (a′) Experimental results for the interference patterns between the inclined plane wave and the quasicrystal beams shown in Figs. 6(a) and 6(a′), respectively. (b), (b′) Theoretical results corresponding to the patterns shown in (a) and (a′), respectively.

tilt angles. These two patterns can be reconstructed with the fields $\psi_8(\rho,\phi)$ with $\varphi_s = s\pi/4$ and $\varphi_s = s\pi/2$, as shown in Figs. 6(b) and 6(b′).

The present crystal and quasicrystal beams generally belong to the complex optical fields. Phase singularities of the complex fields are characterized by isolated dark spots, where phases are ambiguous and amplitudes are zero. Phase singularities are conventionally described in terms of the phase angle field $\Theta(\rho,\phi) = \arctan\{\text{Im}[\psi_q(\rho,\phi)]/\text{Re}[\psi_q(\rho,\phi)]\}$, where $\text{Re}[\psi_q(\rho,\phi)]$ and $\text{Im}[\psi_q(\rho,\phi)]$ are the real and imaginary parts of the field $\psi_q(\rho,\phi)$. The vortices of $\Theta(\rho,\phi)$ are the singularities at which the phase angle of the field $\psi_q(\rho,\phi)$ is undefined. Figures 6(c) and 6(c′) depict the contour plots of phase fields $\Theta(\rho,\phi)$ for the boxed regions shown in Figs. 6(b) and 6(b′) to display the feature of phase singularities. We also employed an inclined plane wave to perform interference with the generated quasicrystal beams for exhibiting the phase structure. Figures 7(a) and 7(a′) depict the experimental results for the interference patterns between the inclined plane wave and the quasicrystal beams shown in Figs. 6(a) and 6(a′). The theoretical patterns are also depicted in Figs. 7(b) and 7(b′) for comparison. The good agreement confirms that the generated waves do indeed have the phase structure of quasicrystal beams. Optical vortex beams that possess orbital angular momentum due to a phase singularity have been extensively used in the study of optical tweezers, trapping and guiding of cold atoms, and entanglement states of photons. Therefore, the generated quasicrystal beams are expected to be potentially beneficial to future applications.

IV. CONCLUSION

In conclusion, we have explored the generation of pseudonondiffracting optical beams related to crystal and quasicrystal structures. We have excellently generated pseudonondiffracting optical beams from low-order quasicrystal to high-order kaleidoscopic structures by using a collimated light to illuminate a high-precision stencil mask with multiple apertures regularly distributed on a ring. We also experimentally found that tilting the mask relative to the optical axis could lead to the generation of various quasicrystal beams with different phase factors. Finally, we employed some experimental quasicrystal beams to manifest the structures of phase singularities.

ACKNOWLEDGMENTS

The authors thank the National Science Council for their financial support of this research under Contract No. NSC-97-2112-M-009-016-MY3.

[1] J. Durnin, J. J. Miceli, and J. H. Eberly, Phys. Rev. Lett. **58**, 1499 (1987).

[2] S. Lopez-Aguayo, Y. V. Kartashov, V. A. Vysloukh, and L. Torner, Phys. Rev. Lett. **105**, 013902 (2010).

[3] V. Garcés-Chávez, D. McGloin, H. Melville, W. Sibbett, and K. Dholakia, Nature (London) **419**, 145 (2002).

[4] X. Tsampoula, V. Garcés-Chávez, M. Comrie, D. Stevenson, M. B. Agate, F. J. Gunn-Moore, C. T. A. Brown, and K. Dholakia, Appl. Phys. Lett. **91**, 053902 (2007).

[5] C. Yu, M. R. Wang, A. J. Varela, and B. Chen, Opt. Commun. **177**, 369 (2000).

[6] Z. Ding, H. Ren, Y. Zhao, J. S. Nelson, and Z. Chen, Opt. Lett. **27**, 243 (2002).

[7] Z. Bouchal, Czech. J. Phys. **53**, 537 (2003).

[8] J. Mikhael, J. Roth, L. Helden, and C. Bechinger, Nature (London) **454**, 501 (2008).

[9] B. Freedman, R. Lifshitz, J. W. Fleischer, and M. Segev, Nat. Mater. **6**, 776 (2007).

[10] N. K. Efremidis, S. Sears, D. N. Christodoulides, J. W. Fleischer, and M. Segev, Phys. Rev. E **66**, 046602 (2002).

[11] J. W. Fleischer, T. Carmon, M. Segev, N. K. Efremidis, and D. N. Christodoulides, Phys. Rev. Lett. **90**, 023902 (2003).

[12] D. N. Neshev, T. J. Alexander, E. A. Ostrovskaya, Y. S. Kivshar, H. Martin, I. Makasyuk, and Z. Chen, Phys. Rev. Lett. **92**, 123903 (2004).

[13] T. Schwartz, G. Baral, S. Fishman, and M. Segev, Nature (London) **446**, 52 (2007).

[14] P. J. Lu and P. J. Steinhardt, Science **316**, 981 (2007).

[15] G. M. Zaslavsky, R. Z. Sagdeev, D. A. Usikov, and A. A. Chernikov, *Weak Chaos and Quasi-Regular Patterns* (Cambridge University Press, Cambridge, UK, 1991).

[16] X. Wang, C. Y. Ng, W. Y. Tam, C. T. Chan, and P. Sheng, Adv. Mater. **15**, 1526 (2003).

[17] H. Martin, E. D. Eugenieva, Z. Chen, and D. N. Christodoulides, Phys. Rev. Lett. **92**, 123902 (2004).

[18] X. Qi, S. Liu, G. Zhang, R. Guo, Z. Liu, L. Zhou, and Y. Li, Appl. Phys. Lett. **91**, 131111 (2007).

[19] J. F. Nye and M. V. Berry, Proc. R. Soc. London A **336**, 165 (1974).

[20] M. S. Soskin and M. V. Vasnetsov, in *Singular Optics*, edited by E. Wolf, Progress in Optics (Elsevier, Amsterdam, 2001), Vol. 42, p. 219.