Production of bright entangled photons from moving optical boundaries

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We discuss a mechanism of generating two separable beams of light with a high degree of entanglement in momentum using a fast and sharp optical boundary. Three regimes of light generation are identified depending on the number of resonant interactions between the optical perturbation and the electromagnetic field. The intensity of the process is discussed in terms of the relevant physical parameters: variation of refractive index and apparent velocity of the optical boundary. Our results suggest a different class of generation entangled light that is robust against thermal degradation by exciting zero point fluctuations using parametric resonant optical modulations.

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I. INTRODUCTION

Many of the theoretical schemes and experimental applications being proposed and developed in the context of quantum information (QI) (including quantum computation and information processing [1], teleportation [2], etc.) rely on the generation of entanglement between different quantum systems. Though entanglement can arise in nature even from the simplest interactions and even at high temperature [3,4], the degree of entanglement achieved is usually very small. An important exception are photons, which combined with their resilience to thermal effects, can be used, for example, to establish quantum communication at long distances [5]. Until now, entangled photons have been produced experimentally via parametric down conversion (PDC), which is in general a nonlinear process with small efficiency [6,7].

This paper is motivated by the need of sources of photonic entanglement with finer brightness and improved contrast [8,9]. In our proposal, high-quality two-photon entangled states are spontaneously emitted out of the vacuum (or a thermal state) by a superluminal modulation of the refractive index of an optical medium, such as a semiconductor where the sudden creation of electron-hole pairs can reduce the refractive index from ∼3.5 to almost 0 [10], or a gas swept by a laser or electron beam and producing a plasma via photoionization [11–13]. Recently, a Gaussian beam was sent into a plasma to induce a superluminal two-photon ionization front and used for optical-to-THz photon conversion [14,15]. We show that similar techniques can generate highly entangled photons with a mean number of pairs that can be made arbitrarily high by increasing the sharpness of the induced refractive index variation and by tuning the apparent velocity of the optical modulation and the phase velocity of the electromagnetic modes (superluminal resonance). For current state-of-art experimental values, our estimates suggest that it is possible to produce photons in excess of 10^{10} s^{-1}. The main limitation comes from the difficulty in producing an optical modulation close enough to the resonance conditions. These results open doors to the efficient generation of entangled photons with very high signal-to-noise ratios via time-dependent optical perturbations, and to the potential application to QI and quantum metrology experiments.

II. OPTICAL MOVING BOUNDARIES

Recently, a series of papers [16–18] introduced the concept of time refraction (TR) to describe how the classical and quantum properties of light are altered by the sudden change of the optical properties of a medium. TR results from the symmetry between space and time, extending the usual concept of refraction into the time domain. Like the Unruh effect [19], the Hawking mechanism [20], and the dynamical Casimir effect [21], the quantum theory of TR predicts the excitation of virtual particles from the turmoil of zero-point fluctuations (ZPFs) and the emission of pairs of real counterpropagating photons, which (as we will show) are highly entangled. The number of pairs emitted is proportional to the variation of the refractive index of light associated with the optical perturbation. For any realistic experimental parameter, the mean photon number produced in the optical domain from the vacuum state is smaller than 1. To overcome this limitation, a different process of excitation of ZPF was proposed in a recent work [22], using a nonaccelerated optical boundary moving with *apparent* superluminal velocity across an optical medium. Like TR, this effect also leads to the emission of photon pairs, but now the moving optical boundary works as a relativistic partial mirror, producing a considerable Doppler shift, altering radically the intensity of the interaction between light and matter, and yielding a potentially measurable number of photons by choosing adequately the velocity of the optical boundary.

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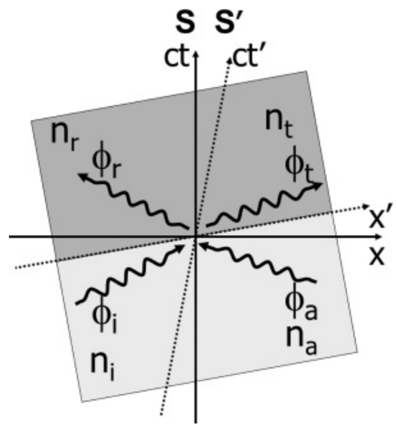

FIG. 1. Space-time schematic diagram of superluminal space-time refraction. In the S frame (bold), the optical perturbation is observed as moving along the x axis (from left to right) with apparent velocity u; whereas in the S' frame (dashed), the optical perturbation alters the refractive index of the medium simultaneously for all points of space at instant $t' = 0$. Both frames can be related using a standard Lorentz boost with $\beta = v_\infty/c = -c/u < 1$.

Extending the results in [22] from a one-dimensional to a three-dimensional geometry, we consider an infinite optical medium swept by an optical perturbation, described as a sharp variation of the refractive index of the medium with *apparent* superluminal velocity $\mathbf{u}$ (see Fig. 1). In this context, the apparent velocity $\mathbf{u}$ describes a delay of the change of refractive index between different points of space and does not refer to an actual velocity of propagation of the optical profile. Hence $u \equiv |\mathbf{u}|$ can take values arbitrarily large, even larger than c.

We describe the process of interaction between the ZPF and the optical perturbation in a reference frame S', with velocity $v_\infty \equiv -c^2/u < c$ relative to the laboratory reference frame S, where this optical boundary is perceived as moving with a infinite velocity: $u' = \lim_{v \to v_\infty} (u+v)/[1+(vu)/c^2] \to \infty$. As a consequence of the relativistic phase invariance, the refractive index of the medium in the S frame and in the S' frame (respectively, n and n') are different [17]

$$n' = n\frac{[\gamma^2(\cos\theta - \beta/n)^2 + \sin^2\theta]^{1/2}}{\gamma^2(1-\beta n\cos\theta)}, \quad (1)$$

where $\beta \equiv v/c = -c/u$, $\gamma = (1-\beta^2)^{-1/2}$, and θ is the angle between the velocity $\mathbf{u}$ and the wave vector $\mathbf{k}$.

In the S' frame, the problem is identical to a TR and can be solved by imposing the continuity of the dielectric displacement and the magnetic induction fields and corresponding field operators [23] during the time discontinuity of the refractive index, or equivalently, by imposing phase-matching conditions at the optical boundary. Back in the S frame, the optical perturbation can be perceived as a four-port device, coupling two initial complex plane-wave modes: $\phi_i(\mathbf{r}) = \exp[-i\mathbf{k}_i \cdot \mathbf{r}]$ and $\phi_a(\mathbf{r}) = \exp[-i\mathbf{k}_a \cdot \mathbf{r}]$ existing for $\mathbf{r} > \mathbf{u}t$, with two final complex plane-wave modes $\phi_t(\mathbf{r}) = \exp[-i\mathbf{k}_t \cdot \mathbf{r}]$ and $\phi_r(\mathbf{r}) = \exp[-i\mathbf{k}_r \cdot \mathbf{r}]$ existing for $\mathbf{r} < \mathbf{u}t$, which satisfy

$$\mathbf{k}_t = k_i\gamma^2[f + \sigma_{it}g_t]\,\mathbf{u}_\parallel + k_i\sin\theta_i\mathbf{u}_\perp, \quad (2)$$

$$\mathbf{k}_r = -k_i\gamma^2[f - \sigma_{ir}g_r]\,\mathbf{u}_\parallel - k_i\sin\theta_i\mathbf{u}_\perp, \quad (3)$$

$$\mathbf{k}_a = -k_i\gamma^2[f + g_i]\,\mathbf{u}_\parallel - k_i\sin\theta_i\mathbf{u}_\perp, \quad (4)$$

where $f \equiv (\cos\theta_i - \beta/n_i)$, $\sigma_{it} \equiv h_i/h_t$, $\sigma_{ir} \equiv h_i/h_r$, $h_{i,t,r} \equiv [\gamma^2(\cos\theta_i - \beta/n_{i,t,r})^2 + \sin^2\theta_i]^{1/2}$, $g_{t,r} \equiv \beta(1-\beta n_{t,r}\cos\theta_i)/n_{t,r}$, and θ_i now is the angle between the velocity of the optical perturbation $\mathbf{u}$ and the wave vector $\mathbf{k}_i$. The different values of the refractive index n_i, n_t, n_r, and n_a for the incident, transmitted, reflected, and anti-incident waves take into account the dispersion of the medium prior to the optical perturbation and after it has passed.

Like Eq. (1), Eqs. (2)–(4) are also derived from the invariance of the phase of light between any two different inertial frames [17] and correspond to a double Doppler shift. For values $\theta_i \neq 0$ and π, the expressions for θ_t and θ_r are calculated as

$$\theta_t = \arctan[\sin\theta_i/\gamma^2(f + \sigma_{it}g_t)], \quad (5)$$

$$\theta_r = \arctan[\sin\theta_i/\gamma^2(f - \sigma_{ir}g_r)]. \quad (6)$$

These expressions correspond to the generalized Fresnel formula for a moving superluminal partial mirror.

Using the continuity conditions for dielectric displacement and the magnetic induction fields [16,23] at time $\mathbf{r} = t\mathbf{u}$, the annihilation and creation operators for these modes can be related as

$$a_i = Aa_t - Ba_r^\dagger, \quad a_a = Aa_r - Ba_t^\dagger, \quad (7)$$

where $A = (1+\alpha^2)/2\alpha$, $B = (1-\alpha^2)/2\alpha$, and $\alpha = [n_i^2 g_i h_i/n_t^2 g_t h_t]^{1/2}$, satisfying $A^2 - B^2 = 1$.

As demonstrated in Refs. [24,25], the two-mode squeezing transformation (7) implies that, after the optical perturbation has passed, an initial vacuum can be expressed in terms of the new eigenstates of the field as

$$|0\rangle_i|0\rangle_a = \sum\nolimits_n C_n|n\rangle_t|n\rangle_r, \quad (8)$$

with $C_n = \sqrt{1-|z|^2}z^n$ and $z = B/A$. Equation (8) implies the emission of photon pairs moving along the different directions of $\mathbf{k}_t$ and $\mathbf{k}_r$, according to Eqs. (2)–(4). The mean photon number for wave vectors $\mathbf{k}_t$ and $\mathbf{k}_r$ is

$$\langle N_t\rangle = \langle N_r\rangle = \frac{|z|^2}{1-|z|^2} = \frac{\left[n_i^2h_ig_i - n_t^2h_tg_t\right]^2}{4n_i^2n_t^2h_ih_tg_ig_t}. \quad (9)$$

According to Eq. (9) the number of photons emitted diverges for $z \to 1$. In the one-dimensional case studied in [22] this could only be achieved if a perfect matching between the velocity of the optical perturbation u and n_i such that $\beta n_i = cn_i/u \to 1$. However, in the three-dimensional case there is an extra degree of freedom corresponding to the angle between the wave vector and the direction of the apparent motion of the optical perturbation, and the condition $z \to 1$ can be achieved for both $\beta n_i\cos\theta_i^{\mathrm{res}} \to 1$ and $\beta n_r\cos\theta_r^{\mathrm{res}} \to 1$. Unlike the case of TR, the photon emission produced by a superluminal optical perturbation is not limited by the maximum variation of refractive index produced by optical perturbation. Instead, when the phase velocity of the waves ϕ_i and ϕ_r along $\mathbf{u}$ are identical to $\mathbf{u}$, corresponding, respectively, to $\beta n_i\cos\theta_i^{\mathrm{res}} \to 1$ and $\beta n_r\cos\theta_r^{\mathrm{res}} \to 1$, the optical perturbation and the waves ϕ_i and ϕ_r move together and can interact for longer times, producing an arbitrarily large number of photons. This process can be described as a form of superluminal resonance. We

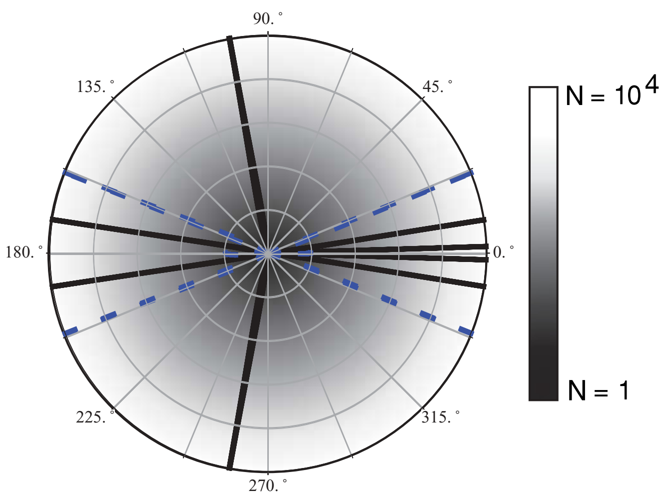

FIG. 2. (Color online) Emission spectrum: Angular distribution of the number of photons emitted by the optical boundary corresponding to a change of refractive index from $n_i = n_a = 1.1$ to $n_t = n_r = 1.5$ and for $\beta = 0.9$ (dashed) and $\beta = 0.99$ (bold). Notice that the emission is predominantly concentrated in a very small solid angle around the resonance angles. With $\beta = 0.9$ there is only the resonance for $\beta n_i \cos\theta_i^{\rm res} \to 1$; whereas with $\beta = 0.99$, both resonances $\beta n_i \cos\theta_i^{\rm res} \to 1$ and $\beta n_r \cos\theta_r^{\rm res} \to 1$ exist.

identify three regimes: (i) for $\beta n_i < 1$ and $\beta n_r < 1$, there are no resonances; (ii) for either $\beta n_i > 1$ or $\beta n_r > 1$, there is only one pair of resonant emission angles; and (iii) for both $\beta n_i > 1$ and $\beta n_r > 1$, two pairs of resonant light are emitted. An extra resonance also exists for $\beta = n$ in media such as plasmas, where the refractive index is lower than 1 [22]; for simplicity we neglect this resonance herein. These resonances can be achieved for a wide range of experimental parameters and configurations. The angular distribution corresponding to Eq. (9) is represented in Fig. 2 where we can clearly identify $\theta_t^{\rm res}$ and $\theta_r^{\rm res}$, calculated from $\theta_i^{\rm res}$ using Eqs. (5) and (6), respectively. Notice that emission is mainly limited to a narrow solid angle, resulting in collimated beams.

III. PHOTONIC ENTANGLEMENT GENERATION

The field can be separated into two subsystems (S and S') corresponding to the two distinct sets of photons emitted, i.e., ϕ_r and ϕ_t. Depending on the initial state of the field, these two subsystems may become entangled after the optical perturbation. We discuss and compare the degree of entanglement between two situation: an initial vacuum and a thermal state (which is the experimental case).

According to Eq. (7), an initial vacuum state is changed into another pure state for which the entanglement entropy $E_{VN}(\rho_{SS'}) \equiv -\mathrm{Tr}[\rho^S \ln \rho^S]$ (with $\rho^S = \mathrm{Tr}_{S'}[\rho_{SS'}]$) is the canonical entanglement measure [26], yielding $E_{VN} = \ln(1 + \langle N_t\rangle)(1 + \langle N_t\rangle) - \langle N_t\rangle \ln\langle N_t\rangle$. Notice that E_{VN} is basically the Shannon entropy introduced by increasing a photon pair in the system. For $z \to 1$, the entanglement diverges as the system approaches the resonance condition and the maximal entanglement state is achieved, i.e.,

$$\lim_{z\to 1}|0\rangle_i|0\rangle_a = \lim_{N\to\infty}\sum_{n=0}^{N}\frac{1}{\sqrt{N}}|n\rangle_t|n\rangle_r. \quad (10)$$

If the system is initially in a thermal state of both wave modes, ϕ_i and ϕ_a, i.e., $\rho_{ia}(\bar{n}) = \rho(\bar{n}) \otimes \rho(\bar{n})$, with

$$\rho(\bar{n}) = \frac{1}{1+\bar{n}}\sum_{n=0}^{\infty}\left(\frac{\bar{n}}{1+\bar{n}}\right)^n |n\rangle\langle n|, \quad (11)$$

where $\bar{n}$ is the thermal mean occupancy, then after the optical perturbation has passed, the state $\rho_{\rm tr}(z)$ describing the ϕ_t and ϕ_r modes becomes $\rho_{\rm tr}(z) = S(z)\rho_{ia}(\bar{n})S(z)^\dagger$, which is a squeezed thermal state [27] and for which E_{VN} is not an adequate entanglement measure [28]. However $\rho_{\rm tr}$ is a Gaussian state, and its entanglement can be completely characterized using continuous-variable methods (see [29] for a review), namely, via the *logarithmic negativity*, $E_N(\rho) = \max[0, -\ln\mu]$, where μ is the smallest symplectic eigenvalue of the Gaussian state $\rho_{\rm tr}$. The expression for μ (see [30] for a derivation) is $\mu = (2\bar{n} + 1)\exp(-2\,\mathrm{arctanh}\, z)$. The latter defines a thermal occupancy n_c, above which all entanglement vanishes, yielding $2\bar{n}_c + 1 = \exp(2\,\mathrm{arctanh}\, z)$. Close to resonance ($z \to 1$) the maximum allowable thermal occupancy diverges $\bar{n}_c \to \infty$; entailing that entanglement extraction from optical boundaries is very robust regarding temperature by choosing a sufficiently high z.

IV. DISCUSSION OF EFFICIENCY

Now we consider an optical perturbation in a frame S of the form $n(x - ut) = n_0 + \delta n\, f[K(x - ut)]$, where K is a spatial scale describing the sharpness and duration of the optical perturbation and $f(x) = 0$ for $x \leqslant 0$, $f(x) = x$ for $0 < x \leqslant 1$, and $f(x) = 1$ for $x > 1$. In the S' frame, the creation and annihilation operators in the interaction picture satisfy [31]

$$\frac{d}{dt'}a_t = \nu(t')a_r^\dagger, \quad \frac{d}{dt'}a_r^\dagger = \nu^*(t')a_t, \quad (12)$$

with $\nu(t') = \frac{1}{2}\exp[2i\psi(t')][\frac{d}{dt'}\ln n']$, where $\psi(t')$ is a phase. The total photon number $N = N_t + N_r$ satisfies

$$\frac{d^2}{dt'^2}(N + 2) = 4|\nu(t')|^2(N + 2). \quad (13)$$

For a small variation of refractive index (i.e., $\delta n \ll n_0$), the total number of photons produced and the maximum and average rate of photon generation from initial thermal states are, respectively,

$$N_{\rm total} \approx (N_0 + 2)\,\cosh(\eta\delta n), \quad (14)$$

$$R_{\max} \equiv \frac{dN}{dt} \approx (N_0 + 2)\eta\delta n K u\,\sinh(\eta\delta n), \quad (15)$$

$$R_{\rm mean} \approx (N_0 + 2)Ku\gamma^{-1}\,\cosh(\eta\delta n), \quad (16)$$

where $\eta \equiv \frac{d}{dn_0}\ln n_0'$. For conditions close to the resonances, $\eta \sim 1/(n_0\Delta)$, where $\Delta \equiv 1 - \beta n\cos\theta$ is the detuning from the resonance conditions.

V. CONCLUSIONS

We presented an emission mechanism of entangled radiation using a sharp optical perturbation with an apparent superluminal velocity. The emission spectrum and the emissivity depend on the apparent velocity and the change of refractive index of the optical perturbation. These results extend those of Ref. [22] from a one-dimensional configuration to one that includes all complex plane-wave modes in a three-dimensional space and is valid for an arbitrary dispersive medium. For our particular configuration, the optimum direction of emission is defined by the resonances $\Delta_i \to 0$ and $\Delta_r \to 0$. The resonance angles θ_t^{res} and θ_r^{res} correspond to both the best radiance and to the optimally entangled photons. From a purely theoretical point of view, this process has considerable advantages over PDC as a source of entangled light, namely, since it is capable of delivering two well separable and highly entangled beams with large intensities. In our case the photons are entangled in momentum, whereas in PDC the photons are entangled in polarization; however, these two types of entanglement can be interconverted [32]. From a more experimental point of view, it is not easy to produce a sharp and sudden optical perturbation at scales inferior to the optical wavelengths to allow the large number of photon pairs necessary to make this process competitive with PDC. A conservative estimate based on parameters from present day experimental demonstrations of superluminal ionization fronts [14,15,33] (with $\beta \approx 0.9995$, $K/c \approx 0.02$ fs, c the speed of light and assuming $\delta n/n_0 \approx 1\%$) predicts photon yields in excess of $R_{\text{max}} \sim 10^{10}$ s^{-1} ($R_{\text{mean}} \sim 10^9$ s^{-1}) for $\Delta \sim 0.01$. Moreover, a recent work has shown that this quantum mechanism of extracting photon pairs out of ZPF can be extended to optical perturbations with arbitrary shape as long as they have an *apparent* superluminal velocity [31]. These results suggest the possibility of generating high-intensity entangled photons via specific time-dependent optical perturbations, including the dynamical Casimir effect.

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