Dependence of Berry's phase on the sign of the *g* factor for conical rotation of a magnetic field, measured without any dynamical phase shift

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Berry's phase for a whole turn in a conical rotation of the magnetic field with a semiangle θ has been clearly manifested free from the dynamical phase shift using the magnetic-field-insensitive two-photon transitions between sodium-ground hyperfine states having different signs of the g factors. The solid angles for states with a positive g factor and with a negative g factor are verified to be $2\pi(1-\cos\theta)$ and $-2\pi(1+\cos\theta)$, respectively, for a right-handed rotation of a magnetic field and a semiangle of $0 \leqslant \theta \leqslant \pi/2$.

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I. INTRODUCTION

Since Berry predicted in 1984 that a particle of any spin in an eigenstate of a magnetic field slowly rotated around a circuit C will acquire a geometrical phase factor in addition to the dynamical phase factor [1], many experimental demonstrations of Berry's phase have been carried out using polarized light [2], nuclear quadrupole resonance [3], neutron interferometers [4,5], and other physical systems [6]. Furthermore, in 1992, Miniatura *et al*. first demonstrated the atomic Berry's phase using a longitudinal Stern-Gerlach atomic interferometer [7]. In most studies, it was verified that Berry's phase is given by $\gamma = -m\Omega(C)$, where m is the spin component along the magnetic field and $\Omega(C)$ is the solid angle, not depending on the magnitude of the g factor. If C is a right-handed circuit around a cone of semiangle θ, the solid angle is $\Omega(C) = 2\pi(1-\cos\theta)$. This solid angle is true for neutrons [4] whose g factors are negative and for atoms with positive atomic g factors. (It should be noted that the sign of the atomic g factor is conventionally reversed to that of an elementary particle.) However, it is defined by the complementary larger solid angle of $\Omega = -2\pi(1+\cos\theta)$ for a negative atomic g factor and the same rotation direction, which is derived using Berry's connection $\langle\psi|\,\frac{d}{dt}\,|\psi\rangle$ [8].

Until now, Tycko pointed out that the spin interaction with a real field depends on the sign of the nuclear g factor [3], and Richardson *et al*. discussed that the solid angle depends on the rotation direction and the sign of the g factor [5]. In 2005, we examined Berry's phase of sodium atoms for the partial rotation of a magnetic field using a two-photon-stimulated Raman atom interferometer, by which two phases for different spin states were compared. The Zeeman sublevels in the $F=1$ state have a negative atomic g factor (hereafter, we simply describe it as g factor), whereas those in the $F=2$ state have a positive g factor. We showed that a sign of Berry's phase for a partial rotation of angle ϕ depends on the sign of the g factor at $\theta = \pi/2$ [9]. Subsequently, we measured Berry's phase for a partial conical rotation of a magnetic field between these states and showed a possibility of the dependence of the solid angle on the sign of the g factor [10], although the experimental data were scattered with large uncertainties for the dynamical phase shift due to the scalar Aharonov-Bhom effect [11]. Thus, the solid angle for a negative g factor has not yet been experimentally demonstrated.

To experimentally verify the dependence of Berry's phase on the sign of the g factor, it should be measured without any dynamical phase shift for a whole turn in a conical rotation of the magnetic field, similarly to experiments on polarized light [2] or a two-level system on a Bloch sphere [12]. In the rotation of a magnetic field, the dynamical phase shift can be ignored by using magnetic-field-insensitive transitions, such as "clock" transitions between two $m=0$ states [9]. Recently, the two-photon microwave-radiofrequency (MW-rf) transition between the ground hyperfine levels of the $|F=1, m_F=-1\rangle$ state to the $|F=2, m_F=1\rangle$ state of ^{87}Rb was found as a magnetic-field-insensitive transition, since the two states undergo the same first-order Zeeman shift at the "magic" magnetic field of 323 μT [13,14].

In this paper, we report the dependence of Berry's phase on the sign of the g factor. We attempted to use the transition between the $|F=1, m_F=-1\rangle$ sodium ground hyperfine state with a negative g factor and the $|F=2, m_F=1\rangle$ state with a positive g factor. The magnetic-field-insensitive transition could be generated easily by the two-photon MW-rf transition at a lower magic magnetic field. Berry's phase for a whole turn of the magnetic field was measured as a function of semiangle θ. The obtained Berry's phase allowed us to verify the solid angles for positive and negative g factors.

II. EXPERIMENTAL METHODS

In a sodium atom, there are three two-photon transitions between the ground hyperfine levels that are insensitive to the magnetic field to the first order. Figure 1 shows a diagram of the ground-state hyperfine levels of ^{23}Na with the Zeeman splitting due to the presence of a magnetic field and the three clock transitions. The transition between $|F=1, m_F=0\rangle$ and $|F=2, m_F=0\rangle$ is a well-known clock transition that can be directly transited by a single photon. These three transitions occur at the two-photon MW-rf transition [15]. The g factors of the $F=1$ and $F=2$ states are -1/2 and 1/2, respectively; thus, the Zeeman splitting for each state is essentially identical. The resonance frequencies are shown in Fig. 1(b), which are calculated using the Breit-Rabi formula [16] with sodium data [17]. The resonance frequency $\nu_{-1,1}$ between the $|1,-1\rangle$ and

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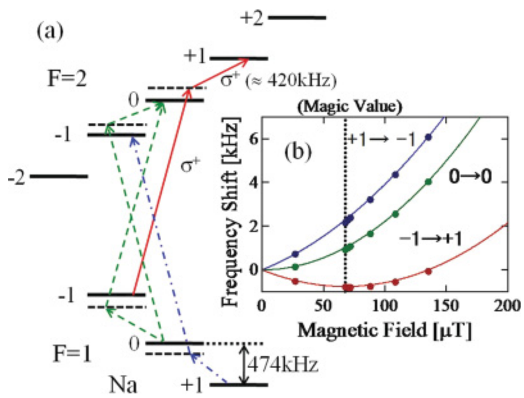

FIG. 1. (Color online) (a) A diagram of the ground hyperfine states of ^{23}Na shown in Zeeman splitting due to the presence of a magic magnetic field of 67.7 μT, together with magnetic-field-insensitive two-photon microwave-radiofrequency transitions. (b) Differential Zeeman shift at low magnetic fields for the $|F=1, m_F=-1\rangle \rightarrow |F=2, m_F=1\rangle$, $|1,0\rangle \rightarrow |2,0\rangle$, and $|1,1\rangle \rightarrow |2,-1\rangle$ transitions. The solid lines are the predicted splitting from the Breit-Rabi formula and open circles are experimental data.

$|2,1\rangle$ states is minimum at $B_0 = 67.7$ μT, which is called magic value, and it is 762 Hz lower than the frequency of clock transition at zero magnetic field; thus, the differential Zeeman shift between the $|1,-1\rangle$ and $|2,1\rangle$ states is nearly independent of the magnetic field around B_0. As a results, the frequency shift for $\nu_{-1,1}$ is only 7 Hz, even if the magnetic field is varied by about 10% around B_0. The strength of the magic magnetic field is easier to generate and is suitable for smooth and adiabatic rotation.

We use two-photon MW-rf excitations to drive these clock transitions. At the magic magnetic field, the Zeeman splitting frequency is 474 kHz. We apply a pulse of microwave radiation in which the frequency is 420 kHz less than that corresponding to the ground-state hyperfine splitting of Na (1.771626 GHz), along with an rf magnetic field around 420 kHz. This connects the $|1,-1\rangle$ state to the $|2,0\rangle$ state via an intermediate virtual state with a detuning of 54 kHz from the $|2,0\rangle$ state by σ^+-σ^+ polarized fields. Other two-photon excitations are shown in Fig. 1.

To measure Berry's phase of an atomic spin for a whole turn of a magnetic field around a cone of semiangle θ, we used a Ramsey atom interferometer composed of the $|1, m_F\rangle$ and $|2, m_F\rangle$ sodium ground hyperfine states [9] coupled with two two-photon MW-rf pulses. Let us suppose that sodium atoms remain at the origin of the x, y, and z coordinates. First, the magnetic field B_x with the magic value is applied in the x direction and the magnetic field B_z is applied in the z direction, as shown in Fig. 2. The resultant magnetic field B is $B = \sqrt{B_x^2 + B_z^2}$, and the semiangle θ from the z axis is $\cos\theta = B_z/\sqrt{B_x^2 + B_z^2}$. During the interrogation time between the two two-photon pulses, the B_x is adiabatically rotated in the x-y plane by a whole turn at a constant strength, then the tip of the resultant magnetic field traces a closed loop C in which the solid angle is $\Omega(C)$. There are two senses of rotation. We define right-handed rotation around $+z$ axis as the positive sense of rotation and left-handed rotation around the $+z$ axis as the negative sense of rotation. Then the two states each acquire

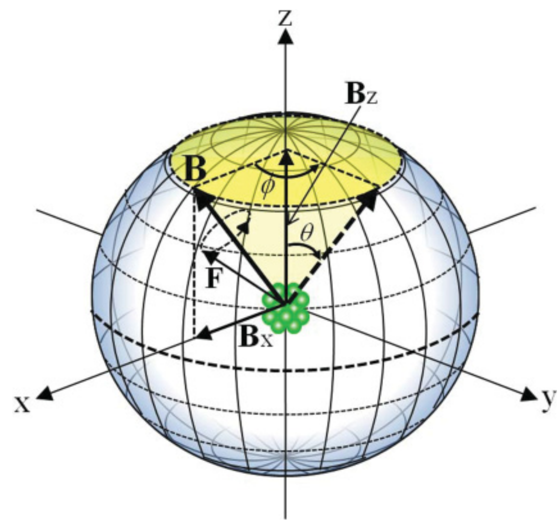

FIG. 2. (Color online) Schematic of Berry's phase in a conical rotation of the magnetic field. $\boldsymbol{B}$ is the total magnetic field, sum of $\boldsymbol{B}_x$, which is the magnetic field rotating in the x-y plain, and $\boldsymbol{B}_z$, which is the magnetic field along z axis. Total angular-momentum vector $\boldsymbol{F}$ of atom precesses around $\boldsymbol{B}$. The sense of precession depends on the sign of the g factor (figure shows the case of $g > 0$). θ is the semiangle of $\boldsymbol{B}$ from $+z$ axis and ϕ rotation angle of $\boldsymbol{B}$. The right-handed rotation around the $+z$ axis is defined as positive rotation sense. A whole turn: $\phi = 2\pi$.

Berry's phase and dynamical phase due to the fluctuation of the magnetic field. The phase of the interference fringes shifts by the difference between Berry's phase of each state, while the dynamical phase shifts between the $|1,-1\rangle$ and $|2,1\rangle$ states should be canceled out.

III. EXPERIMENTAL RESULTS

The experimental apparatus and setup were described in detail in a previous paper [11]. Sodium atoms were trapped in a dual-operated magneto-optical trap (MOT) [18] and cooled by polarization-gradient cooling. The temperature of the sodium atoms was about 200 μK and the number of trapped atoms was 10^9 atoms with a peak density of 10^{11} atoms/cm^3. A few milliseconds after the release of atoms from the trap, a quantization magnetic field of 67.6 ± 0.1 μT was applied in the x direction, and all atoms were initialized to the $F = 1$ state by optical pumping. The three magnetic sublevels of the $|1,1\rangle$, $|1,0\rangle$, and $|1,-1\rangle$ states were populated with equal-population probabilities. A dipole antenna, which generates MW with a frequency of 1.771206 GHz, was set parallel to the x direction near the magneto-optical trap; thus, the polarization direction of the MW is perpendicular to the quantization magnetic field (σ-polarization). An rf field was produced by a loop antenna near the MOT and swept at a frequency around 420 kHz. To resolve the three magnetic-field-insensitive transitions separated by a few kHz, the width of these MW and rf pulses was set to 2 ms, and their field strength was optimized to maximize the strength of the two-photon spectra. The spectra are shown in Fig. 3(a). The three peaks corresponding to the transitions $|1,-1\rangle$ to $|2,1\rangle$, $|1,0\rangle$ to $|2,0\rangle$, and $|1,1\rangle$ to $|2,-1\rangle$ are clearly resolved. The resonance frequencies of the three peaks are plotted in the Fig. 1(b), together with theoretical

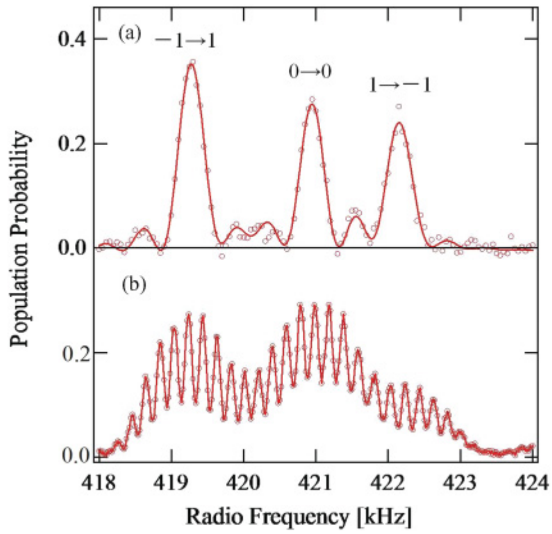

FIG. 3. (Color online) Spectra of the |1, −1⟩ → |2, 1⟩, |1, 0⟩ →|2, 0⟩, and |1, 1⟩ → |2, −1⟩ two-photon transitions at the magnetic field of 67.5 μT. (a) Spectra excited by a two-photon pulse with a pulse width of 2 ms. (b) Ramsey fringes excited by two two-photon pulses with pulse widths of 0.75 ms and pulse durations of 4.25 ms.

curves obtained from the Breit-Rabi formula. The agreement between experiment and theory is reasonably good.

The time-domain atom interferometer used in the experiment was composed of two two-photon MW-rf pulses with pulse widths of 0.75 ms, which were separated by 4.25 ms. The interrogation time was 5 ms. Under a constant magnetic field, the Ramsey fringes with a cycle of 200 Hz can be clearly seen in Fig. 3(b). The visibility is about 0.5 for the |1, −1⟩-to-|2, 1⟩ transition.

To rotate the magnetic field around the z axis, magnetic fields along the x and y axes were produced by two mutually orthogonal Helmholtz coils [19]. The coils were driven by alternating currents with a relative phase shift of 90°. The strength of the magnetic field was 67.5 μT, which corresponds to a Larmor frequency of 470 kHz. The magnetic field was rotated around the z axis by a whole turn for 5 ms. During the rotation, the amplitude of each alternating current was adjusted so that the Zeeman frequency shifts of the resonance for magnetic-field-sensitive transitions were 67.5 ± 0.4 μT. The rotation frequency of 200 Hz was sufficient to satisfy the adiabatic condition. To change the semiangle θ from the z axis, the magnetic field B_z parallel to the z axis was applied before rotation. The strength of B_z was changed from 0 to ±40 μT, which corresponds to a change from $\pi/3$ to $2\pi/3$. The resultant magnetic field at $\pi/3$ and $2\pi/3$ became 1.15 times larger than that at $\theta = \pi/2$, which yields a frequency shift of at most 17 Hz.

A few of the central Ramsey fringes of the two-photon MW-rf transition between the |1, −1⟩ and |2, 1⟩ states are shown in Fig. 4, where Fig. 4(a) shows fringes under a constant magnetic field of 67. 5 μT, Fig. 4(b) shows those under a whole rotation in the negative direction with a semiangle of 0.465π, and Fig. 4(c) shows those under a whole rotation in the negative direction with a semiangle of 0.552π. The size of fringes decreases as the semiangle increases. The phase in

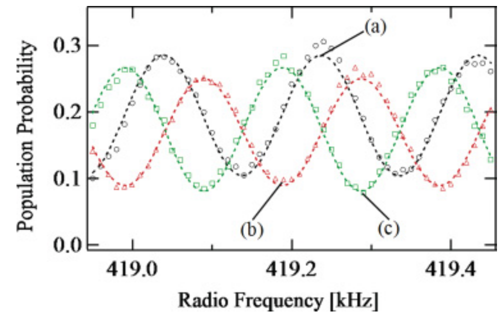

FIG. 4. (Color online) Observed central Ramsey fringes of the |1, −1⟩ → |2, 1⟩ transition under (a) a constant magnetic field, (b) a whole rotation to the positive sense at $\theta = 0.457\pi$, and (c) a whole rotation to the negative sense at $\theta = 0.544\pi$.

Fig. 4(b) is shifted by $(0.53\pm0.01)\pi$ from the phase without rotation, but that in Fig. 4(c) is shifted by $(0.48\pm0.01)\pi$.

Thus, we measured Berry's phases of the three transitions between the |1, −1⟩ and |1, 1⟩ states, the |1, 1⟩ and |1, −1⟩ states, and the |1, 0⟩ and |2, 0⟩ states as reference, for a whole turn of the magnetic field around a cone of semiangle θ from $\pi/3$ to $2\pi/3$ for positive and negative rotations. The results are summarized in Fig. 5. The phase shift observed for the transition between |1, −1⟩ and |2, 1⟩ for positive rotation and that for the transition between |1, 1⟩ and |2, −1⟩ for negative rotation are well fitted by the curve $\Delta\gamma = 4\pi\cos\theta$. On the other hand, the data for the transition from |1, −1⟩ to |2, 1⟩ for negative rotation are well fitted by $\Delta\gamma = -4\pi\cos\theta$. The data for the transition from |1, 0⟩ to |2, 0⟩ is displaced from 0 with increasing semiangle, because the resonance frequency is shifted by the second-order Doppler shift that depends on the total strength of the magnetic field. On the other hand, the frequency shift of the transition from |1,−1⟩ to |2, 1⟩ is so small around the magic magnetic field that we could obtain Berry's phase free from the dynamical phase shift.

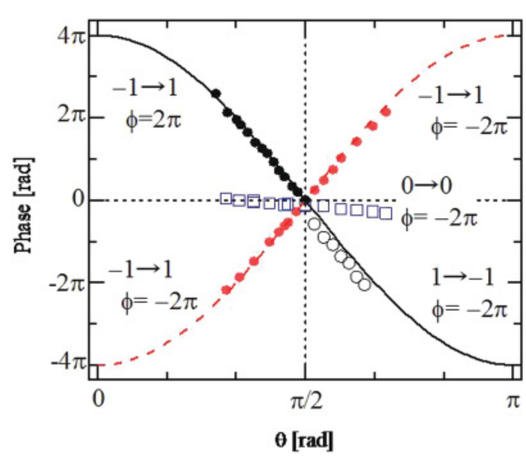

FIG. 5. (Color online) Observed Berry's phase using atom interferometer as a function of θ for several situations denoted in the figure. m_F-$m_F{}'$ represents the |1, m_F > → |2, $m_F{}'$ > transition. $\phi = +2\pi$ and -2π represents a whole rotation to the positive sense and to the negative sense, respectively. A solid line is a $4\pi\cos\theta$ curve and a dashed line is a $-4\pi\cos\theta$ curve.

IV. DISCUSSION

The solid angles for positive and negative g factors are calculated by a path integral of Berry's connection [8]. First, we define the rotation axis of the magnetic field and θ ($<\pi/2$) to be the semiangle between the direction of the magnetic field and the rotation axis. The right-handed rotation of the magnetic field is defined as the positive sense of rotation, and the left-handed rotation is defined as the negative sense of rotation. Then, the solid angles for positive and negative g factors are given by

$$0 \leqslant \theta \leqslant \pi/2 \quad : \quad \begin{aligned} \Omega_{g>0} &= 2\pi(1-\cos\theta) \\ \Omega_{g<0} &= -2\pi(1+\cos\theta). \end{aligned} \tag{1}$$

When θ becomes larger than $\pi/2$, the direction of the rotation axis reverses; thus, the sense of the rotation of the magnetic field reverses. The semiangle is newly defined by $\theta' = \pi - \theta$; thus, the formulas for Ω are described as follows in terms of θ:

$$\pi/2 \leqslant \theta \leqslant \pi \quad : \quad \begin{aligned} \Omega_{g>0} &= -2\pi(1+\cos\theta) \\ \Omega_{g<0} &= 2\pi(1-\cos\theta). \end{aligned} \tag{2}$$

Namely, the formulas for $\Omega_{g>0}$ and $\Omega_{g<0}$ are exchanged. In either case, Berry's phase is given by multiplying the magnetic quantum number m_F. As a result, the difference between Berry's phases for the m_F and $m_F{}'$ states measured by the atom interferometer is

$$\Delta\gamma = \gamma - \gamma' = -m_F\Omega(C) + m'_F\Omega'(C). \tag{3}$$

Consequently, Berry's phase between |2, 1> and |1, -1> for a whole positive rotation of the magnetic field ($\phi = 2\pi$) when $0 \leqslant \theta \leqslant \pi/2$ is given by

$$\Delta\gamma = -2\pi(1-\cos\theta) - \{-2\pi(1+\cos\theta)\} = 4\pi\cos\theta,$$

whereas for $\pi/2 \leqslant \theta \leqslant \pi$,

$$\Delta\gamma = 2\pi(1+\cos\theta) - \{2\pi(1-\cos\theta)\} = 4\pi\cos\theta.$$

Thus, we obtain the equation $\Delta\gamma = 4\pi\cos\theta$ for any θ. Similarly, Berry's phase between $|2,\ 1\rangle$ and $|1,\ -1\rangle$ for a whole negative rotation is given by $\Delta\gamma = -4\pi\cos\theta$, and that between $|2,\ -1\rangle$ and $|1,\ 1\rangle$ for a whole negative rotation is $\Delta\gamma = 4\pi\cos\theta$.

In the case of photons in a helically wound optical fiber, the fact that Berry's phases for right-handed or left-handed circular polarization are $\gamma = -2\pi(1-\cos\theta)$ or $\gamma = 2\pi(1-\cos\theta)$ means that their polarizations correspond to the spin component ±1 [2], [20] with a positive atomic g factor.

V. CONCLUSION

In conclusion, we have demonstrated Berry's phase of a spin for a whole rotation of a magnetic field free from the dynamical phase shift using a sodium magnetic-field-insensitive transition. The experimental results show the clear dependence of Berry's phase on the sign of the g factor. The results verify that the solid angle for a positive g factor is $\Omega_{g>0} = 2\pi(1-\cos\theta)$, and that for a negative g factor is $\Omega_{g<0} = -2\pi(1+\cos\theta)$ for a positive rotation and a semiangle of $0 \leqslant \theta \leqslant \pi/2$. Consequently, the sign of the g factor of particle and the sense of rotation direction of the magnetic field should be considered whenever Berry's phase is examined or utilized in any physical systems.

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