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Robust and scalable scheme to generate large-scale entanglement webs

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We propose a robust and scalable scheme to generate an N-qubit W state among separated quantum nodes (cavity-QED systems) by using linear optics and postselections. The present scheme inherits the robustness of the Barrett-Kok scheme [S. D. Barrett and P. Kok, Phys. Rev. A **71**, 060310(R) (2005)]. The scalability is also ensured in the sense that an arbitrarily large N-qubit W state can be generated with a quasipolynomial overhead $\sim 2^{O[(\log_2 N)^2]}$. The process to breed the W states, which we introduce to achieve the scalability, is quite simple and efficient and can be applied for other physical systems.

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Introduction. So far tremendous efforts have been paid for experimental realizations of quantum-information processing (QIP), and, for example, control of a few qubits has been performed in cavity QED, ion traps, etc. It, however, seems difficult to increase the number of qubits dramatically within a single physical system. In order to realize large-scale QIP, we have to develop a way to integrate individual physical systems scalably. Furthermore, for communication purposes, quantum information has to be shared among separated quantum nodes. To meet these requirements, *distributed* QIP, where stationary qubits are entangled by using flying qubits (photons), seems to be very promising [1–4]. A lot of protocols have been proposed so far for remote entangling operations and probabilistic two-qubit gates [5–7]. The Barrett-Kok scheme is particularly promising, since it is fully scalable and robust against experimental imperfections [6]. It is further studied for generating graph states efficiently [8]. Experiments of the remote entangling operations (or probabilistic two-qubit gates) between separated qubits have also been done in both atomic ensembles [9] and trapped single atoms [10]. They are important ingredients for fault-tolerant distributed quantum computation [6,11–13].

Multipartite entanglement is not only a key ingredient for quantum communication but also an important clue to understand the nature of quantum physics. There are a lot of classes of multipartite entanglement, for example, GHZ (Greenberger-Horne-Zeilinger) states [14], cluster states [15], and W states [16]. Among them, the W states,

$$|W_N\rangle = \frac{1}{\sqrt{N}}(|100\cdots 0\rangle + |010\cdots 0\rangle \cdots + |000\cdots 1\rangle),$$

are quite robust in the sense that any pairs of qubits are still entangled, even if the rest of the qubits are discarded [17]. This weblike property is very fascinating as a universal resource, i.e., *entanglement webs*, for quantum communication. There are several protocols which use the W states for quantum key distribution, teleportation, leader election, and information splitting [18]. Furthermore, inevitable decoherence in sharing the W states can be counteracted by using a scheme of purification [19]. The preparation of the W states by using optics has been discussed so far extensively both theoretically and experimentally [20]. It has been also discussed in other systems, such as cavity QED and ion traps [21]. Nevertheless none of them seems to be fully scalable. That is, the overhead required for sharing an N-qubit W state scales exponentially in the number of qubits N or the W state is prepared in a single system, which cannot be used for quantum communication among separated quantum nodes.

In this paper, we develop a *robust and scalable* scheme to generate the N-qubit W state by using separated cavity-QED systems and linear optics. The present scheme is scalable in the sense that an arbitrarily large N-qubit W state can be generated among separated quantum nodes with only a quasipolynomial overhead $\sim 2^{O[(\log_2 N)^2]}$. In the following, we first develop an efficient way to generate the four-qubit W state $|W_4\rangle$ by following the concept of the Barrett-Kok scheme [6], which is quite robust against the experimental imperfections. The success probability to obtain the $|W_4\rangle$ is significantly high to be $1/2$. Then, by using the four-qubit W states as *seeds*, we can *breed* an arbitrarily large W state in an economical way, where the two $|W_N\rangle$'s are converted to one $|W_{2(N-1)}\rangle$ probabilistically by accessing only two qubits. In contrast to classical webs, where a local connection does not result in a global web structure, this property of entanglement webs is a genuine quantum phenomenon. Even if the conversion fails, the two $|W_{N-1}\rangle$'s are left and can be recycled. This breeding method is quite simple and economical and can be applied to other physical systems, such as polarization qubits in optics [20].

Four-qubit W state (*seeding*). We consider four three-level atoms, each of which is embedded in a separated cavity. The two long-lived states of the atom, $|0\rangle$ and $|1\rangle$, are used as a qubit, where only the state $|1\rangle$ is coupled to the excited state $|e\rangle$, whose transition frequency is equal to that of the cavity mode (see Fig. 1). The output fields of the cavities are mixed with 50:50 beam splitters (BSs) and measured by photodetectors. The effective Hamiltonian of the system is given by

$$H = \sum_{i=1}^{4} \frac{g_i}{2}(|1\rangle_{ii}\langle e|\hat{c}_i^\dagger + \text{H.c.}) - i\sum_{i=1}^{4}\kappa_i \hat{c}_i^\dagger \hat{c}_i,$$

where g_i denotes the coupling between the $|1\rangle_i \leftrightarrow |e\rangle_i$ transition and the ith cavity mode $\hat{c}_i$. The cavity photon leaks to the output mode with rate $2\kappa_i$ ($\kappa_i > g_i$), which is treated as the non-Hermitian term by following the quantum jump approach [22]. For simplicity, the cavity parameters are set to $g_i = g$ and $\kappa_i = \kappa$ ($i = 1,2,3,4$). As shown in Fig. 1, the output modes are mixed by using the four 50:50 BSs. Thus

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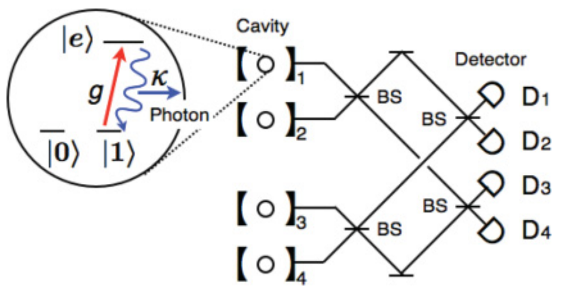

FIG. 1. (Color online) Three-level atoms are embedded in cavities. The two long-lived states $|0\rangle$ and $|1\rangle$ are used as a qubit, and the transition between the states $|1\rangle \leftrightarrow |e\rangle$ is coupled to the cavity mode. The output modes are mixed with four 50:50 BSs and measured by photodetectors D_i.

the modes $\hat{a}_i$ of the four detectors D_i are given in terms of the cavity modes $\hat{c}_i$ by

$$\hat{a}_1 = (\hat{c}_1 + \hat{c}_2 + \hat{c}_3 + \hat{c}_4)/2, \quad \hat{a}_2 = (\hat{c}_1 - \hat{c}_2 + \hat{c}_3 - \hat{c}_4)/2,$$
$$\hat{a}_3 = (\hat{c}_1 + \hat{c}_2 - \hat{c}_3 - \hat{c}_4)/2, \quad \hat{a}_4 = (\hat{c}_1 - \hat{c}_2 - \hat{c}_3 + \hat{c}_4)/2.$$

The procedure to obtain the four-qubit W state $|W_4\rangle$ is as follows. We first prepare the initial state of the atoms as $|\Psi(0)\rangle = (|0\rangle + |e\rangle)^{\otimes 4}/4$ by using π pulses. Then, we wait for a sufficiently long time t_w to detect photons. Before proceeding to the second round, each qubit is flipped as $|0\rangle \leftrightarrow |1\rangle$, and the state $|1\rangle$ is excited to $|e\rangle$ by a π pulse. Then we wait again for t_w to detect photons. If three- and single-detector clicks, or vice versa, are observed at the first and second rounds, respectively, the $|W_4\rangle$ is obtained up to unimportant phase factors, which can be removed by using local operations.

Let us see in detail how the $|W_4\rangle$ is generated and calculate the success probability. For concreteness, we consider the case, where the D_1, D_2, and D_3 are clicked at t_1, t_2, and t_3 ($t_1 < t_2 < t_3$), respectively, in the first round. In the second round, the fourth detector is clicked at t_4. The state conditioned on the first three clicks is given up to normalization as

$$\begin{aligned}|\Psi(t_1,t_2,t_3)\rangle &= (2\kappa)^{3/2}\hat{a}_3 e^{-iH(t_3-t_2)}\hat{a}_2 e^{-iH(t_2-t_1)}\hat{a}_1 e^{-iHt_1}|\Psi(0)\rangle \\ &= \frac{(2\kappa)^{3/2}}{8}\alpha(t_3)\alpha(t_2)\alpha(t_1)[\mathcal{W}(|1,0\rangle,|0,0\rangle) \\ &\quad + \alpha(t_3)\mathcal{W}(|1,0\rangle,|1,1\rangle) + \beta(t_3)\mathcal{W}(|1,0\rangle,|e,0\rangle)],\end{aligned}$$

where $|a,b\rangle$ ($a \in \{0,1,e\}$ and $b \in \{0,1\}$) indicates the states of the atom $|a\rangle$ and photon $|b\rangle$, respectively, for the combination $\mathcal{W}(|A\rangle,|B\rangle) \equiv (|A\rangle|A\rangle|A\rangle|B\rangle - |A\rangle|A\rangle|B\rangle|A\rangle - |A\rangle|B\rangle|A\rangle|A\rangle + |B\rangle|A\rangle|A\rangle|A\rangle)/2$. The coefficients $\alpha(t)$ and $\beta(t)$ are the solutions to the Schrödinger equation:

$$\alpha(t) = -ig/(2\sqrt{\kappa^2 - g^2})(-e^{\omega_+ t} + e^{\omega_- t}),$$
$$\beta(t) = g^2/(4\sqrt{\kappa^2 - g^2})(-e^{\omega_+ t}/\omega_+ + e^{\omega_- t}/\omega_-),$$

where $\omega_\pm = (-\kappa \pm \sqrt{\kappa^2 - g^2})/2$. The probability of such an event is given by

$$p(t_1,t_2,t_3) = |\langle\Psi(t_1,t_2,t_3)|\Psi(t_1,t_2,t_3)\rangle|^2.$$

For the sufficiently long t_w ($\gg 1/|\omega_\pm|$), the states $|e,0\rangle$ and $|1,1\rangle$ decay to $|1,0\rangle$ incoherently. The postmeasurement state at t_w is given by

$$\rho(t_w) = \mathcal{N}[\rho_{\mathcal{W}}(|1,0\rangle,|0,0\rangle) + |\alpha(t_{\mathrm{w}})|^2|1,0\rangle\langle 1,0|^{\otimes 4}],$$

where $\rho_{\mathcal{W}}(|A\rangle,|B\rangle) = \mathcal{W}(|A\rangle,|B\rangle)\mathcal{W}(|A\rangle,|B\rangle)^\dagger$ and $\mathcal{N} = (2\kappa)^3|\alpha(t_3)\alpha(t_2)\alpha(t_1)|^2/[64p(t_1,t_2,t_3)]$.

Before proceeding to the second round, each qubit is flipped as $|0\rangle \leftrightarrow |1\rangle$, and the state $|1\rangle$ is excited to $|e\rangle$ similarly to the first round. Then, the initial state of the second round is given by

$$\rho'(0) = \mathcal{N}[\rho_{\mathcal{W}}(|0,0\rangle,|e,0\rangle) + |\alpha(t_{\mathrm{w}})|^2|0,0\rangle\langle 0,0|^{\otimes 4}].$$

Since the first term has exactly one excitation, by observing the single detector click at t_4, the second term is removed in this round. Finally we obtain the four-qubit W state $|W_4\rangle$ for the atoms. The joint probability for the first three clicks and the second single click is calculated as

$$\begin{aligned}p(t_1,t_2,t_3,t_4) &= p(t_4|t_1,t_2,t_3)p(t_1,t_2,t_3) \\ &= \mathrm{Tr}[2\kappa\hat{a}_4^\dagger\hat{a}_4 e^{-iHt_4}\rho'(0)e^{iHt_4}]p(t_1,t_2,t_3) \\ &= (2\kappa)^4|\alpha(t_1)\alpha(t_2)\alpha(t_3)\alpha(t_4)|^2/256.\end{aligned}$$

For sufficiently long t_w ($\gg 1/|\omega_\pm|$), the success probability is calculated as

$$\prod_{i=1}^{4}\int_0^{t_{\mathrm{w}}} dt_i \frac{(2\kappa)^4}{256}|\alpha(t_1)\alpha(t_2)\alpha(t_3)\alpha(t_4)|^2 = \frac{1}{256},$$

where the sum over the orderings of t_1, t_2, and t_3 is also taken. By considering the cases for three detector clicks, (D_1,D_1,D_1), (D_1,D_1,D_2), and so on, we obtain the total success probability $p = 1/2$, which is unexpectedly high. This success probability can also be understood by the fact that the initial state $(|0\rangle + |1\rangle)^{\otimes 4}/4$ contains two types of the W states (i.e., $|0001\rangle\cdots$ and $|1110\rangle\cdots$) with each probability 1/4. Then in the present setup, we can fully extract the W states by virtue of the highly symmetric detector modes. This method inherits the robustness of the Barrett-Kok scheme [6]; the detector inefficiency and photon loss do not deteriorate the fidelity, but only decrease the success probability. The success probability scales like $p = (\eta_d\eta_l)^4/2$, where η_d and $1 - \eta_l$ denote the detector efficiency and photon loss rate, respectively. Other imperfections such as decoherence of the qubits, detector dark counts, and mode mismatchings would not deteriorate the fidelity crucially for a specific physical system such as the nitrogen-vacancy (NV)-diamond system, as discussed in Ref. [6].

The above process to prepare the $|W_4\rangle$ is viewed as a single concatenation of entangling operation, $|1\rangle \to |10\rangle + |01\rangle$ and $|0\rangle \to |00\rangle$. It can be extended straightforwardly to generate an N-qubit ($N = 2^L$ with an integer L) W state $|W_N\rangle$ with probability $N/2^{N-1}$ by using a similar setup. The detector modes are given by $\hat{a}_j^{(L)} = A_{ij}^{(L)}\hat{c}_i/\sqrt{N}$ in terms of an $N \times N$ matrix $A^{(L)}$ generated recursively by

$$A^{(L+1)} = \begin{pmatrix} A^{(L)} & A^{(L)} \\ A^{(L)} & -A^{(L)} \end{pmatrix},$$

where $A^{(0)} = 1$. Then single and $N-1$ clicks, or vice versa, at the first and second rounds, respectively, result in the $|W_N\rangle$.

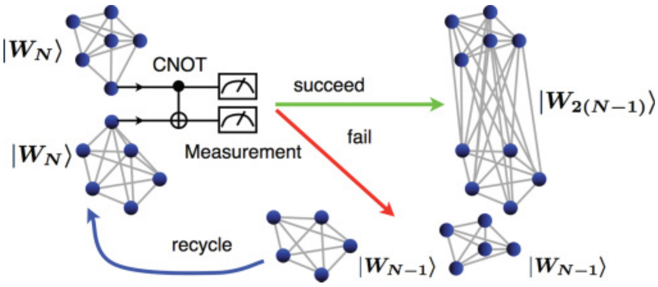

FIG. 2. (Color online) Economical breeding. Two $|W_N\rangle$'s, which are depicted symbolically as circles connected with lines, are converted to a $|W_{2(N-1)}\rangle$ probabilistically. Even if the conversion fails, the two $|W_{N-1}\rangle$'s are left and can be recycled.

More generally, if we observe m and $N-m$ clicks at the first and second rounds, respectively, we can obtain the Dicke-symmetric state [23]:

$$|D_{m,N-m}\rangle = \sum_i \mathcal{S}_i(|0\rangle^{\otimes m}|1\rangle^{\otimes N-m}))/\sqrt{C^m_{N-m}},$$

where $\{\mathcal{S}_i\}$ denotes the set of all distinct combinations of the qubits and $C^m_{N-m} = N!/[m!(N-m)!]$. With the increasing number of qubits N, however, the success probability $\sim 2^{-O(N)}$ diminishes exponentially.

Economical breeding. We next show that the four-qubit W states are sufficient to generate an arbitrarily large W state with a quasipolynomial overhead, introducing an economical breeding (Fig. 2). Suppose that we have obtained the N-qubit W states:

$$|W_N\rangle = \frac{1}{\sqrt{N}}|1\rangle^{(a)}|0_{N-1}\rangle + \sqrt{\frac{N-1}{N}}|0\rangle^{(a)}|W_{N-1}\rangle,$$

where the qubit labeled by (a) is used as an ancilla for the breeding and $|0_n\rangle \equiv |0\rangle^{\otimes n}$. Then, the two N-qubit W states can be rewritten as

$$\begin{aligned}|W_N\rangle|W_N\rangle = &\frac{1}{N}|11\rangle^{(a)}|0_{2(N-1)}\rangle \\ &+\frac{\sqrt{N-1}}{N}|10\rangle^{(a)}|W_{N-1}\rangle|0_{N-1}\rangle \\ &+\frac{\sqrt{N-1}}{N}|01\rangle^{(a)}|0_{N-1}\rangle|W_{N-1}\rangle \\ &+\frac{N-1}{N}|00\rangle^{(a)}|W_{N-1}\rangle|W_{N-1}\rangle,\end{aligned}$$

where the two ancilla qubits in the W states are moved to the first two-qubit Hilbert space labeled by (a). Here, we perform a controlled-NOT (CNOT) gate between the two ancilla qubits and measure the second ancilla qubit in the Z basis. If the measurement outcome is 1, the postmeasurement state is given by

$$\frac{1}{\sqrt{2}}(|11\rangle^{(a)}|W_{N-1}\rangle|0_{N-1}\rangle + |01\rangle^{(a)}|0_{N-1}\rangle|W_{N-1}\rangle).$$

The probability for obtaining such an outcome is $(N-1)/N^2$. Next, by measuring the first ancilla qubit in the X basis and performing local operations properly depending on the outcome, we can convert the two N-qubit W state to the $2(N-1)$-qubit W state:

$$\frac{1}{\sqrt{2}}(|W_{N-1}\rangle|0_{N-1}\rangle + |0_{N-1}\rangle|W_{N-1}\rangle) = |W_{2(N-1)}\rangle.$$

This indicates a good property of entanglement webs; a local connection produces a global web structure.

Alternatively, if the outcome of the first measurement for the second ancilla qubit is 0, we have

$$\frac{|10\rangle^{(a)}|0_{2(N-1)}\rangle + (N-1)|00\rangle^{(a)}|W_{N-1}\rangle|W_{N-1}\rangle}{\sqrt{N^2-2N+2}}.$$

Then, by measuring the first ancilla qubit in the Z basis with the outcome 0, the two $|W_{N-1}\rangle$'s are left, which can be recycled to generate the $|W_{2(N-2)}\rangle$. The joint probability to obtain such outcomes as (0,0) is $(N-1)^2/N^2$.

Notice in the above that, in order to grow the size of the W state, $2(N-1) > N$ is required, that is, $N \geqslant 3$. Thus starting from the four-qubit W states, we can breed an arbitrarily large W state by repeating the conversion process. With an even number of qubits, we can also obtain the W state with an odd number of qubits as byproducts when the conversion fails.

In the cavity-QED setup such as in Fig. 1, instead of the above procedure (CNOT and measurements), the original Barrett-Kok scheme can be used to project the ancilla qubits to the subspace spanned by $\{|10\rangle^{(a)}, |01\rangle^{(a)}\}$. Then, if the projection is successful with probability $(N-1)/N^2$, the two $|W_N\rangle$'s are converted to the $|W_{2(N-1)}\rangle$. In the failure case, then, if the ancilla qubits (atoms) are confirmed to be in the $|00\rangle^{(a)}$ by measuring them directly, the two $|W_{N-1}\rangle$'s are left for recycling. Even when the detector inefficiency and photon loss are considered, the conversion probability is diminished by only $(\eta_d\eta_l)^2$.

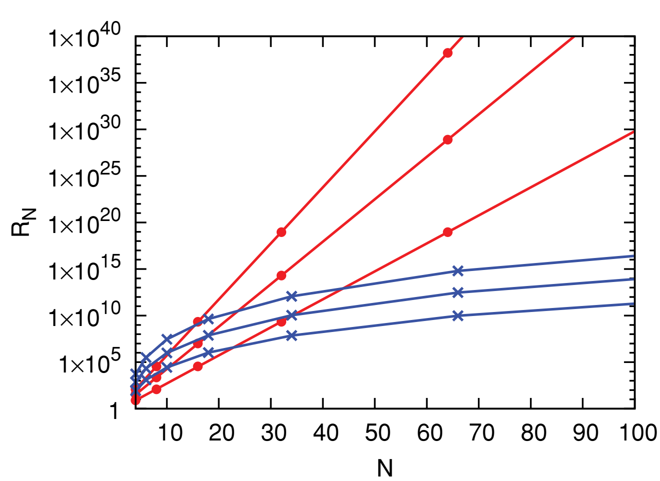

FIG. 3. (Color online) The overheads R_N for the concatenated entangling (red ○) and the breeding (blue ×), respectively, are plotted as functions of the number of qubits N, where $\eta_d\eta_l = 0.5, 0.7, 1$ from top to bottom.

The success probability for the breeding sequence $|W_4\rangle \rightarrow \cdots |W_{N_k}\rangle \rightarrow \cdots |W_N\rangle$ is calculated ($\eta_d \eta_l = 1$ in the ideal case) as

$$p_N = \frac{1}{2} \prod_{k=1}^{K} \frac{2^k + 1}{(2^k + 2)^2},$$

where $K = \log_2(N-2) - 1$ means the number of conversions required to breed the $|W_N\rangle$ and $N_k = 2^{k+1} + 2$ satisfies $N_{k+1} = 2(N_k - 1)$. The overhead $R_N = 4 \times 2^K / p_N$ scales like $2^{O[(\log_2 N)^2]}$ for $N \gg 1$, which is quasipolynomial in the number of qubits N. This is because, although the success probability of the conversion $|W_N\rangle|W_N\rangle \rightarrow |W_{2(N-1)}\rangle$ decreases as $O(1/N)$, the size of the W state grows exponentially with the number of conversions $O(\log_2 N)$. (The overhead will be somewhat improved by recycling.) On the other hand, if we generate the $|W_N\rangle$ by the concatenated entangling with $A^{(L)}$ as mentioned before, the overhead $R_N \sim 2^{O(N)}$ is exponential. Furthermore, the number of total clicks in the breeding is $4 + 2K = 3 + 2\log_2(N-2)$. Thus the detector inefficiency η_d and photon loss $1 - \eta_l$ do not upset the scalability in the breeding scheme though they require somewhat more resources. In Fig. 3, the overheads R_N for the concatenated entangling (red ○) and the breeding (blue ×), respectively, are plotted as functions of the number of qubits N, where $\eta_d \eta_l = 0.5, 0.7, 1$ from top to bottom. As by-products in breeding the $|W_N\rangle$, the $|W_{N-2M}\rangle$ ($1 \leqslant M \leqslant N/2 - 1$) can also be obtained with probability $(N-2M)p_N/N$ and resources $4 \times 2^K[N/(N-2M)]/p_N$ by recycling.

Discussion and conclusion. We have considered a robust and scalable scheme to generate large-scale entanglement webs. We have first introduced an efficient way to generate the four-qubit W state by following the Barrett-Kok's concept, which provides a significantly high success probability of 1/2. Then, by using the four-qubit W states as seeds, we have developed an economical breeding method to generate an arbitrarily large W state with a quasipolynomial overhead. The breeding method is quite simple and exploits a unique property of entanglement webs. That is, a global web structure can be constructed only by a local connection. This provides a different perspective on multipartite entanglement.

Note added in proof. Recently, we became aware of Ref. [24], which uses the breeding method for generating the W states of polarization qubits.

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