Wave-particle duality and polarization properties of light in single-photon interference experiments

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We consider superposition of two states of light polarized along mutually orthogonal directions. We show that partial polarization of the superposed light may be interpreted as a manifestation of the wave-particle duality.

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Quantum systems (quantons [1]) possess properties of both particles and waves. Bohr's correspondence principle [2] suggests that these two properties are mutually exclusive. In other words, depending on the experimental situation, a quanton will behave as a particle or as a wave. In the third volume of his famous lecture series [3], Feynman emphasized that this *wave-particle duality* may be understood from a Young's two-pinhole interference experiment [4]. In such an experiment, a quanton may arrive at the detector along two different paths. If one can determine which path the quanton traveled, then no interference fringe will be found (i.e., the quanton will show complete particle behavior). On the other hand, if one *cannot* obtain any information about the quanton's path, then interference fringes of unit visibility will be obtained (i.e., the quanton will show complete wave behavior), assuming that the intensities at the two pinholes are the same. In the intermediate case, when one has partial "which-path information" (WPI), fringes of visibility lesser than unity are obtained, even if the intensities at the two pinholes are equal. For the sake of brevity, we will use the term "best circumstances" [5] to refer to the situation when intensities at the two pinholes are equal (or to equivalent situations in other interferometric setups). The relationship between fringe visibility and WPI has been investigated in Refs. [7–9]. It has been established that a quantitative measure of WPI and fringe visibility obey a certain inequality [10].

The aim of this Brief Report is to show that it is not only the fringe visibility, but also the polarization properties of the superposed light, which may depend on WPI in an interference experiment. In particular, it will be established that the degree of polarization and a measure of WPI obey an inequality, with equality holding under the "best circumstances".

Polarization properties of light have been extensively studied in the framework of classical theory (see e.g., [11–14]). The foundation of this subject was laid down by Stokes [15] as he formulated the theory of polarization in terms of certain measurable parameters, now known as Stokes parameters (see e.g., [12], p. 348). In the late 1920s, Wiener [16–18] showed that a correlation matrix, namely the coherency matrix, can be used to analyze polarization properties of light. Wolf [19] used the correlation matrix formulation for systematic studies of polarization properties of statistically stationary light beams within the framework of classical theory. Later, some publications (see e.g., [20–22]) addressed this problem by the use of quantum-mechanical techniques. A discussion of quantum-mechanical analogs of the Stokes parameters can be found in Ref. [13], Appendix L. Recently, another quantum-mechanical analysis of polarization properties of optical beams has been presented [23].

We will mainly use the formulation of Ref. [23]. According to that formulation, polarization properties (based on first-order correlations [24]) of light at a space-time point $(\mathbf{r},t)$ may be characterized by a correlation matrix [21], namely the so-called quantum-polarization matrix

$$\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t) \equiv [G_{ij}^{(1)}(\mathbf{r},t;\mathbf{r},t)] = \mathrm{Tr}\{\widehat{\rho}\widehat{E}_i^{(-)}(\mathbf{r},t)\widehat{E}_j^{(+)}(\mathbf{r},t)\},\quad i=x,y;\quad j=x,y. \tag{1}$$

Here $\widehat{E}_i^{(+)}$ and $\widehat{E}_i^{(-)}$ are the i components of the positive and negative frequency parts of the quantized electric-field operator, respectively, and $\widehat{\rho}$ represents the density operator. A quantitative measure of polarization properties of photons, at a space-time point $(\mathbf{r},t)$, is given by the degree of polarization [Eq. (31) of Ref. [23]],

$$\mathscr{P}(\mathbf{r},t) \equiv \sqrt{1-\frac{4\mathrm{Det}\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t)}{\left\{\mathrm{Tr}\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t)\right\}^2}}, \tag{2}$$

where Det and Tr denote the determinant and the trace, respectively. This quantity is always bounded by zero and unity, i.e., $0 \leqslant \mathscr{P}(\mathbf{r},t) \leqslant 1$. It is to be noted that $\mathscr{P}(\mathbf{r},t)$ is expressed in terms of the trace and the determinant of the matrix $\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t)$, and hence is invariant under unitary transformations. When $\mathscr{P}(\mathbf{r},t)=0$, the light is completely unpolarized at the space-time point $(\mathbf{r},t)$, and when $\mathscr{P}(\mathbf{r},t)=1$, the light is completely polarized at that space-time point. In intermediate cases, $0<\mathscr{P}(\mathbf{r},t)<1$, the light is said to be partially polarized.

Suppose, now, that $|\psi_1\rangle$ and $|\psi_2\rangle$ represent two normalized single-photon states (eigenstates of the number operator), so that

$$\langle\psi_1|\psi_2\rangle = \langle\psi_2|\psi_1\rangle = 0, \tag{3a}$$
$$\langle\psi_1|\psi_1\rangle = \langle\psi_2|\psi_2\rangle = 1. \tag{3b}$$

We will consider superposition of the two states in some interferometric arrangements, where a photon may travel along two different paths. Suppose that $|\psi_{\mathrm{ID}}\rangle$ represents a state of

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light, which is formed by a coherent superposition of the two states $|\psi_1\rangle$ and $|\psi_2\rangle$, i.e., that

$$|\psi_{\rm ID}\rangle = \alpha_1 |\psi_1\rangle + \alpha_2 |\psi_1\rangle , \quad |\alpha_1|^2 + |\alpha_2|^2 = 1, \tag{4}$$

where α_1 and α_2 are, in general, two complex numbers. In this case, a photon may be in the state $|\psi_1\rangle$ with probability $|\alpha_1|^2$, or in the state $|\psi_2\rangle$ with probability $|\alpha_2|^2$, but the two possibilities are intrinsically *indistinguishable*. The density operator $\widehat{\rho}_{\rm ID}$ will then have the form

$$\begin{aligned}\widehat{\rho}_{\rm ID} = {}& |\alpha_1|^2 |\psi_1\rangle\langle\psi_1| + |\alpha_2|^2 |\psi_2\rangle\langle\psi_2| \\ &+ \alpha_1^*\alpha_2 |\psi_2\rangle\langle\psi_1| + \alpha_2^*\alpha_1 |\psi_1\rangle\langle\psi_2| .\end{aligned} \tag{5}$$

In the other extreme case, when the state of light is due to an incoherent superposition of the two states, the density operator $\widehat{\rho}_{\rm D}$ will be given by the expression

$$\widehat{\rho}_{\rm D} = |\alpha_1|^2 |\psi_1\rangle\langle\psi_1| + |\alpha_2|^2 |\psi_2\rangle\langle\psi_2| . \tag{6}$$

Here $|\alpha_1|^2$ and $|\alpha_2|^2$ again represent the probabilities that the photon will be in state $|\psi_1\rangle$ or in state $|\psi_2\rangle$, but now the two possibilities are intrinsically *distinguishable*. Mandel [7] showed that, in any intermediate case, the density operator

$$\begin{aligned}\widehat{\rho} = {}& \rho_{11} |\psi_1\rangle\langle\psi_1| + \rho_{12} |\psi_1\rangle\langle\psi_2| + \rho_{21} |\psi_2\rangle\langle\psi_1| \\ &+ \rho_{22} |\psi_2\rangle\langle\psi_2|\end{aligned} \tag{7}$$

can be uniquely expressed in the form

$$\widehat{\rho} = \mathscr{I}\widehat{\rho}_{\rm ID} + (1-\mathscr{I})\widehat{\rho}_{\rm D}, \quad 0 \leqslant \mathscr{I} \leqslant 1. \tag{8}$$

Mandel defined $\mathscr{I}$ as the *degree of indistinguishability*. If $\mathscr{I} = 0$, the two paths are completely distinguishable, i.e., one has complete WPI; and if $\mathscr{I} = 1$, they are completely indistinguishable, i.e., one has no WPI. In the intermediate case $0 < \mathscr{I} < 1$, the two possibilities may be said to be partially distinguishable. Evidently, $\mathscr{I}$ provides a measure of WPI.

According to Eqs. (7) and (8), one can always express $\widehat{\rho}$ in the form

$$\begin{aligned}\widehat{\rho} = {}& |\alpha_1|^2 |\psi_1\rangle\langle\psi_1| + |\alpha_2|^2 |\psi_2\rangle\langle\psi_2| + \mathscr{I}(\alpha_1^*\alpha_2 |\psi_2\rangle\langle\psi_1| \\ &+ \alpha_2^*\alpha_1 |\psi_1\rangle\langle\psi_2|).\end{aligned} \tag{9}$$

Clearly, the condition of "best circumstances" requires that $|\alpha_1| = |\alpha_2|$.

For the sake of simplicity, let us assume that $|\psi_1\rangle$ and $|\psi_2\rangle$ are of the form

$$|\psi_1\rangle = |1\rangle_x |0\rangle_y , \tag{10a}$$
$$|\psi_2\rangle = |0\rangle_x |1\rangle_y , \tag{10b}$$

where the two states are labeled by the same (vector) mode $\mathbf{k}$, and x, y are two mutually orthogonal directions, both perpendicular to the direction of $\mathbf{k}$ [for the sake of brevity, $\mathbf{k}$ is not displayed in Eq. (10)]. Evidently, $|\psi_1\rangle$ represents the state of a photon polarized along the x direction, and $|\psi_2\rangle$ represents a state polarized along the y direction.

In the present case, one may express $\widehat{E}_i^{(+)}(\mathbf{r},t)$ in the form

$$\widehat{E}_i^{(+)}(\mathbf{r},t) = Ce^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\widehat{a}_i, \quad i = x,y, \tag{11}$$

where the operator $\widehat{a}_i$ represents annihilation of a photon in mode $\mathbf{k}$, polarized along the i axis, and C is a constant. From Eqs. (1), (9), and (11), one can readily show that the quantum-polarization matrix $\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t)$ has the form

$$\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t) = |C|^2\begin{pmatrix} |\alpha_1|^2 & \mathscr{I}\alpha_1^*\alpha_2 \\ \mathscr{I}\alpha_1\alpha_2^* & |\alpha_2|^2 \end{pmatrix}. \tag{12}$$

From Eqs. (2) and (12), and using the fact $|\alpha_1|^2 + |\alpha_2|^2 = 1$, one then finds the degree of polarization to be given by the expression

$$\mathscr{P} = \sqrt{(|\alpha_1|^2 - |\alpha_2|^2)^2 + 4|\alpha_1|^2|\alpha_2|^2\mathscr{I}^2}. \tag{13}$$

It follows from Eq. (13) by simple calculations that

$$\mathscr{P}^2 - \mathscr{I}^2 = (1-\mathscr{I}^2)(2|\alpha_1|^2 - 1)^2. \tag{14}$$

Since $0 \leqslant \mathscr{I} \leqslant 1$, one has

$$\mathscr{P} \geqslant \mathscr{I}. \tag{15}$$

Therefore, the degree of polarization of the output light in a single-photon interference experiment is always greater or equal to the degree of indistinguishability.

Let us now assume that the condition of "best circumstances" holds, i.e., one has $|\alpha_1|^2 = |\alpha_2|^2$. Equation (13) then reduces to

$$\mathscr{P} = \mathscr{I}. \tag{16}$$

Let us now discuss the physical interpretation of this result. If one has complete which-path information (i.e., $\mathscr{I} = 0$), then from Eq. (16) it follows that the degree of polarization of the light emerging from the interferometer is equal to zero. Therefore, when a photon behaves like a particle in a single-photon interference experiment, the light generated by superposition is completely unpolarized, under the best circumstances. In the other extreme case, when one has no which-path information, i.e., when a photon does *not* display any particle behavior, the output light will be completely polarized. Any intermediate case will produce partially polarized light.

We summarize our result by saying that, in a single-photon interference experiment, the polarization properties of the light emerging from the interferometer depend on whether a photon behaves like a particle or like a wave. Clearly such a phenomenon is completely quantum mechanical in nature and cannot be interpreted by the use of classical theory.

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