Stationary entanglement achievable by environment-induced chain links

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We investigate the possibility of chaining qubits by letting pairs of nearest-neighbor qubits dissipate into common environments. We then study entanglement dynamics within the chain and show that steady-state entanglement can be achieved.

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I. INTRODUCTION

It is nowadays well established that entanglement represents a fundamental resource for quantum information tasks [1]. However, being a purely quantum feature, it is fragile with respect to environmental contamination. Notwithstanding that, the possibility of achieving entangled states as stationary ones in open quantum systems has been put forward in many different contexts (for those concerning qubit systems, see, e.g., Refs. [2,3]). The subject has attracted a lot of attention, until a recent striking experiment on long-lived entanglement [4]. The works on this topic can be considered as falling into two main categories: one where all qubits are plunged in the same environment [2] and the other where each qubit is plunged in its own environment [3]. In the former case, the environment can provide an indirect interaction between otherwise decoupled qubits and thus a means to entangle them. In the latter case, a direct interaction between qubits is needed to create entanglement, and to maintain it, one usually has to also exploit other mechanisms (state resetting, driving, etc.).

Here we consider a hybrid situation as depicted in Fig. 1. It represents a sort of spin-$\frac{1}{2}$ chain dimerized by environments. In practice, each environment induces a chain link between contiguous qubits. Hence, we can expect that a simple dissipative dynamics in such a configuration is able to establish entanglement along the chain without the need to exploit any other mechanism. Actually, we will show, for the case of three qubits, the possibility of achieving stationary entanglement for each qubit pair. The amount of entanglement results is strongly dependent on the initial (separable) state. Also the dependence from the chain boundary conditions (open or closed) will be analyzed, as well as a left-right asymmetry in the qubit-environment interaction.

The layout of this paper is the following. In Sec. II we introduce the model relying on physical motivations and we discuss the general dynamical properties. In Sec. III we restrict our attention to the three-qubit case and investigate the entanglement dynamics in the open boundary condition. In Sec. IV we analyze the same system but with closed boundary conditions. Concluding remarks are presented in Sec. V.

II. THE MODEL

The model of Fig. 1 can be motivated by physically considering two-level atoms inside cavities connected by fibers [5]. In such a scheme, each atom-qubit can be thought of as exchanging energy with the optical modes supported by the fiber. In turn, the latter can be modeled as an environment. Thus each qubit dissipates energy through two environments (one on the left and the other on the right). It happens that two contiguous qubits dissipate energy into the same environment. Then this environment mediates the interaction between the contiguous qubits.

More specifically, let us consider, at the ith site of a chain, a qubit described by ladder operators $\sigma_i^{\pm}$ satisfying the usual spin-$\frac{1}{2}$ algebra $[\sigma_i^+,\sigma_i^-]=\sigma_i^z$. Let us also consider, at the ith site of a chain, radiation modes described by ladder operators $b_{i,j},b_{i,j}^\dagger$ satisfying the usual bosonic algebra $[b_{i,j},b_{i,j'}^\dagger]=\delta_{j,j'}$. Then, the interaction Hamiltonian reads

$$H_I=\sum_i\sum_j \sigma_i^-(b_{i-1,j}^\dagger+b_{i,j})+\text{H.c.}. \tag{1}$$

By considering $b_{i,j}$ as environment operators for the ith qubit, we can use standard techniques [6] to arrive at the following master equation:

$$\frac{\partial\rho}{\partial t}=\sum_i \mathcal{L}_{i,i+1}\rho,$$
$$\mathcal{L}_{i,i+1}\rho=(\sigma_i^-+\sigma_{i+1}^-)\rho(\sigma_i^++\sigma_{i+1}^+) - \tfrac{1}{2}\{(\sigma_i^++\sigma_{i+1}^+)(\sigma_i^-+\sigma_{i+1}^-),\rho\}, \tag{2}$$

where $\{,\}$ denotes the anticommutator and we have assumed a unit decay rate. Since we are interested in the steady state, we have to notice that given a master equation written in the standard Linbladian form,

$$\frac{\partial\rho}{\partial t}=-i[H,\rho]+\sum_i[L_i\rho L_i^\dagger-\tfrac{1}{2}\{L_i^\dagger L_i,\rho\}], \tag{3}$$

the uniqueness of the stationary solution is guaranteed if the only operators commuting with Hamiltonian H and every Lindblad operator L_i are multiples of identity [7].

In the case of Eq. (2), there is no Hamiltonian term and the σ_i^- commute with Lindblad operators. Hence the steady state may not be unique, that is, it may depend on the initial conditions. Due to that, we need to study the full dynamics of the system.

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III. THE THREE-QUBIT CASE WITH OPEN BOUNDARY CONDITIONS

We restrict our attention to a chain of three sites. We first consider open boundary conditions. Then, the dynamics will be described by a master equation that can be easily derived from Eq. (2),

$$\frac{\partial\rho}{\partial t} = \gamma\mathcal{L}_{1,2}\rho + (1-\gamma)\mathcal{L}_{2,3}\rho = \gamma\{2(\sigma_1^- + \sigma_2^-)\rho(\sigma_1^+ + \sigma_2^+) - [(\sigma_1^+ + \sigma_2^+)(\sigma_1^- + \sigma_2^-),\rho]\} + (1-\gamma)\{2(\sigma_2^- + \sigma_3^-) \times \rho(\sigma_2^+ + \sigma_3^+) - [(\sigma_2^+ + \sigma_3^+)(\sigma_2^- + \sigma_3^-),\rho]\}. \quad (4)$$

Here we have considered the possibility, for each qubit, of having an asymmetric decay rate on the left and right environments. This has been accounted for by the real factors γ and $(1-\gamma)$, with the assumption $0 < \gamma < 1$. Clearly the symmetric situation is recovered when $\gamma = 1/2$.

By arranging the density matrix (expressed in the computational basis $\{|e\rangle,|g\rangle\}^{\otimes 3}$) as a vector $\mathbf{v}$ (e.g., $\rho_{i,j} = \mathbf{v}_{8(i-1)+j}$), the master equation (4) can be rewritten as a linear set of differential equations,

$$\dot{\mathbf{v}}(t) = M\mathbf{v}(t), \quad (5)$$

where M is a 64×64 matrix of constant coefficients given by

$$\begin{aligned} M = {} & 2\gamma(\sigma^- \otimes I_2 \otimes I_2 + I_2 \otimes \sigma^- \otimes I_2) \otimes (\sigma^- \otimes I_2 \otimes I_2 + I_2 \otimes \sigma^- \otimes I_2) \\ & - \gamma(\sigma^+ \otimes I_2 \otimes I_2 + I_2 \otimes \sigma^+ \otimes I_2)(\sigma^- \otimes I_2 \otimes I_2 + I_2 \otimes \sigma^- \otimes I_2) \otimes I_8 \\ & - \gamma I_8 \otimes (\sigma^+ \otimes I_2 \otimes I_2 + I_2 \otimes \sigma^+ \otimes I_2)(\sigma^- \otimes I_2 \otimes I_2 + I_2 \otimes \sigma^- \otimes I_2) \\ & + 2(1-\gamma)(I_2 \otimes \sigma^- \otimes I_2 + I_2 \otimes I_2 \otimes \sigma^-) \otimes (I_2 \otimes \sigma^- \otimes I_2 + I_2 \otimes I_2 \otimes \sigma^-) \\ & - (1-\gamma)(I_2 \otimes \sigma^+ \otimes I_2 + I_2 \otimes I_2 \otimes \sigma^+)(I_2 \otimes \sigma^- \otimes I_2 + I_2 \otimes I_2 \otimes \sigma^-) \otimes I_8 \\ & - (1-\gamma) I_8 \otimes (I_2 \otimes \sigma^+ \otimes I_2 + I_2 \otimes I_2 \otimes \sigma^+)(I_2 \otimes \sigma^- \otimes I_2 + I_2 \otimes I_2 \otimes \sigma^-), \end{aligned} \quad (6)$$

with I_n being the $(n \times n)$-dimensional identity matrix and $\sigma^+ = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$, $\sigma^- = \left(\begin{smallmatrix} 0 & 0 \\ 1 & 0 \end{smallmatrix}\right)$. Then, the set of differential Eq. (5) can be converted into a set of algebraic equations via the Laplace transform, $\tilde{\mathbf{v}}(s) = \int_0^\infty \exp(-st)\mathbf{v}(t)$, i.e.,

$$s\tilde{\mathbf{v}}(s) - \mathbf{v}(0) = M\tilde{\mathbf{v}}(s). \quad (7)$$

By decoupling these equations, one finds that the Laplace transforms of the density-matrix elements are rational functions of polynomials, and the inverse Laplace transformation can be performed analytically. The results are not explicitly reported because the expressions are too cumbersome.

Having the density matrix of the system, we can study the entanglement dynamics for each qubit pair of the system. To quantify the amount of entanglement between each of the qubits, we use the concurrence [8]. We recall that to find the concurrence of a bipartite system described by the density matrix ρ, the following steps should be taken:

(i) Find the complex conjugate of the density matrix in the computational basis and denote it by ρ^*.

(ii) Define $\tilde{\rho} := (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, where $\sigma_y = i(\sigma^- - \sigma^+)$.

(iii) Find the square root of the eigenvalues of $\rho\tilde{\rho}$ and sort them in decreasing order: $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$.

(iv) The concurrence is given by

$$C = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}. \quad (8)$$

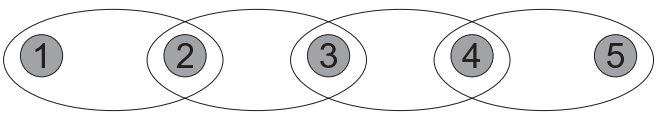

FIG. 1. Environments (ellipses) inducing chain links between contiguous qubits (gray circles).

A. Entanglement dynamics

Figure 2 shows the evolution of entanglement between each qubit pair for the $|eee\rangle$ initial state. As can be seen in this figure, when all the qubits are initially in an excited state, it takes a longer time for the first and the third qubits to become entangled compared to the time needed to generate entanglement between the first and second, or second and third, qubits. As a consequence, for the qubits that are not nearest neighborhood, we have a *sudden birth of entanglement*, i.e., it suddenly becomes nonzero at times greater than zero (it does not smoothly increase starting from the initial time) [9].

If we start with different initial states, the entanglement behaves differently in time. The left panel of Fig. 3 shows the value of concurrence in time for $|eeg\rangle$ as the initial state.

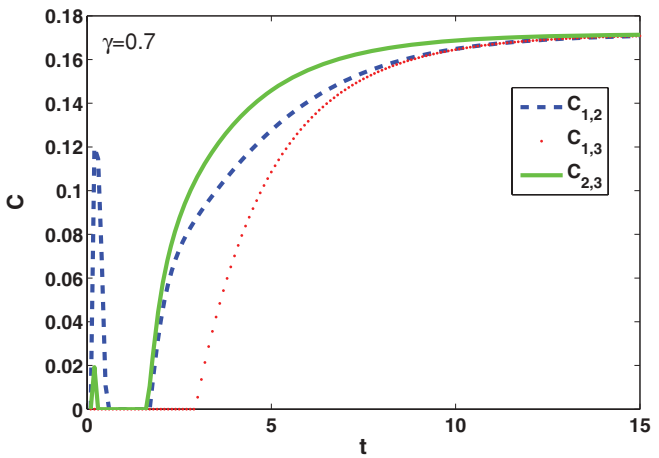

FIG. 2. (Color online) Qubit-pair concurrence vs time for the $|eee\rangle$ initial state. The dashed blue line shows $C_{1,2}$, the solid green line shows $C_{2,3}$, and the dotted red line shows $C_{1,3}$.

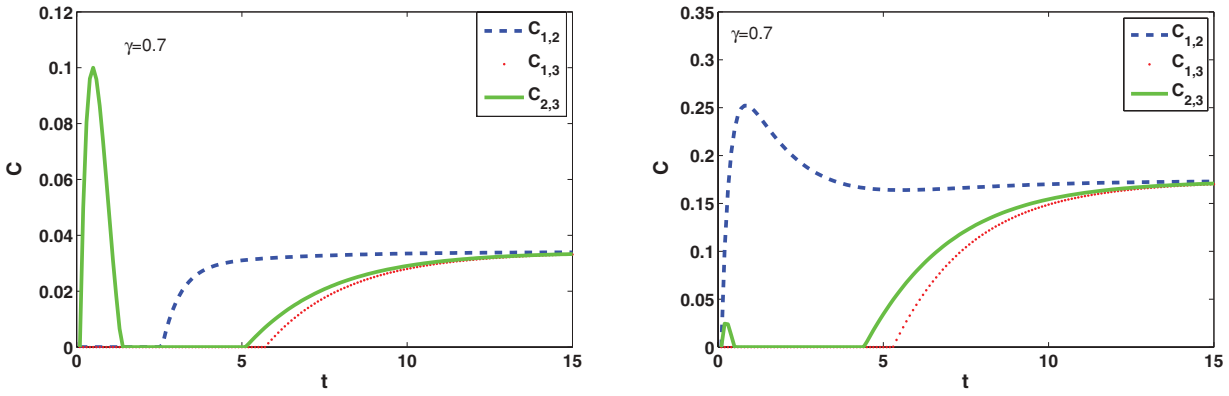

FIG. 3. (Color online) Qubit-pair concurrence vs time for the $|eeg\rangle$ (left) and $|ege\rangle$ (right) initial states. The dashed blue line shows $C_{1,2}$, the solid green line shows $C_{2,3}$, and the dotted red line shows $C_{1,3}$.

Like the previous case, it shows that the time needed by the first and third qubits to become entangled is longer than the time needed by the nearest-neighborhood qubits. Moreover, we still have the entanglement sudden-birth phenomenon, but this time it manifests not only for distant qubits (first and third), but also for the first and second qubits. The interesting point in this figure is that the entanglement generation between the nearest neighbors is quicker if they are initially prepared in the $|eg\rangle$ rather than the $|ee\rangle$ state. The other possibility with two excitations in the initial state is $|ege\rangle$, for which the time evolution of entanglement in shown in the right panel of Fig. 3. This time the entanglement sudden-birth phenomenon only manifests for distant qubits (first and third).

A big difference appears if the number of excitations of the initial state reduces to one. Figure 4, left and right panels, shows entanglement evolution for the initial states $|egg\rangle$ and $|geg\rangle$, respectively. As can be seen in this figure, entanglement is generated between each qubit pair from the beginning, no matter how far they are from each other, i.e., we no longer have the entanglement sudden-birth phenomenon.

Finally, in the case of the $|ggg\rangle$ initial state, there is no entanglement at any time because this state represents a fixed point of the Liouvillian superoperator on the right-hand side of the master equation (4), or, in other words, $M\mathbf{v} = 0$ with $\mathbf{v}_j = \delta_{j,64}$.

B. Stationary entanglement

Taking the limit $t \to \infty$ in the density-matrix elements, we arrive at the following general form for the steady state:

$$\rho_s = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f & 0 & -f & f & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -f & 0 & f & -f & 0 \\ 0 & 0 & 0 & f & 0 & -f & f & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1-3f \end{pmatrix}, \quad (9)$$

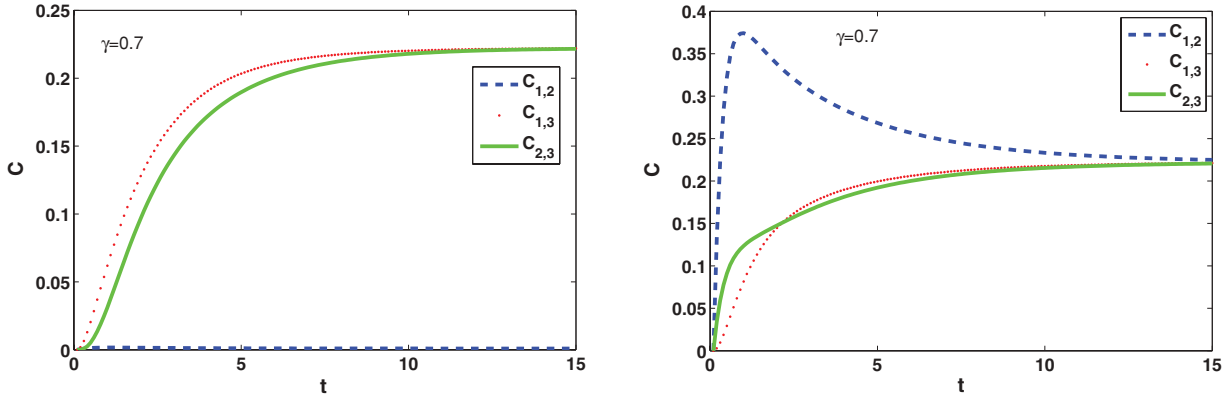

FIG. 4. (Color online) Qubit-pair concurrence vs time for the $|egg\rangle$ (left) and $|geg\rangle$ (right) initial states. The dashed blue line shows $C_{1,2}$, the solid green line shows $C_{2,3}$, and the dotted red line shows $C_{1,3}$.

where $f \in \mathbb{R}$ is determined by the initial state of the system. In particular, we have the following correspondence:

$$\begin{aligned} f &= \frac{24 - 19\gamma + 19\gamma^2}{216 + 81\gamma - 81\gamma^2}, \quad |eee\rangle \\ f &= \frac{4(4 - 5\gamma + \gamma^2)}{27(8 + 3\gamma - 3\gamma^2)}, \quad |eeg\rangle \\ f &= \frac{4(4 + 5\gamma - 5\gamma^2)}{27(8 + 3\gamma - 3\gamma^2)}, \quad |ege\rangle \\ f &= \frac{1}{9}, \quad |egg\rangle, \quad |geg\rangle, \quad |gge\rangle \\ f &= \frac{4\gamma(3 + \gamma)}{27(8 + 3\gamma - 3\gamma^2)}, \quad |gee\rangle \\ f &= 0. \quad |ggg\rangle \end{aligned} \tag{10}$$

To find the amount of entanglement in each qubit pair at the steady state, we first write the reduced density matrices,

$$\rho_{1,2} = \rho_{2,3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & f & -f & 0 \\ 0 & -f & f & 0 \\ 0 & 0 & 0 & 1-2f \end{pmatrix},$$

$$\rho_{1,3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & f & f & 0 \\ 0 & f & f & 0 \\ 0 & 0 & 0 & 1-2f \end{pmatrix}. \tag{11}$$

Then, it is easy to show that the concurrence becomes

$$C_{12} = C_{13} = C_{23} = 2f. \tag{12}$$

Figure 5 shows the stationary entanglement vs γ for different initial states. For initial states such as $|egg\rangle$, $|geg\rangle$,and $|gge\rangle$, the entanglement does not depend on γ. For the $|eee\rangle$ initial state, by increasing the difference in the dissipation rates, more entanglement will be induced in the system. On the contrary, for $|ege\rangle$, the maximum value of entanglement is achieved when the dissipation rates into the two environments are the same. As is expected, the entanglement for the initial states $|eeg\rangle$ and $|gee\rangle$ are equal at $\gamma = 1/2$. In these cases, the maximum value can be attained when there is one excitation in the initial state.

IV. THE THREE-QUBIT CASE WITH CLOSED BOUNDARY CONDITIONS

We now study the chain of three sites with closed boundary conditions. In this case, the master equation (2) reads

$$\begin{aligned} \frac{\partial \rho}{\partial t} = \gamma \mathcal{L}_{1,2}\rho + \mu \mathcal{L}_{2,3}\rho + \nu \mathcal{L}_{3,1}\rho = \gamma\{2(\sigma_1^- + \sigma_2^-)\rho(\sigma_1^+ + \sigma_2^+) - [(\sigma_1^+ + \sigma_2^+)(\sigma_1^- + \sigma_2^-), \rho]\} + \mu\{2(\sigma_2^- + \sigma_3^-)\rho(\sigma_2^+ + \sigma_3^+) \\ - [(\sigma_2^+ + \sigma_3^+)(\sigma_2^- + \sigma_3^-), \rho]\} + \nu\{2(\sigma_3^- + \sigma_1^-)\rho(\sigma_3^+ + \sigma_1^+) - [(\sigma_3^+ + \sigma_1^+)(\sigma_3^- + \sigma_1^-), \rho]\}, \end{aligned} \tag{13}$$

where the possibility, for each qubit, of having an asymmetric decay rate on the left and right environments is accounted for by the real factors γ, μ, and ν ($0 \leqslant \gamma, \mu, \nu \leqslant 1$). Clearly the symmetric situation is recovered when $\gamma = \mu = \nu$ (=1/2).

We recall that for finding the stationary state for the open boundary condition, we have solved the equation

$$[\gamma \mathcal{L}_{1,2} + (1 - \gamma)\mathcal{L}_{2,3}]\rho = 0. \tag{14}$$

It resulted in a nonunique steady state. A special case occurs when we have $|ggg\rangle$ as the initial state. This is invariant under the action of $[\gamma \mathcal{L}_{1,2} + (1 - \gamma)\mathcal{L}_{2,3}]$ and therefore the steady state is the *pure* state $|ggg\rangle$ itself. For other possible initial states, the steady state is a mixed state characterized by a parameter f depending on the initial state [see Eq. (9)]. For the closed boundary conditions, we should solve the equation

$$(\gamma \mathcal{L}_{1,2} + \mu \mathcal{L}_{2,3} + \nu \mathcal{L}_{3,1})\rho = 0 \tag{15}$$

to find the steady state. By comparing it with Eq. (14), it becomes clear that it puts more constraints on the steady state, hence we expect it to have a more restricted form than the one in Eq. (9).

It is easy to check that $|ggg\rangle$ is also invariant under the action of $(\gamma \mathcal{L}_{1,2} + \mu \mathcal{L}_{2,3} + \nu \mathcal{L}_{3,1})$, so it will be one of the possible steady states of the system with a closed boundary condition. Actually it is the unique one. To show that, we again arrange the density matrix (expressed in the computational basis $\{|e\rangle, |g\rangle\}^{\otimes 3}$) as a vector $\mathbf{v}$ (e.g., $\rho_{i,j} = \mathbf{v}_{8(i-1)+j}$). The master equation (13) can be rewritten as a linear set of differential equations,

$$\dot{\mathbf{v}}(t) = M\mathbf{v}(t), \tag{16}$$

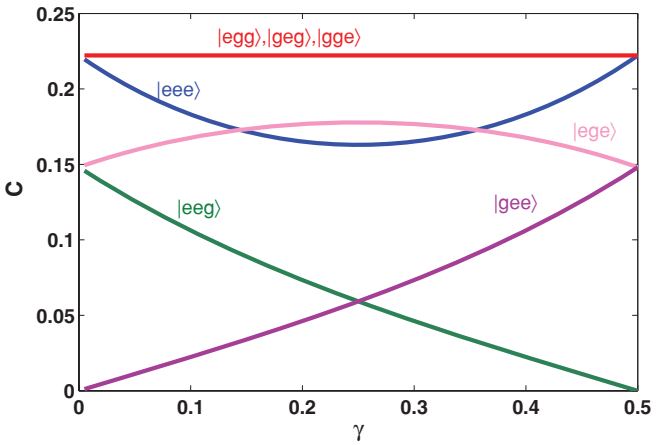

FIG. 5. (Color online) Steady-state entanglement vs γ for different initial states.

where M is a 64×64 matrix of constant coefficients given by

$$\begin{aligned} M = {} & 2\gamma(\sigma^- \otimes I_2 \otimes I_2 + I_2 \otimes \sigma^- \otimes I_2) \otimes (\sigma^- \otimes I_2 \otimes I_2 + I_2 \otimes \sigma^- \otimes I_2) \\ & - \gamma(\sigma^+ \otimes I_2 \otimes I_2 + I_2 \otimes \sigma^+ \otimes I_2)(\sigma^- \otimes I_2 \otimes I_2 + I_2 \otimes \sigma^- \otimes I_2) \otimes I_8 \\ & - \gamma I_8 \otimes (\sigma^+ \otimes I_2 \otimes I_2 + I_2 \otimes \sigma^+ \otimes I_2)(\sigma^- \otimes I_2 \otimes I_2 + I_2 \otimes \sigma^- \otimes I_2) \\ & + 2\mu(I_2 \otimes \sigma^- \otimes I_2 + I_2 \otimes I_2 \otimes \sigma^-) \otimes (I_2 \otimes \sigma^- \otimes I_2 + I_2 \otimes I_2 \otimes \sigma^-) \\ & - \mu(I_2 \otimes \sigma^+ \otimes I_2 + I_2 \otimes I_2 \otimes \sigma^+)(I_2 \otimes \sigma^- \otimes I_2 + I_2 \otimes I_2 \otimes \sigma^-) \otimes I_8 \\ & - \mu I_8 \otimes (I_2 \otimes \sigma^+ \otimes I_2 + I_2 \otimes I_2 \otimes \sigma^+)(I_2 \otimes \sigma^- \otimes I_2 + I_2 \otimes I_2 \otimes \sigma^-) \\ & + 2\nu(I_2 \otimes I_2 \otimes \sigma^- + \sigma^- \otimes I_2 \otimes I_2) \otimes (I_2 \otimes I_2 \otimes \sigma^- + \sigma^- \otimes I_2 \otimes I_2) \\ & - \nu(I_2 \otimes I_2 \otimes \sigma^+ + \sigma^+ \otimes I_2 \otimes I_2)(I_2 \otimes I_2 \otimes \sigma^- + \sigma^- \otimes I_2 \otimes I_2) \otimes I_8 \\ & - \nu I_8 \otimes (I_2 \otimes I_2 \otimes \sigma^+ + \sigma^+ \otimes I_2 \otimes I_2)(I_2 \otimes I_2 \otimes \sigma^- + \sigma^- \otimes I_2 \otimes I_2). \end{aligned} \tag{17}$$

In this case, it is possible to see that

$$M\mathbf{v} = 0 \tag{18}$$

admits a unique solution $\mathbf{v}_j = \delta_{j,64}$, corresponding to $|ggg\rangle$, for all values of γ, μ, and ν. By having the extra constraints for the closed boundary conditions, no parameter is left free to characterize the steady state. Actually in this case, all excitations of the initial state, whichever it is, dissipate to the environment. Hence no entanglement survives at stationary conditions. Then, comparing the steady states for the open and closed boundary conditions, we can loosely speak about entanglement frustration because it is the addition of an extra bath (interaction 1–3) that prevents the state from having entanglement. Likewise, in [10], it is the addition of extra spins (or modes) that prevents maximally entangled states.

V. CONCLUSION

In this work, we have first considered a spin-$\frac{1}{2}$ chain dimerized by environments. By means of a dissipative mechanism, each environment induces a chain link between contiguous qubits. Then we have studied the possibility of having long-lived entanglement without resorting to any other mechanism. In particular, for the case of a three-qubit chain with an open boundary condition, we have classified the amount of stationary entanglement according to some initial (separable) states. Here we have also shown the appearance of the entanglement sudden birth. On the contrary, for the case of a three-qubit chain with a closed boundary condition, we have proven the impossibility of stationary entanglement. This fact can be interpreted as the entanglement frustration phenomenon [10], induced in this context by the imposed periodic boundary conditions.

The proposed scheme lends itself to be extended to $n > 3$ sites, where one can evaluate how entanglement scales as a function of the distance between the two qubits. Moreover, in the limit of large n, these results could also be useful for studying possible connections between quantum phase transitions and reservoir properties.

Finally, the discussed method of generating entanglement seems economical and offers interesting perspectives for the generation of the so-called graph states (useful for measurement-based quantum computation) [11], when one considers network topologies more complicated than a simple chain.

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