Generating and stabilizing the Greenberger-Horne-Zeilinger state in circuit QED: Joint measurement, Zeno effect, and feedback

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In a solid-state circuit QED system, we extend the previous study of generating and stabilizing a two-qubit Bell state [Phys. Rev. A **82**, 032335 (2010)] to a three-qubit GHZ state. In a dispersive regime, we employ the homodyne *joint readout* for multiple qubits to infer the state for further processing, and in particular we use it to stabilize the state directly by means of an *alternate-flip-interrupted* Zeno (AFIZ) scheme. Moreover, the state-of-the-art feedback action based on the filtered current enables not only a deterministic generation of the pre-GHZ state in the initial stage, but also a fast recovery from occasional error in the later stabilization process. We show that the proposed scheme can maintain the state with high fidelity if the efficient quantum measurement and rapid single-qubit rotations are available.

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I. INTRODUCTION

Quantum entanglement is novel and useful because it exhibits correlations that have no classical analog, and it is one of the key ingredients for quantum technology applications such as quantum teleportation, quantum cryptography, quantum dense coding, and quantum computation [1]. The conventional approach to generate quantum entanglement is via unitary two-qubit gates, using the necessary qubit-qubit interactions. Interestingly, instead of employing entangling gates, one can exploit quantum measurement as an alternative means to achieve a similar goal. In both cavity and circuit QED systems, promising ideas along this route were proposed to *probabilistically* create entangled states by means of the homodyne measurement alone [2–5].

The circuit QED system [6–8], a solid-state analog of the conventional quantum optics cavity QED, is a promising solid-state quantum computing architecture. This architecture couples superconducting electronic circuit elements, which serve as the qubits, to harmonic oscillator modes of a microwave resonator, which serve as a "quantum bus" that mediates interqubit coupling and facilitates quantum measurement for the qubit state. Moreover, quantum measurement in this system can be carried out by operating in the dispersive limit, i.e., with a detuning between the resonator and the qubit much larger than their coupling strength. In this limit, the qubit-resonator interaction induces a qubit-state-dependent shift on the resonator's frequency. Then, by measuring the resonator output voltage with a homodyne scheme, information about the qubit state is obtained.

With these advantages, the schemes proposed in Refs. [4,5] for using measurement to create, respectively, two- and three-qubit entangled states in the circuit QED system are attractive. However, in addition to the drawback of being *probabilistic*, the *measurement-only* approach cannot stabilize the generated state. To resolve this problem, the technique of quantum feedback control may emerge as a possible route [9]. In a recent work [10], a feedback scheme was analyzed for the creation and stabilization of the two-qubit Bell states. Owing to using the dispersive joint readout of multiple qubits and performing proper feedback, the scheme leads to results superior to some previous ones. For instance, it enhances the concurrence to values higher than 0.9, by noting the 0.31 obtained in Ref. [11]; and also it avoids the experimental difficulty in the jump-based feedback [12] or the complexity for a state-estimation feedback [2,13,14].

In this work, we extend the study of generating and stabilizing the two-qubit Bell state in Ref. [10] to the three-qubit GHZ state. To our knowledge, unlike the feedback control of a two-qubit Bell state [10–12,15,16], schemes for stabilizing a three-qubit GHZ state are not well investigated. In Ref. [2] elegant analysis is carried out for preparing the Dicke state of an ensemble of atoms (qubits) in a cavity, by means of projective measurement in a dispersive limit, and as well by using feedback to make the scheme *deterministic*. However, the ability of stabilizing the generated Dicke state against decoherence is not demonstrated. In the present work, we plan to exploit the advantages of the homodyne *joint readout* in a dispersive regime for multiple qubits, to infer the state for further processing, and in particular to stabilize the target state directly by means of an *alternate-flip-interrupted* Zeno (AFIZ) scheme. Also, the state-of-the-art feedback action properly designed according to the measurement current enables both a *deterministic* generation of the pre-GHZ state in the initial stage and a fast recovery from occasional error in the later stabilization process.

Before proceeding to the details of the proposed scheme, we first briefly outline the control efficiency. For the deterministic generation of the pre-GHZ state, keeping track of the joint measurement information together with simple individual qubit rotations in our scheme will either lead to a direct subsequent success of target state generation with probability $1/2$, being higher than the $1/4$ given by the naive rerunning scheme, which additionally needs the difficult "data-clearing" procedures, or avoid clearing the wrong state before rerunning the generation scheme. More importantly, the AFIZ stabilization protocol, based only on an alternate but regular qubit flips and continuous measurement, can maintain very high fidelity (higher than 90%) for a considerably long time (much longer than the single-qubit decoherence time). In principle, if the continuous measurement can approach the effect of

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fast repeated strong projective measurement, i.e., the ideal Zeno effect, the AFIZ scheme can stabilize the pre-GHZ state for an arbitrarily long time. Meanwhile, another remarkable advantage of the AFIZ scheme is that it greatly simplifies the unitary manipulations on qubits, compared to either the state-based or current-based continuous feedback. Moreover, associated with the AFIZ scheme, an auxiliary alarm to the failure of AFIZ control, which may occur *occasionally* owing to the finite strength of the measurement, can restart the fast deterministic generation of the target state by simply using two-qubit flips, one-qubit ($\pi/2$) rotation, and two projective measurements.

II. MODEL AND FORMALISM

We consider a specific circuit QED system consisting of three superconducting qubits coupled to the fundamental mode of a microwave resonator cavity. The qubits, the resonator cavity, and their mutual coupling are well described by the Jaynes-Cummings Hamiltonian [6,17]:

$$H = \omega_r a^\dagger a + \mathcal{E}(a^\dagger + a) + \sum_{j=1}^{3}\left[\frac{\Omega_j}{2}\sigma_j^z + g_j(\sigma_j^- a^\dagger + \sigma_j^+ a)\right]. \tag{1}$$

The operators $\sigma_j^-(\sigma_j^+)$ and $a(a^\dagger)$ are, respectively, the lowering (raising) operators for the jth qubit and the resonator cavity photon (hereafter we will call it a *cavity photon* for simplicity). ω_r is the frequency of the cavity photon, and Ω_j and g_j are the jth qubit transition energy and its coupling strength to the cavity photon, respectively. In this work we consider a three-qubit setup as shown schematically in Fig. 1, which can result in the required sign, $\mathrm{sgn}(g_1) = \mathrm{sgn}(g_2) = -\mathrm{sgn}(g_3)$. The $\mathcal{E}$ term in Eq. (1) stands for a microwave driving to the resonator cavity that is employed for the task of measurement. Explicitly, $\mathcal{E} = \epsilon e^{-i\omega_m t} + \text{c.c.}$, where the driving frequency can differ from the cavity photon frequency, i.e., $\Delta_r \equiv \omega_r - \omega_m \neq 0$. Moreover, we will focus on a dispersive limit measurement [6–8], i.e., with the energy detuning $\Delta_j = \omega_r - \Omega_j$ much larger than g_j. In this limit, the canonical transformation, $H_{\text{eff}} \simeq U^\dagger H U$, where $U = \exp[\sum_j \lambda_j(a\sigma_j^+ - a^\dagger\sigma_j^-)]$ with $\lambda_j = g_j/\Delta_j$, yields (in a *joint rotating frame* with the driving frequency ω_m with respect to both the cavity photon and each individual qubit)

$$H_{\text{eff}} \simeq \Delta_r a^\dagger a + (\epsilon^* a + \epsilon a^\dagger) + \sum_{j=1}^{3}(\omega_j + \chi_j)\frac{\sigma_j^z}{2} + \sum_{j=1}^{3}\chi_j a^\dagger a \sigma_j^z, \tag{2}$$

where $\omega_j = \Omega_j - \omega_m$ and $\chi_j = g_j^2/\Delta_j$. Here we have neglected the virtual cavity-photon-mediated effective coupling between qubits, as is appropriate for sufficient detuning between qubits [5].

In the circuit-QED system, the measurement is typically performed via a homodyne detection of the transmitted microwave photons. The photon's leakage from the resonator cavity is described by a Lindblad term $\kappa\mathcal{D}[a]\rho$ in the master equation, where κ is the leakage rate and the Lindblad superoperator acting on the reduced density matrix ρ is defined by $\mathcal{D}[a]\rho = a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}$. However, conditioned on the output homodyne current, i.e., $I_{\text{hom}}(t) = \kappa\langle a + a^\dagger\rangle_c(t) + \sqrt{\kappa}\xi(t)$, there will be an additional unraveling term in the conditional master equation, $\mathcal{H}[a]\rho_c\xi(t)$. Here $\langle(\cdots)\rangle_c(t) \equiv \mathrm{Tr}[(\cdots)\rho_c(t)]$ with $\rho_c(t)$ the conditional density matrix, and $\mathcal{H}[a]\rho_c \equiv a\rho_c + \rho_c a^\dagger - \mathrm{Tr}[(a + a^\dagger)\rho_c]\rho_c$. The Gaussian white noise $\xi(t)$, which has the ensemble average properties $E[\xi(t)] = 0$ and $E[\xi(t)\xi(t')] = \delta(t - t')$, stems from the quantum-jump-related stochastic nature.

In this work, we assume a *strongly damped* resonator cavity, which enables us to adiabatically eliminate the cavity photon degree of freedom [4,18]. Qualitatively, observing the effective coupling $\sum_{j=1}^{3}\chi_j a^\dagger a \sigma_j^z$ in Eq. (2), we can understand that the fluctuation of the photon number will cause a pure dephasing backaction onto the qubits, with a joint dephasing operator $J_z = \sum_{j=1}^{3}\delta_j\sigma_j^z$, where $\delta_j = \chi_j/\bar{\chi}$ and $\bar{\chi} = \sum_{j=1}^{N}\chi_j/N$. Thus, we can expect the following results after adiabatic elimination of the photon's degree of freedom: (i) the dephasing term $\sim \mathcal{D}[J_z]\rho_c$, (ii) the unraveling term $\sim \mathcal{H}[J_z]\rho_c\xi(t)$, and (iii) the homodyne current $I_{\text{hom}}(t) \sim \langle J_z\rangle_c(t) + \xi(t)$. Indeed, following the standard procedures of adiabatic elimination [4,18], an *effective* quantum trajectory equation (QTE) involving only the degrees of freedom of qubits can be obtained as

$$\dot{\rho}_c = \mathcal{L}\rho_c + \frac{\Gamma_d}{2}\mathcal{D}[J_z]\rho_c + \frac{\sqrt{\Gamma_m}}{2}\mathcal{H}[J_z]\rho_c\xi(t), \tag{3}$$

in which the Liouvillian is defined as

$$\mathcal{L}\rho_c = -i\left[\sum_j \frac{\omega_j + \chi_j}{2}\sigma_j^z + \bar{\chi}|\alpha|^2\sum_j \delta_j\sigma_j^z, \rho_c\right] + \sum_j(\gamma_j + \gamma_{pj})\mathcal{D}[\sigma_j^-]\rho_c + \sum_j \frac{\gamma_{\phi j}}{2}\mathcal{D}[\sigma_j^z]\rho_c.$$

Here we have also assumed a resonant driving, i.e., $\Delta_r = 0$. In Eq. (3), γ_j and $\gamma_{\phi j}$ are the relaxation and dephasing rates caused by the surrounding environments, respectively. Since the external dephasing can be strongly suppressed by proper design of the superconducting qubits, we will thus neglect it in our following simulations. The γ_{pj} terms, with $\gamma_{pj} = \kappa\lambda_j^2$, stem from the so-called Purcell effect, describing an *indirect* qubit decoherence induced by the damping of the cavity photons. Since we assumed mutually distinct

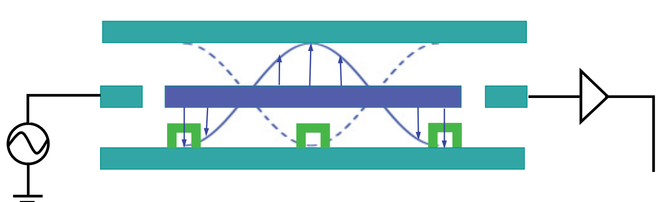

FIG. 1. (Color online) Schematic diagram of the circuit QED with three qubits, together with a microwave transmission measurement in a dispersive limit. The Cooper-pair box qubits are fabricated inside a superconducting transmission-line resonator and are capacitively coupled to the voltage standing wave.

frequencies of qubits, we can treat the Purcell effect under secular approximation, i.e., neglecting the cross-terms in $\mathcal{D}[\sum_j g_j\sigma_j^-]\rho_c$. (The cross-terms characterize *interference* between the radiation from different qubits and become important only if the qubit frequencies are sufficiently close.) With these considerations, the Purcell-effect-induced and other environment-caused decoherences can be equally treated. Therefore, we combine them by $\gamma = \gamma_j + \gamma_{pj}$. Moreover, the measurement-backaction-induced dephasing rate $\Gamma_d = 8|\alpha|^2\bar{\chi}^2/\kappa$, with $\alpha = -2i\epsilon/\kappa$. And finally, the information-gain rate Γ_m in Eq. (3) is in general related to the backaction dephasing rate in terms of the quantum efficiency, $\eta = \Gamma_m/(2\Gamma_d)$.

III. FILTERING THE OUTPUT CURRENT

After adiabatic elimination of the cavity degree of freedom, we obtain an effective measurement operator $J_z = \sum_{j=1}^{3}\delta_j\sigma_j^z$, which has a number of discrete eigenvalues. However, in the practical homodyne measurement, the output record is the homodyne current, which we rewrite as $dI_{\text{hom}}(t) = \sqrt{\Gamma_m}\langle J_z\rangle_c(t)dt + dW(t)$, where $dW(t) = \xi(t)dt$ is the Wiener increment that has the statistical properties $E[dW(t)] = 0$ and $E[dW(t)dW(s)] = \delta(t-s)dt$. In this context, we remind readers that we are actually employing a series of photons by using their transmission through the cavity to measure the qubit state. Accordingly, the stochastic Wiener increment in the homodyne current just characterizes the very weak (partial) random collapse caused by the individual measuring photons. It is the sum of the $\langle J_z\rangle_c(t)$ term plus the Wiener increment that corresponds to the measurement record *after* the (infinitesimal) partial collapse, while $\langle J_z\rangle_c(t)$, from its definition $\langle J_z\rangle_c(t) = \text{Tr}[J_z\rho_c(t)]$, exposes the information of state *before* the infinitesimal collapse.

Of great interest and very usefully, instead of knowing $\langle J_z\rangle_c(t)$ from the usual ensemble measurement by repeating a large number of realizations (noting that the quantum mechanical expectation indicates a statistical or ensemble-average interpretation), we can approximately obtain it in the context of continuous weak measurement by averaging the homodyne current over a properly chosen window of time. This has some similarity to the *ergodic* assumption in statistical physics, where the average along time is assumed to replace the ensemble average. In practice, we can low-pass filter the homodyne current over a small time window [19] and get a smoothed signal as $\bar{I}_{\text{hom}}(t) = \frac{1}{\mathcal{N}}\int_{t-T}^{t} e^{-\gamma_{\text{ft}}(t-\tau)}dI_{\text{hom}}(\tau)$, where γ_{ft} is a low-pass filtering parameter, and the factor $\mathcal{N}$ normalizes the smoothed signal to a maximum magnitude of unity. A numerical test shows that $\bar{I}_{\text{hom}}(t)$ indeed coincides with $\langle J_z\rangle_c(t)$ satisfactorily. Thus, in the numerical simulations of this work, we simply use $\langle J_z\rangle_c(t)$ as a state indicator to guide our feedback manipulations. In particular, after the qubits have experienced sufficient transmission measurement by a large number of photons and fully collapse onto one eigenstate of the measurement operator, we can in principle unambiguously infer it from the filtered current $\langle J_z\rangle_c(t)$.

Finally, we remark that, in regard to feedback design, $\langle J_z\rangle_c(t)$ in the homodyne current $dI_{\text{hom}}(t)$ is informative about the state and is thus useful, yet the $dW(t)$ term is harmful. In other words, the $dW(t)$ term is harmful the homodyne current has *distinct roles* when using the current to update the state *versus* to perform feedback. In doing the former, it is necessary, whereas in doing the latter, it is useless and should better be erased. This understanding was demonstrated in recent work [10].

IV. DETERMINISTIC GENERATION OF THE PRE-GHZ STATE

Since our control target, say, the GHZ state $|000\rangle + |111\rangle$, can be easily obtained by a simple flip of a single qubit (i.e., the third one) from $|001\rangle + |110\rangle$, which may be accordingly named a *pre-GHZ* state, we can aim to control this pre-GHZ state instead. From the QTE formalism outlined above in a dispersive regime, we see that an effective measurement operator reads $J_z = \sum_j \delta_j\sigma_j^z$. Noting that the consequence of a quantum measurement is to collapse an arbitrary state onto one of the eigenstates of the measurement operator, we may design $J_z = \sigma_1^z + \sigma_2^z + 2\sigma_3^z$, which makes the pre-GHZ state be its eigenstate with eigenvalue $J_z = 0$. In practice, this can be realized by setting the dispersive shifts $\chi_1{:}\chi_2{:}\chi_3 = 1{:}1{:}2$.

More specifically, let us start with an initially separable state:

$$\begin{aligned}|\Psi_i\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_1 \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_2 \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_3 \\ &= \frac{1}{\sqrt{8}}[|000\rangle + |111\rangle + (|010\rangle + |100\rangle) \\ &\quad + (|011\rangle + |101\rangle) + (|001\rangle + |110\rangle)]. \end{aligned} \tag{4}$$

Performing the above designed homodyne measurement, in an individual single realization, would collapse $|\Psi_i\rangle$ *stochastically* onto one of the eigenstates of J_z. According to the principle of quantum projective measurement, $|\Psi_i\rangle$ would collapse onto the pre-GHZ state $|001\rangle + |110\rangle$ with probability 1/4, as a result of getting the record $J_z = 0$. However, there are probabilities of getting other results. That is, the state would collapse onto $|011\rangle + |101\rangle$ or $|100\rangle + |010\rangle$ with probability 1/4, depending on the result being $J_z = 2$ or -2. It may also collapse onto $|111\rangle$ or $|000\rangle$ with probability 1/8 if one gets $J_z = 4$ or -4.

What we described above is in fact a *measurement alone* scheme to generate the pre-GHZ state *stochastically*. Below we show that proper current-based feedback manipulations can make the scheme *deterministic*. If, unfortunately, we did not get the result $J_z = 0$, we may adopt the following distinct procedures based on the specific measurement results obtained: (i) If the result is $J_z = 2$, which indicates the state $|011\rangle + |101\rangle$ projected out, we perform a σ_x flip on the first qubit and a $\pi/2 - \sigma_y$ rotation on the third qubit. Noting that $|011\rangle + |101\rangle$ can be rewritten as $(|01\rangle + |10\rangle) \otimes |1\rangle$, it is clear that the above rotations will transform it to $|000\rangle + |111\rangle + |001\rangle + |110\rangle$, which then has a new probability of 1/2 in the successive measurement to be collapsed onto the pre-GHZ state $|001\rangle + |110\rangle$. (ii) Similarly, for the result $J_z = -2$, a σ_x flip on the first qubit and a $3\pi/2 - \sigma_y$ rotation on the third one can be performed to achieve the same goal as described in (i). (iii) If the measurement result is $J_z = 4$ or -4, which indicates the state $|111\rangle$ or $|000\rangle$ obtained, we then apply a $\pi/2 - \sigma_y$ or a

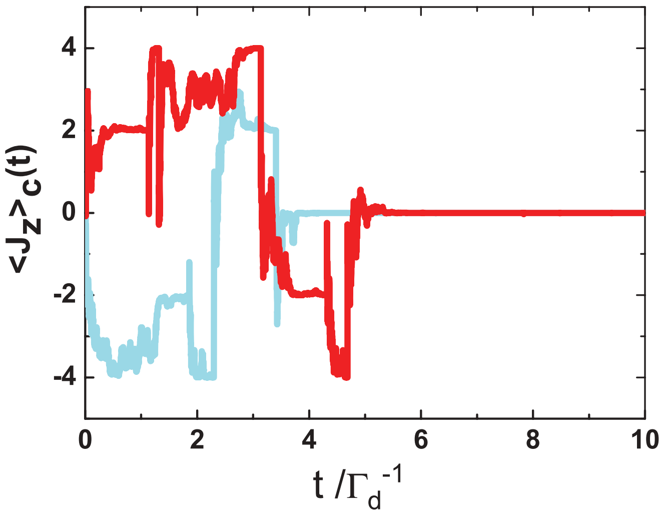

FIG. 2. (Color online) Two representative quantum trajectories showing the deterministic generation of the pre-GHZ state.

$3\pi/2 - \sigma_y$ rotation on each qubit, making the state return back to the initial one $(|0\rangle + |1\rangle)_1 \otimes (|0\rangle + |1\rangle)_2 \otimes (|0\rangle + |1\rangle)_3$, which allows us to rerun the generating procedures.

In Fig. 2 we show two representative quantum trajectories that are *deterministically* guided to the pre-GHZ state. The quantity $\langle J_z\rangle_c(t)$ plotted here is an appropriate indicator for the finally collapsed state. Here we remark that our present scheme of entangled state generation is *efficient*. Generally speaking, a probabilistic scheme of quantum information processing needs to recycle the process with the same initial state. This will require a procedure of clearing the *unwanted data* (qubit states), which is, unfortunately, not an easy job in quantum system compared to its classical counterpart. On the contrary, in addition to that we do not require "data clearing," our scheme in case (ii) needs only two single-bit rotations, and the subsequent success probability of projection (i.e., 1/2) is higher than the recycling scheme, which has a success probability 1/4. Even in the worst case (iii), the manipulation of qubit rotations is identical to rerunning the generation procedure, but the process of "data clearing" is avoided.

V. STABILIZATION USING FLIP-INTERRUPTED ZENO PROJECTION AND FEEDBACK

In the above, we discuss an efficient *deterministic* scheme to generate the pre-GHZ state. However, under the unavoidable influence of the surrounding environment, this state will degrade if we do not provide proper active protections. Below we first propose an *alternate-flip-interrupted* Zeno (AFIZ) stabilization scheme to enhance considerably the lifetime of the pre-GHZ state, then design additional manipulation to prevent the state from degradation. Actually, the pre-GHZ state is an eigenstate of the measurement operator J_z. Then, if one performs a continuous observation (measurement) on it, the quantum Zeno effect would attempt to freeze the state. Using the Zeno effect to stabilize the quantum state is an interesting topic in quantum physics and particularly in quantum computing [20,21]. However, as we will see shortly, generalization of the Zeno protection from single to multiple qubits will suffer more complexities. As a major contribution of the present work, we will see that the proposed AFIZ stabilization scheme can greatly improve the control quality.

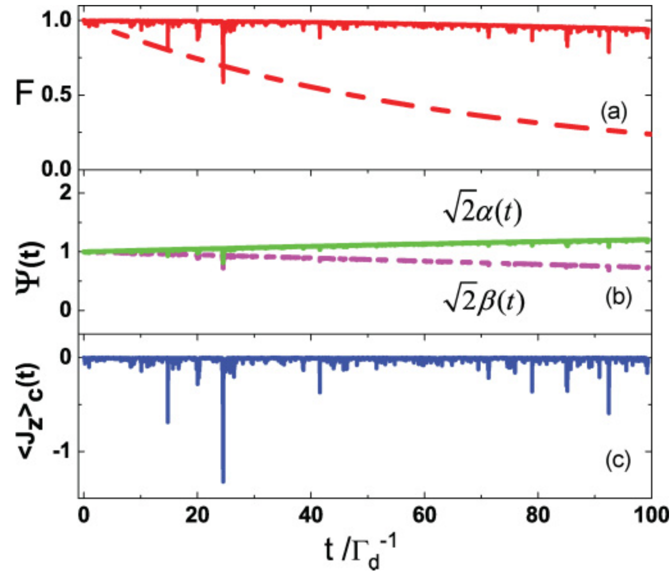

FIG. 3. (Color online) (a) State fidelity under the conventional quantum Zeno (not the AFIZ) stabilization for the pre-GHZ state, showing the result (solid line) much better than the uncontrolled one (dashed line). (b) Detailed inspection for the Zeno pulled-back state in (a), $|\Psi(t)\rangle = \alpha(t)|001\rangle + \beta(t)|110\rangle$, showing a gradual deviation from the target state $|\Psi_T\rangle = (|001\rangle + |110\rangle)/\sqrt{2}$. (c) Unconscious output current for the changing state $|\Psi(t)\rangle$. Single-qubit decoherence rate: $\gamma = 0.01\Gamma_d$.

In Fig. 3(a) we plot the state fidelity under Zeno protection against the one in the absence of such protection, where the Zeno effect is *automatically* realized via the continuous J_z-type measurement as discussed above. We see that, provided the measurement strength is much stronger than the decoherence rate, the effect of Zeno stabilization is obvious. Regarding the underlying mechanism of Zeno stabilization for a multiple-qubit state, it is analogous to that for a single qubit. That is, while the environment is causing the state to move away from the target state to other unwanted states, the relatively strong continuous measurement is at the same time pulling it back. Since our target state is a superposition of $|001\rangle$ and $|110\rangle$, it should, however, be more fragile than the *one-component* state in regard to the Zeno stabilization.

We may understand this point better as follows. Owing to coupling with environment, the qubits would experience an entangling evolution with the environment, for an infinitesimal interval of time. Then the measurement projects the qubit state back. Since each individual qubit couples to the environment independently, this pull-back action via measurement from entangling with the environment cannot guarantee the *reorganized* components $|001\rangle$ and $|110\rangle$ with exactly *unchanged* superposition weights. And, unfortunately, the changed superposition amplitudes cannot be distinguished by the measurement, since the arbitrary superposition of $|001\rangle$ and $|110\rangle$ is the eigenstate of the measurement operator and will result in the same output current. In Figs. 3(b) and 3(c) we show, respectively, the gradual change of the superposition amplitudes away from the initial value $1/\sqrt{2}$ and the corresponding output current.

The Zeno-effect-induced pull-back action, in terms of quantum measurement language, corresponds to a null-result record of spontaneous emission of the qubits. Conditioned on the null result of spontaneous emission, an effective Hamiltonian governing the qubits state evolution reads $\tilde{H}_{\rm qu} = H_{\rm qu} - i\frac{\gamma}{2}\sum_{j=1}^{3}\sigma_j^{+}\sigma_j$, where $H_{\rm qu}$ stands for the qubits Hamiltonian contained in Eq. (2). This effective Hamiltonian acting on the target pre-GHZ state $|\Psi_T\rangle = (|001\rangle + |110\rangle)/\sqrt{2}$, or more generally on $|\Psi\rangle = \alpha|001\rangle + \beta|110\rangle$, leads to an effective evolution as $|\Psi(t)\rangle = (\alpha e^{-\gamma t/2}|001\rangle + \beta e^{-\gamma t}|110\rangle)/||\cdot||$, where $||\cdot||$ denotes the normalization factor. We have checked that this effective evolution is in perfect agreement with the results from numerical simulation, e.g., the one shown in Fig. 3(b). Then, favorably, we get an insight that the change of the superposition amplitudes is owing to the *unbalance* of qubit states "0" and "1" in the components $|001\rangle$ and $|110\rangle$. Based on this observation, quite simply, if we flip simultaneously each of all the three qubits after the Zeno stabilization continuing for a time period τ, the pre-GHZ state will be restored from the flipped state after an equal time interval τ. In practice, the time interval τ can be chosen as a few Γ_d^{-1}, since on such a short timescale the variations of the state amplitudes α and β are negligibly small, provided $\gamma \ll \Gamma_d$. We can expect and will demonstrate in the following that this *alternate evolution*, which is called in this work an *alternate-flip-interrupted* Zeno (AFIZ) scheme, is capable of stabilizing the pre-GHZ state very efficiently. As an interesting remark, this AFIZ scheme has a certain similarity to the *refocus* technique in the echo physics in quantum optics or nuclear magnetic resonance, where a similar flip is manipulated to cause an inverse evolution toward the initial state, i.e., to produce an echo.

Unfortunately, to implement the above AFIZ protection in practice, there should exist very small but nonzero probabilities to collapse the qubits' state to $|000\rangle$ and a mixture of $|100\rangle$ and $|010\rangle$, owing to the finite strength of the measurement. (i) For the result of $|000\rangle$, the output current would trigger a feedback action as described in the deterministic generation scheme, which will force the state rapidly back to the pre-GHZ state under the guided efficient projection of measurement. (ii) For the mixture of $|100\rangle$ and $|010\rangle$, the output current will also trigger a feedback action as described in the deterministic generation scheme. As a result, besides being projected to $|000\rangle$ and $|111\rangle$, which will be further guided to the pre-GHZ state, a mixed state with $|001\rangle$ and $|110\rangle$ will be filtered out by the measurement. Very unfortunately, this mixed state is *not* the pre-GHZ state, but with the same zero output current. To eliminate this error, one can perform a flip action on the third qubit. Then, a mixed state with $|000\rangle$ and $|111\rangle$ is formed, and the rapid deterministic generation procedures will be triggered.

In Fig. 4 we show the numerical result of stabilizing the pre-GHZ state, based on the measurement and feedback schemes described above. The stabilization dynamics is illustrated by both the state fidelity and the output current. We see that at most times the current is zero, only interrupted occasionally by jumps between 0, ± 2, and ± 4. The flat current indicates the stage of Zeno stabilization, during which the quality of the pre-GHZ state is maintained at a desirable high level, with a state fidelity larger than 0.9.

The state fidelity of a single quantum trajectory has a certain stochasticity, owing to the measurement-induced quantum jumps. We may follow the conventional way to employ the fidelity of the ensemble average state as a reliable figure of merit to characterize the control quality. Figure 5(a) shows the results of ensemble average fidelity, for $\gamma = 10^{-2}\Gamma_d$ and $10^{-3}\Gamma_d$, respectively. We notice that, even for $\gamma = 10^{-2}\Gamma_d$, which in most cases such as the one- or two-qubit feedback control is taken as a tolerable error rate, the average fidelity can be higher than 0.9, while for a smaller error rate such as $\gamma = 10^{-3}\Gamma_d$ the average fidelity can reach nearly unity, and the individual quantum trajectory also shows a perfect control result as illustrated in Fig. 5(b). In addition, in Fig. 5(c) we

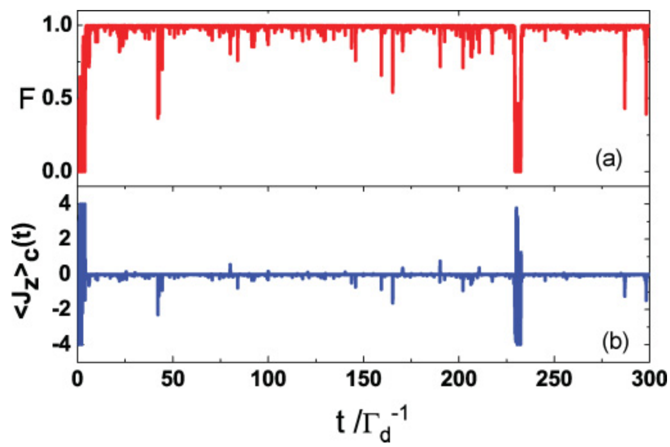

FIG. 4. (Color online) (a) Fidelity of the pre-GHZ state under the combined AFIZ-plus-feedback stabilization. (b) The corresponding output current. Single-qubit decoherence rate: $\gamma = 0.01\Gamma_d$.

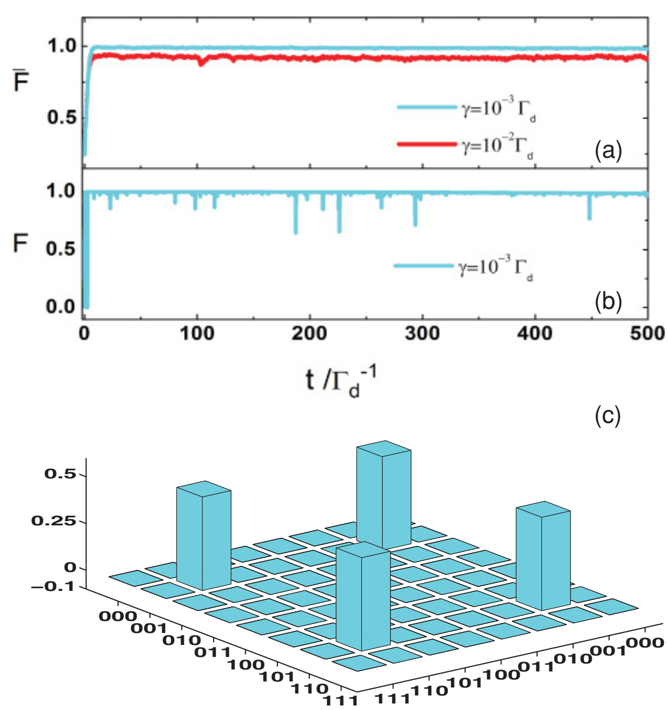

FIG. 5. (Color online) (a) Average fidelity of the pre-GHZ state over 1000 quantum trajectories. (b) Fidelity of an individual realization with $\gamma = 0.001\Gamma_d$, showing perfect control result under this even weaker decoherence when compared to $\gamma = 0.01\Gamma_d$ in Fig. 4. (c) The full state density matrix at a specific time in (b).

present the full density matrix for a representative state during the Zeno stabilization stage, which clearly reveals the quality of the protected pre-GHZ state.

VI. CONCLUDING REMARKS

Finally, we make a number of remarks before summarizing the work. (i) Some approximations are involved, for instance, the rotating-wave approximation contained in the Jaynes-Cummings model Eq. (1), the effective Hamiltonian Eq. (2) in a dispersive regime, and the adiabatic elimination of cavity photons leading to Eq. (3). In the recent work by Liu *et al.* [10], all these approximations are properly justified. (ii) The main problem of doing feedback in circuit QED is the lack of efficient homodyne detection. Currently, the way to perform homodyne and heterodyne detection is to first amplify the signal before mixing it on a nonlinear circuit element of some kind. As a consequence, the extra noise added by the amplifier will reduce the quantum efficiency and prohibit quantum-limited feedback. It seems that this situation is to be changed quickly, for instance, by developing Josephson parametric amplifiers that can be realized in superconducting circuits [22]. (iii) In our numerical simulation, we did not explicitly include the nonunit quantum efficiency in the homodyne detection of the field. After adiabatic elimination of the cavity photon degree of freedom, the nonunit quantum efficiency of homodyne detection will reduce the effective information-gain rate Γ_m in Eq. (3). This implies an emergence of an extra nonunraveling dephasing term in the quantum trajectory equation. However, for the present particular study, this term results only in dephasing among the pre-GHZ state and the others in the single quantum-trajectory realizations. In addition to simple intuitive expectation, we have numerically examined that lowering the quantum efficiency by some acceptable amount does not obviously change the results. (iv) Experimental verification of the GHZ state is of great interest and is analyzed theoretically in the recent work by Bishop *et al.* [5]. In order to observe a violation of the Bell-Mermin inequality, a relatively high signal-to-noise ratio in performing the measurement is required, which is unfortunately beyond the existing scope of experiment. But, optimistically, the situation is expected to change in the near future by fast experimental progress.

To summarize, we have presented a promising quantum control scheme for deterministic generation and stabilization of a three-qubit GHZ state in the solid-state circuit QED system. The scheme largely depends on a joint readout of multiple qubits in a dispersive regime, which enables us not only to infer the state for further processing, but also to stabilize the target state directly by means of an alternate-flip-interrupted Zeno (AFIZ) projection. The proposed scheme was demonstrated by quantum trajectory simulations, which show satisfactory control effects.

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