Symmetry breaking and multipeaked solitons in inhomogeneous gain landscapes

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We address one-dimensional soliton formation in a cubic nonlinear medium with two-photon absorption and transversally inhomogeneous gain landscape consisting of a single or several amplifying channels. Existence of the solitons requires certain threshold gain while the properties of solitons strongly depend on whether the number of the amplifying channels is odd or even. In the former case, an increase of the gain leads to symmetry breaking, which occurs through the pitchfork bifurcation, and to emergence of a single or several coexisting stable asymmetric modes. In the case of an even number of amplifying channels, we have found only asymmetric stable states.

The emergence of localized nonlinear patterns supported by localized gain has recently attracted increasing attention. There have been reported stable one-dimensional (1D) structures in media described by the cubic complex Guinzburg-Landau equation with linear losses and either one [1] or two [2] highly localized "hot spots." Stable solitons were also found in periodic lattices with active channels and nonlinear two-photon absorption [3] and at the surface of a periodic medium [4]. Stable patterns may exist even in two-dimensional (2D) settings, in layered structures of the planar waveguides [5], and in 2D lattices [6].

Localized gain significantly changes the physics of emergent nonlinear patterns. In addition to standard constraints imposed by the balance between the dissipation and gain, it introduces a new spatial scale—the width of the gain domain. This suggests a possibility of existence of more sophisticated structures than the simplest symmetric and/or antisymmetric dissipative solitons. In particular, it is of fundamental interest to exploit the phenomenon of symmetry breaking, which is known to be generic for conservative nonlinear systems possessing a characteristic spatial scale. These are, for example, systems governed by the nonlinear Schrödinger equation with a symmetric double-well potential [8,9]. Self-trapping in one of the two identical channels has already been observed experimentally in nonlinear optics [10] and in a Bose-Einstein condensate [11]. Symmetry breaking in a dual-core dissipative fiber with parametric gain and linear losses, which occurs through the interplay of the nonlinearity and coupling between two cores defined by the modulation of the linear refractive index, was reported in [7]. Asymmetric dissipative solitons were reported only in 2D systems, e.g., in the cubic-quintic Ginzburg-Landau equation [12] and in media with saturable gain and absorption [13]. Here we report a principally different scenario of symmetry breaking. It occurs in a medium without any conservative potential or modulation of the refractive index, but having nonlinear dissipation and localized gain.

Our setting is related to dissipative solitons, observed at a wavelength 1319 nm in self-focusing electrically pumped waveguides fabricated on an InP substrate [14]. In such structures the two-photon absorption usually ranges from 10^{-1} to 10^{-2} cm/GW while localized gain can be implemented by using segmented striplike electrodes or spatially localized optical pumping. Solitons can be excited in 0.6-μm-thick planar guiding layers with a length of several millimeters by input beams with typical waists of a few micrometers at gain levels of about 70 cm^{-1} [14]. The considered model is also relevant for a description of Bose-Einstein condensates of quasiparticles in the presence of nonresonant pumping [15].

We consider the propagation of laser radiation in a focusing cubic medium with strong two-photon absorption and transversally inhomogeneous gain described by the equation for the dimensionless light field amplitude q:

$$i \frac{\partial q}{\partial \xi} = -\frac{1}{2} \frac{\partial^2 q}{\partial \eta^2} + i p_i R(\eta) q - |q|^2 q - i\alpha |q|^2 q, \quad (1)$$

where ξ and η are the normalized longitudinal and transverse coordinates, respectively; $p_i > 0$ is the gain parameter; $R(\eta)$ describes the transverse gain profile (with amplitude normalized to one); and $\alpha > 0$ is the strength of two-photon absorption. We consider gain landscapes containing an integer number n of periods of $\cos^2 \eta$. For example, to model a gain with an odd number of amplifying channels we set $R(\eta) = \cos^2 \eta$ for $|\eta| \leqslant \eta_n$, where $\eta_n = n\pi/2$, and $R(\eta) = 0$ for $|\eta| > \eta_n$, and we vary p_i, α, and n. Assuming the characteristic transverse scale to be $3 \, \mu$m, we estimate the longitudinal scale (the diffraction length) to be $\sim$170 μm at a wavelength of 1.32 μm. Then $p_i = 1$ corresponds to the linear gain $\sim$60 cm^{-1} and $\alpha = 1$ corresponds to the two-photon absorption coefficient ≈ 0.017 cm/GW. The linear absorption is supposed to be compensated by the gain for $|\eta| > \eta_n$.

Dissipative solitons of Eq. (1) can be searched for in the form $\psi = w(\eta) e^{ib\xi}$, where b is the propagation constant and $w(\eta) = w_r + i w_i = u(\eta) e^{i\theta(\eta)}$ is a complex amplitude, with real, w_r, and imaginary, w_i, parts. The modulus u and phase θ solve the equations

$$bu = \frac{u_{\eta\eta}}{2} + u^3 - \frac{j^2}{2u^3}, \quad j_\eta = 2 p_i R(\eta) u^2 - 2\alpha u^4, \quad (2)$$

where $j(\eta) \equiv \theta_\eta u^2$ can be referred to as a current density. We are interested in localized solutions with $u, j \to 0$ at $|\eta| \to \infty$, which can be obtained for $b > 0$ (more specifically with the exponentially decaying asymptotics $u \sim \exp[-(2b)^{1/2}|\eta|]$ and $j \sim \exp[-4(2b)^{1/2}|\eta|]$).

Localized modes of the system (2) form if a focusing nonlinearity counterbalances diffractive broadening, i.e., when the equation

$$bU = -\frac{1}{2}\int u_\eta^2 d\eta - \int \frac{j^2}{u^2}d\eta + \int u^4 d\eta, \qquad (3)$$

where $U = \int u^2 d\eta$ is the energy flow, is satisfied and when the nonlinear losses integrally compensate the spatially inhomogeneous gain, i.e., when

$$p_i \int R(\eta)u^2 d\eta = \alpha \int u^4 d\eta. \qquad (4)$$

The above formulas allow one to argue for the possible existence of two different types of modes at the same parameters of the system. Consider the limit of high amplification, $p_i \to \infty$. Assuming that the solution amplitude $\mathcal{A} = \max\{u\}$ grows and the width ℓ decreases, the relation (4) suggests the scaling $\mathcal{A} \sim 1/\ell \sim \sqrt{p_i/\alpha}$. This allows us to deduce from (4) the estimate $U \approx (\alpha/p_i)\int u^4 d\eta \sim \sqrt{p_i/\alpha}$, valid subject to the assumption that the soliton's maximum is placed exactly at $\eta = 0$ where the pump has the maximal value, i.e., valid for a symmetric mode. For a crude guess of the proportionality coefficient in this estimate we use the ansatz $u \approx \nu/\cosh(\nu\eta)$ [which corresponds to neglecting the current j in the first of Eq. (2), which strictly speaking can be done only in the vicinity of $\eta = 0$]. This allows us to obtain from (4) for the symmetric mode $\nu = \sqrt{3p_i/2\alpha}$ and thus $U \sim 2\nu = \sqrt{6p_i/\alpha}$ and $b \sim \nu^2/2 = 3p_i/4\alpha$.

However, constraint (4) admits yet another scaling where a solution width grows with p_i. Then for wide solutions, i.e., at $\ell \gg \eta_0 = \pi$, from Eq. (4) the relation $p_i \sim \alpha \mathcal{A}^2 \ell$ follows. On the other hand, now $U \sim \mathcal{A}^2\ell$, i.e., $U \sim p_i/\alpha$. Thus, unlike in the previous estimate, now we are restricted neither by the position of the maximum of the mode nor by the symmetry of its shape. Moreover, in the corresponding solution, the diffraction term $u_{\eta\eta} \sim \mathcal{A}/\ell^2$ cannot be compensated by the Kerr nonlinearity $u^3 \sim \mathcal{A}^3$ alone, and the role of the current distribution, i.e., of j^2/u^3, becomes crucial (reducing the impact of the Kerr nonlinearity). Notice that the major influence of the current occurs not at the origin (where for the symmetric solutions it is exactly zero) but at some intermediate point η_* defined by the condition $j_\eta(\eta_*) = 0$. Thus if a solution with the suggested scaling exists, it should have asymmetric shape, with the maximum located in the vicinity of the point η_* (at least in the limit $p_i \to \infty$).

Further information about the maximal field amplitude $\mathcal{A}$ can be obtained from Eq. (2). Indeed, for $\eta > \eta_n$ the current is decaying, $j_\eta = -2\alpha u^4 < 0$, and is directed outward from the gain domain: $j > 0$ (since j tends to zero as $\eta \to \infty$). This means that the maxima of $|j|$ are achieved at some points located inside the gain domain, i.e., $|\eta_*| < \eta_n$. In such points the amplitude of the field is given by $u_*^2 = p_i R(\eta_*)/\alpha$ (where notice that η_* itself depends on the gain coefficient). Considering the symmetric one-peak mode in the case of one gain channel [i.e., when functions $u(\eta)$ and $j(\eta)$ feature only
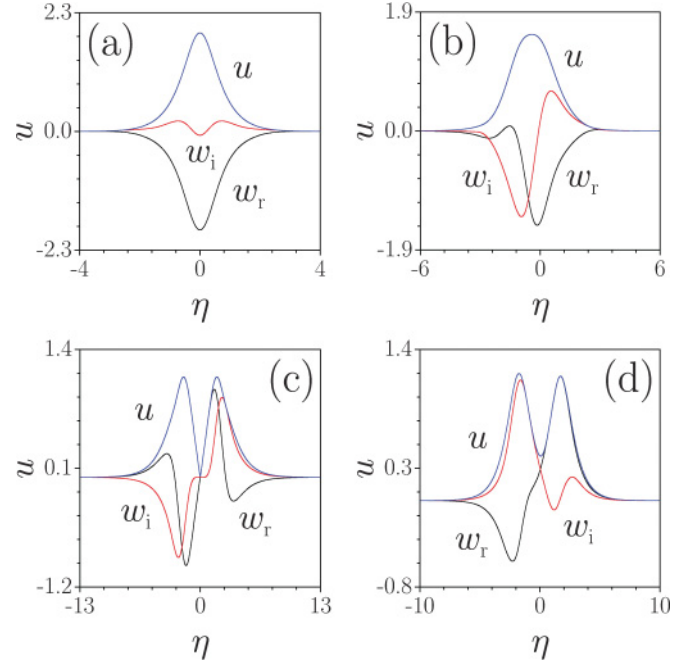

FIG. 1. (Color online) Profiles of symmetric (a) and asymmetric (b) one-peak solitons at $n = 1$, $p_i = 3.5$, $\alpha = 1.2$ and of (c) symmetric and (d) asymmetric two-peak solitons at $n = 2$, $p_i = 2.5$, $\alpha = 1.8$. The modes in panels (a) and (b) correspond to circles in Figs. 3(a) and 3(b). Hereafter all quantities are plotted in arbitrary dimensionless units.

a single maximum], one has the two maxima of j_η at $\pm\eta_*$ and hence $j_\eta > 0$ in the interval $|\eta| < \eta_*$. Therefore the amplitude of the field is bounded by the interval $u_*^2 \leqslant \mathcal{A}^2 \leqslant p_i R/\alpha$.

The above prediction of symmetric and asymmetric modes was confirmed in simulations (Fig. 1). We observed that while the growth of zero background is suppressed at large η, the light concentrates inside the amplifying channels. Strictly speaking this feature is typical for symmetric modes. A maximum of an asymmetric mode is shifted from the gain peak and the width of the mode grows with p_i, according to the estimates presented above. This broadening of the soliton leads to the situation where an appreciable part of the light energy concentrates outside the gain channel for large p_i.

The observed asymmetry in the field modulus remains relatively small for all considered p_i (which is most clearly visible in $w_{r,i}$ distributions), but it becomes more pronounced in systems with a larger number of channels (see Fig. 2). The number of peaks of stable solitons coincides with the number of amplifying channels. In Fig. 2 we show the symmetric and two coexisting asymmetric modes for the case of three amplifying channels (and we obtained similar solitons in landscapes with n up to 20).

Asymmetric states in a system where gain landscape is symmetric and all other parameters are uniform, i.e., *symmetry breaking*, is an unexpected result. Indeed, unlike in conservative systems, the understanding of the phenomenon cannot be related to energetic arguments. Our system also does not allow for reduction to a simpler discrete model, as happens, say, in the case of a double-well potential. Moreover, in our case the symmetry breaking occurs even for a single
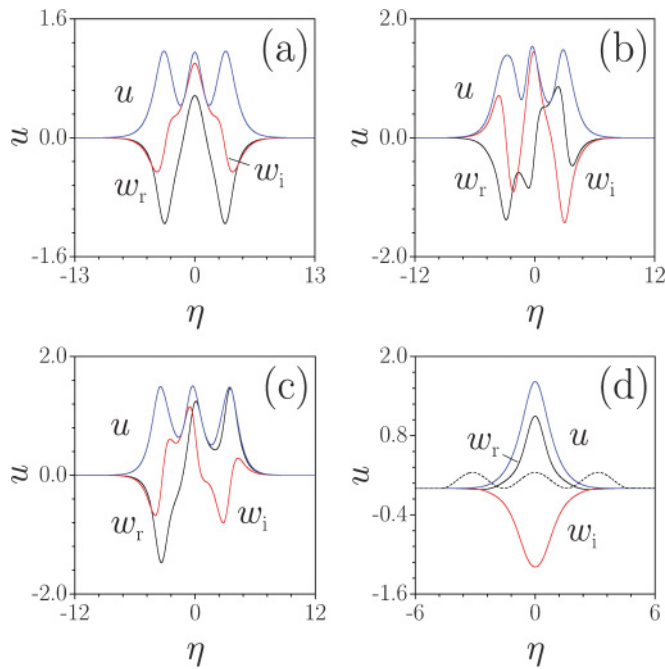
FIG. 2. (Color online) (a) Profiles of a symmetric three-peak soliton at $n = 3$, $p_i = 2.5$ [corresponding to the circle in Fig. 3(c)] and (b) and (c) two coexisting asymmetric three-peak solitons at $n = 3$, $p_i = 4.0$. (d) One-peak soliton at $n = 3$, $p_i = 4.0$. The dashed line schematically shows the gain landscape. In all cases $\alpha = 1.8$.
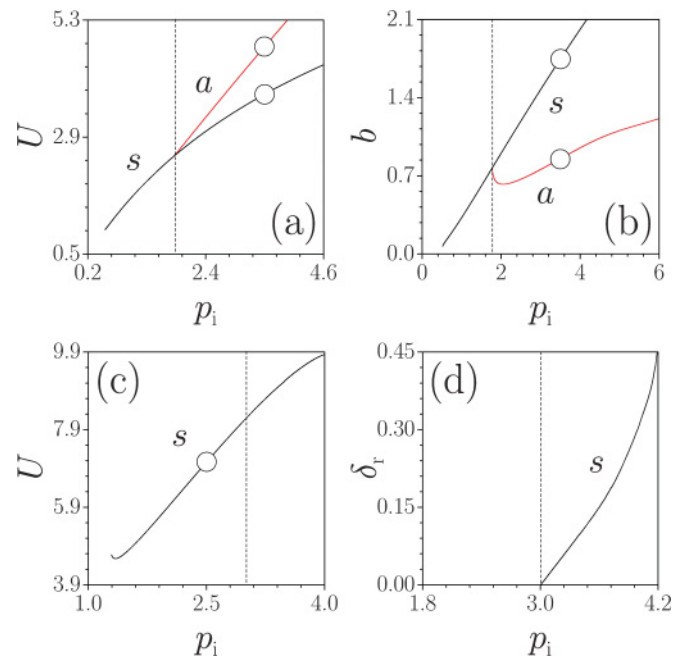

FIG. 3. (Color online) Energy flow (a) and propagation constant (b) vs p_i for symmetric (s) and asymmetric (a) one-peak solitons at $n = 1$, $\alpha = 1.2$. Energy flow (c) and perturbation growth rate (d) vs p_i for symmetric three-peak solitons at $n = 3$, $\alpha = 1.8$. The dashed lines indicate the borders of stability domains $p_i = p_i^{cr}$ for symmetric modes.

gain channel, in contrast to conservative systems (where at least two potential minima are required).

We performed a numerical study of the whole branches of the solutions and studied their stability (Fig. 3). In Figs. 3(a) and 3(b) for $n = 1$ we observe two branches of the solutions (noting that the propagation constant b is not a free parameter): One branch corresponds to symmetric solitons, while the other branch that bifurcates from the symmetric branch at a certain value $p_i = p_i^{cr}$ corresponds to asymmetric solutions (having smaller amplitudes and larger widths as compared to the symmetric ones). The dependencies $U(p_i)$ and $b(p_i)$ for both branches well reproduce the estimates presented above. A linear stability analysis of the modes is performed by plugging the perturbed field $q = (w + ve^{\delta\xi})e^{ib\xi}$ into Eq. (1) and performing a linearization around w.

For odd numbers of amplifying channels, exactly at the bifurcation point p_i^{cr} the branch of symmetric solutions loses its stability, while the stable asymmetric branch emerges [see the dashed lines in Figs. 3(a) and 3(b)]. Since the asymmetric modes appear in pairs (corresponding to the left and right shifts of the maximum from the origin) at the point where the symmetric mode becomes unstable we deal with the pitchfork bifurcation.

For small p_i symmetric solitons broaden dramatically and may expand far beyond the region with gain (since there is always a flow of energy outside the amplifying region). An increase of p_i results in growth of the peak amplitude and progressive localization of the soliton inside the amplifying domains. According to the above estimates for $n = 1$ and for sufficiently small α the energy flow and propagation constant of a symmetric soliton are monotonically increasing

functions of p_i [Figs. 3(a) and 3(b)]. Note that while for small values of α the symmetric one-peak solitons can be found even for $p_i \to 0$, for moderate and high α values such solitons exist only above a certain minimal value of the gain coefficient p_i^{low} [see Fig. 4(a)].

For even n the symmetric modes appear unstable in the whole domain of existence, and the only stable modes are asymmetric ones. In this case the dependencies $U(p_i)$ for symmetric and asymmetric modes do not overlap and no bifurcations occur. Except for stability, other properties of modes supported by even and odd numbers of amplifying channels are similar. Gain with multiple amplifying channels
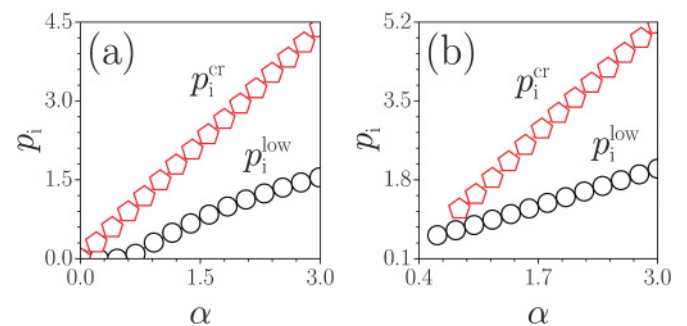

FIG. 4. (Color online) The domain of existence ($p_i \geqslant p_i^{low}$) and stability domain ($p_i^{cr} \geqslant p_i \geqslant p_i^{low}$) for one-peak (a) and three-peak (b) solitons on the plane (α, p_i). For small $\alpha < 0.6$ symmetric one-peak solitons can be obtained even when $p_i \to 0$ but for moderate and high nonlinear losses they exist only above certain minimal gain $p_i = p_i^{low}$

also supports solitons with the number of peaks smaller than the number of channels [Fig. 2(d)].

In dissipative multipeak solitons both w_r and w_i change their signs in neighboring channels with gain, while the field amplitude u is nonzero even in the regions between the channels. This follows from (2). Indeed, let us assume that at some point $\tilde{\eta}$ the field is zero, i.e., $u(\tilde{\eta}) = 0$. Since $u(\eta)$ is nonnegative, in the vicinity of $\tilde{\eta}$ we have $u(\eta) = O((\eta - \tilde{\eta})^2)$ and $j_\eta = O((\eta - \tilde{\eta})^4)$. Expanding $u(\eta)$ and $j(\eta)$ in a Taylor series in the vicinity of $\tilde{\eta}$ we find subsequently that all the expansion coefficients are zero, which means that if u becomes zero at some point, then $u(\eta) \equiv 0$ and $j(\eta) \equiv 0$.

The critical gain at which the bifurcation occurs increases almost linearly with α, so that the domain of stability of symmetric solitons, $p_i^{cr} \geqslant p_i \geqslant p_i^{\text{low}}$, expands with α [Fig. 4(a)]. In the case of symmetric multipeak solitons the energy flow grows with p_i monotonically, except for the narrow region close to the threshold $p_i = p_i^{\text{low}}$, below which no multipeak solitons can be found [Fig. 3(c)]. Multipeak solitons are stable in the region adjacent to p_i^{low}, but an increase of p_i results in their destabilization at $p_i = p_i^{cr}$ [dashed line in Fig. 3(c)]. A typical dependence of perturbation growth rate δ_r on gain parameter for $n = 3$ is shown in Fig. 3(d). The stability domain of a symmetric three-peak soliton expands almost linearly with increasing α [Fig. 4(b)].

Destabilization of symmetric multipeak states is accompanied by the appearance of several stable branches of asymmetric multipeak solitons. Thus the stable asymmetric modes depicted in Figs. 2(b) and 2(c) and the corresponding symmetric unstable three-peak mode (not shown) coexist for the same values of p_i, α.

With increase of the number of the gain channels the picture becomes even richer. When the number of channels is odd, the number of asymmetric modes that can be simultaneously stable for fixed p_i and α values increases. This feature is indicated by the presence of several stable attractors (multistability) in multichannel landscapes. The critical value of the gain coefficient (i.e., the bifurcation point) p_i^{cr} also grows with n, reaching however a certain saturation value. In particular, for $\alpha = 1.5$ this value is about 2.59 and it is reached already at $n = 7$.

Summarizing, we reported the symmetry breaking of a dissipative soliton supported by a single or multiple amplifying channels embedded in a cubic medium with nonlinear losses, which occurs through the loss of the stability of the symmetric family at a point of the pitchfork bifurcation. Since inside the stability domains solitons are attractors with sufficiently large basin they can be excited with a variety of regular or noisy input patterns. For Gaussian inputs and a single amplifying channel stationary solitons may form already after propagation over 20–30 diffraction lengths. In the case of multiple gain channels the bifurcation leads to the appearance of several stable asymmetric modes. While the bifurcation type is the same as one leading to the appearance of self-trapped states in a conservative double-well potential [9], here we deal with a purely dissipative phenomenon and the system does not possesses any characteristic scale related to its conservative part.

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