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Fundamental quantum limits on phase-insensitive linear amplification and phase conjugation in a practical framework

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We present experimentally testable quantum limitations on the phase-insensitive linear amplification and phase conjugation with respect to the transformation of a Gaussian-distributed set of coherent states following the footing to assess the success of continuous-variable quantum teleportation and quantum memory devices. The results enable us to compare the real device with the quantum-limited device via the feasible input of coherent states.

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An important role of theoretical physics is to derive fundamental limitations on the performance of physical devices for manipulating the states of a physical system. The controllability of the physical states over the existence of the quantum noise and its connections to quantum measurement are the central objective in a wide area of quantum physics [1]. An elementary operation for signal processing is amplification and its quantum limitation is generally determined based on the canonical commutation relation [2]. A pertinent approach is optimal cloning of quantum states so as to address the limitation on amplifying quantum information [3–5]. Those limitations are thought to be in the reach of experiments [1,6,7].

Any physical process is described by a completely positive trace-preserving map referred to as a quantum channel [8,9]. We often use the average fidelity as a figure of merit to estimate the performance of the process in quantum-information science. The fidelity represents the probability that the system after the process is in the desired state. The problems to find the quantum-limit phase-insensitive linear amplifier and to optimize the cloning map for coherent states are equivalent when the figure of merit is the trace norm [10] or the joint fidelity [5,11]. In the case of the most familiar amplification limit, the figure of merit is the ratio of the signal-to-noise ratios of the input and output fields [2]. Besides the amplification, an interesting quantum-state manipulation is the phase conjugation [12,13]. It corresponds to the universal NOT gate for qubit states [14] and to a transposition map for finite- and infinite-dimensional states [13].

In addition to the quantum limitations, it is also fundamental to establish classical limitations for asserting the nonclassicality or quantum coherence of the devices.

In this regard, an important benchmark is to outperform classical measure-and-prepare (MP) schemes [11,15–19]. Surpassing the classical limit achieved by classical MP schemes is a proof of entanglement because a MP scheme is an entanglement breaking channel [20]. It is known that the optimal fidelity of the phase conjugation can be achieved by a classical device [12–14] and that the Gaussian phase-conjugation (time-reversal) map belongs to the entanglement breaking channel [21].

To experimentally test the performance of the quantum device, an accessible input state is the coherent state. It is theoretically simple to determine the classical (or other physical) limitation assuming a uniform set of input states because the figure of merit has a covariant property [5] and the group theoretical treatment is useful [22]. In the case of uniform distribution, the quantum limitations were determined for the amplification [2,5,10,11] and for the phase conjugation [12,13]. However, neither testing the input-output relation for every coherent state nor assuming the displacement covariant property for the real device is feasible. In practice, the available power of the input field is limited and the linearity of the transformation is supposed to hold only on a limited range of the input variable. The covariant approach is inconsistent with these conditions since it implies an ever-increasing input power and an everlasting linearity. In the case of optical or atomic continuous-variable quantum-information processing [9], the amplitude of the input coherent states has to be much smaller than the total number of the particles in the so-called local oscillator field.

As a feasible figure of merit, the fidelity averaged over a Gaussian-distributed set of coherent states is employed associated with the quantum teleportation and memory for continuous-variable states [15,16]. Due to the Gaussian prior distribution, one can test the performance without concerning the contributions of impractically high-amplitude coherent states. On this footing, the classical limit of the average fidelity is determined for unit gain devices [15,16] and for the devices with the effect of loss or amplification [11,18]. The well-known quantum teleportation [23] and quantum memory [24] protocols serve as amplifiers via the gain control mechanism, and it is important to show in what extent such a real device is approximating the quantum-limited device. However, the quantum limitation on the amplification has been left open [11].

To give a solid foundation for experiments, it is crucial to address the quantum limitations under experimentally testable frameworks. It is worth noting that the classical capacity for bosonic quantum channels has been derived under the energy constraint [25].

In this Rapid Communication, we consider the quantum limits of the phase-insensitive linear amplification and phase conjugation in terms of the average fidelity with respect to the Gaussian-distributed set of coherent states. We derive a tight

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quantum-limit fidelity for the phase-insensitive amplification task and show that this fundamental limit is achieved by the known Gaussian amplifier. We also derive a tight quantum-limit fidelity for the phase conjugation task and show that this limit is achieved by a classical MP device.

In what follows the state vector with the Greek letter "α" denotes the coherent state and the state vector with the Roman letter "n" denotes the number state (e.g., we write the coherent state in the number basis as $|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\alpha^n|n\rangle/\sqrt{n!}$). When we work on the state with two modes, we call the first system A and the second system B.

Let us define the average fidelity of the physical process $\mathcal{E}$ for the transformation task on the coherent states $\{|\sqrt{N}\alpha\rangle\} \rightarrow \{|\sqrt{\eta}\alpha\rangle\}$ with $N,\eta > 0$ by

$$F_{N,\eta,\lambda}(\mathcal{E}) := \int p_\lambda(\alpha)\langle\sqrt{\eta}\alpha|\mathcal{E}(|\sqrt{N}\alpha\rangle\langle\sqrt{N}\alpha|)|\sqrt{\eta}\alpha\rangle d^2\alpha, \tag{1}$$

where the prior distribution of a symmetric Gaussian function with the inverse width of $\lambda > 0$ is given by

$$p_\lambda(\alpha) := \frac{\lambda}{\pi}\exp(-\lambda|\alpha|^2). \tag{2}$$

This distribution describes the uniform distribution in the limit $\lambda \rightarrow 0$. The fidelity represents the average probability that the input state $|\sqrt{N}\alpha\rangle$ is exactly transformed into the corresponding target state $|\sqrt{\eta}\alpha\rangle$ by the process $\mathcal{E}$. When $\eta/N \geqslant 1$, the transformation task implies the amplification of the coherent-state amplitude with the gain factor η/N. When $\eta = N = 1$, the task is referred to as the unit-gain task and the fidelity estimates how well the input coherent state is retrieved at the output port. When $\eta/N < 1$ the transformation suggests amplitude dumping. This is the case for practical transmission and storage processes, and the loss of fidelity can be seen as a deviation from the ideal lossy channel. When N and η are positive integers the task may be called N-to-η cloning where the fidelity implies how well the transformation from N copies $|\alpha\rangle^{\otimes N}$ to η copies $|\alpha\rangle^{\otimes\eta}$ can be achieved. The *quantum-limit fidelity* is defined as an upper limit of the average fidelity $F(\mathcal{E})$ achieved by the completely positive trace-preserving map $\mathcal{E}$. We call the limit *tight* if the fidelity limit is achieved by a completely positive trace-preserving map. Note that, from Eqs. (1) and (2), by changing the integral parameter we can verify the identity

$$F_{N,\eta,\lambda} = F_{\frac{N}{\eta},1,\frac{\lambda}{\eta}}, = F_{1,\frac{\eta}{N},\frac{\lambda}{N}}. \tag{3}$$

Quantum optimal phase-insensitive linear amplifier. Let us consider the amplification task $\{|\alpha\rangle\} \rightarrow \{|\sqrt{\eta}\alpha\rangle\}$ with the gain $\eta > 1$. In the following we show that the fidelity $F_{1,\eta,\lambda}$ is bounded above by $\frac{1+\lambda}{\eta}$ for sufficiently small λ and that this bound is achieved by the known Gaussian amplifier. Note that the tight quantum-limit fidelity of attenuation task with $\eta \in [0,1]$ is unity [11].

Proof. Let us consider the following integration [26] with the parameters $s \geqslant 0$, $0 \leqslant \kappa \leqslant 1$, and $0 \leqslant \xi < 1$

$$J_{\mathcal{E}}(s,\kappa,\xi) := \int d^2\alpha p_s(\alpha)\,\langle\alpha|_A\,\langle\kappa\alpha^*|_B\mathcal{E}_A \otimes I_B(|\psi_\xi\rangle\langle\psi_\xi|)|\kappa\alpha^*\rangle_B|\alpha\rangle_A, \tag{4}$$

where $|\psi_\xi\rangle = \sqrt{1-\xi^2}\sum_{n=0}^{\infty}\xi^n|n\rangle|n\rangle$ is the two-mode squeezed state and I represents the identity process. The integration can be connected to the average fidelity by

$$J_{\mathcal{E}}(s,\kappa,\xi) = \frac{s(1-\xi^2)}{\lambda}F_{N,1,\lambda}(\mathcal{E}), \tag{5}$$

where the parameters are supposed to satisfy the following relations

$$\lambda = s + (1-\xi^2)\kappa^2, \tag{6}$$

$$\sqrt{N} = \kappa\xi. \tag{7}$$

From the condition $s \geqslant 0$ with Eqs. (6) and (7), we have

$$\frac{\lambda}{1-\xi^2} \leqslant N + \lambda. \tag{8}$$

We proceed to consider the upper bound of $J_{\mathcal{E}}$ instead of the upper bound of the fidelity $F(\mathcal{E})$. For any physical process with the complete positivity and trace-preserving condition, $\rho_{\mathcal{E}} := \mathcal{E}\otimes I(|\psi_\xi\rangle\langle\psi_\xi|)$ is a density operator. Then, the maximum of $J_{\mathcal{E}}$ with respect to the optimization of the process $\mathcal{E}$ is no larger than the maximum achieved by the optimization of the density operator $\rho_{\mathcal{E}}$ over the set of the whole physical states. Thus we have

$$\sup_{\mathcal{E}} J_{\mathcal{E}}(s,\kappa,\xi) \leqslant \max_{\rho_{\mathcal{E}}}\mathrm{Tr}[\rho M] = \|M\|, \tag{9}$$

where we define

$$M := \int p_s(\alpha)|\alpha\rangle\langle\alpha| \otimes |\kappa\alpha^*\rangle\langle\kappa\alpha^*|d^2\alpha,$$

and $\|\cdot\| := \max_{\langle u|u\rangle=1}\langle u|\cdot|u\rangle$ stands for the maximum eigenvalue.

Since M is a two-mode Gaussian state, its maximum eigenvalue is given from the symplectic eigenvalues of its covariance matrix [27]. Let us define the covariance matrix of a density operator on the two-mode field ρ

$$\gamma_\rho := \langle\hat{R}\hat{R}^t + (\hat{R}\hat{R}^t)^t\rangle_\rho - 2\langle\hat{R}\rangle\langle\hat{R}^t\rangle_\rho,$$

where $\hat{R} := (\hat{x}_A,\hat{p}_A,\hat{x}_B,\hat{p}_B)^t$ is the set of the quadrature operators of the mode A and mode B whose elements satisfy the canonical commutation relations $[\hat{x}_A,\hat{p}_A] = i$ and $[\hat{x}_B,\hat{p}_B] = i$. Then, the covariance matrix of the operator M is calculated to be

$$\gamma_M = \mathbb{1}_4 + \frac{2}{s}\begin{pmatrix}\mathbb{1}_2 & \kappa Z\\ \kappa Z & \kappa^2\mathbb{1}_2\end{pmatrix},$$

where $\mathbb{1}_4 := \mathrm{diag}(1,1,1,1)$, $\mathbb{1}_2 := \mathrm{diag}(1,1)$, and $Z := \mathrm{diag}(1,-1)$. To diagonalize this matrix we define a matrix $U(r)$ corresponding to the two-mode squeezing operator $\hat{U}_r := e^{-i(\hat{x}_A\hat{p}_B+\hat{x}_B\hat{p}_A)r} = e^{(\hat{a}^\dagger\hat{b}^\dagger-\hat{a}\hat{b})r}$ through the transformation

$$\hat{U}^\dagger\hat{R}\hat{U} = \begin{pmatrix}\cosh r\mathbb{1}_2 & \sinh rZ\\ \sinh rZ & \cosh r\mathbb{1}_2\end{pmatrix}\hat{R} =: U(r)\hat{R}.$$

When the squeezing parameter satisfies $\tanh 2r = 2\kappa/(1+s+\kappa^2)$ the covariance matrix is diagonalized as $U(-r)\gamma_M U^t(-r) = \mathrm{diag}(\nu_+,\nu_+,\nu_-,\nu_-)$ where the symplectic eigenvalues are determined to be

$$\nu_\pm = [\sqrt{(1+s+\kappa^2)^2-4\kappa^2} \pm (1-\kappa^2)]/s.$$

Therefore, the diagonal form of M is the product of the thermal states $T(\bar{n}_+) \otimes T(\bar{n}_-)$ with the mean photon numbers $\bar{n}_\pm = (\nu_\pm - 1)/2$ where the thermal state with the mean photon number $\bar{n}$ is defined by

$$T(\bar{n}) := \frac{1}{1+\bar{n}} \sum_{n=0}^{\infty} \left(\frac{\bar{n}}{1+\bar{n}} \right)^n |n\rangle\langle n|.$$

This implies the following form of the maximum eigenvalue with the help of Eqs. (6) and (7):

$$\|M\| = 4/[(\nu_+ + 1)(\nu_- + 1)] = \frac{2s}{N + \lambda + 1 + \sqrt{(N+\lambda+1)^2 - 4N/\xi^2}}.$$

Using this relation, Eqs. (5), (8), and (9), we have

$$\sup_{\mathcal{E}} F_{N,1,\lambda}(\mathcal{E}) \leqslant \frac{\lambda}{s(1-\xi^2)} \|M\| \leqslant \frac{2(N+\lambda)}{N+\lambda+1+\sqrt{(N+\lambda-1)^2}} = \begin{cases} N+\lambda & \text{if } (N+\lambda) \leqslant 1, \\ 1 & \text{if } (N+\lambda) > 1. \end{cases} \quad (10)$$

By taking the replacement $(N,\lambda) \to (1/\eta, \lambda/\eta)$ and using the identity of Eq. (3), we obtain the upper bound of the fidelity for the amplification task

$$\sup_{\mathcal{E}} F_{1,\eta,\lambda}(\mathcal{E}) \leqslant \begin{cases} \frac{1+\lambda}{\eta} & \text{if } \eta \geqslant 1+\lambda, \\ 1 & \text{if } \eta < 1+\lambda. \end{cases} \quad (11)$$

Next we consider the attainability of this bound. The Gaussian amplifier with the gain $g = \cosh^2 r \geqslant 1$ is defined by $\mathcal{A}_g(\rho) := \mathrm{Tr}_B[U_r \rho \otimes |0\rangle\langle 0|_B U_r^\dagger]$. It transforms the coherent state as $\mathcal{A}_g(|\alpha\rangle\langle\alpha|) = \frac{1}{\pi(g-1)} \int e^{-\frac{|\beta|^2}{g-1}} |\sqrt{g}\alpha + \beta\rangle\langle\sqrt{g}\alpha + \beta| d^2\beta$. This implies

$$F_{1,\eta,\lambda}(\mathcal{A}_g) = \frac{\lambda}{\lambda g + |\sqrt{g} - \sqrt{\eta}|^2} = \frac{\lambda}{(\lambda+1)\left(\sqrt{g} - \frac{\sqrt{\eta}}{\lambda+1}\right)^2 + \frac{\lambda\eta}{\lambda+1}} \leqslant \frac{1+\lambda}{\eta},$$

where the equality is achieved when $g = \eta/(1+\lambda)^2 \geqslant 1$. Therefore, the upper part of Eq. (11) is saturated by the Gaussian phase-insensitive amplifier if the distribution is sufficiently flat so as to satisfy $\eta \geqslant (1+\lambda)^2$. ■

In the limit of $\lambda \to 0$, our fidelity reproduces the quantum limit for the case of the uniform distribution $F_o = 1/\eta$ [5,11]. As we can see, the fidelity value $(1+\lambda)/\eta$ always exceeds the uniform limit F_o, and thus a naive comparison of the experimental fidelity with F_o gives an illegal result or an overestimation on how well the experimental device is approximating the quantum-limited device. In contrast, our result includes the effect of the finite distribution λ and enables a legitimate estimation toward the fundamental quantum limitation.

Optimal phase conjugator. Let us consider the phase-conjugation task $\{|\sqrt{N}\alpha\rangle\} \to \{|\alpha^*\rangle\}$ with $N > 0$ and define the fidelity $F^*_{N,1,\lambda}(\mathcal{E}) := \int d^2\alpha p_\lambda(\alpha) \langle\alpha^*|\mathcal{E}(|\sqrt{N}\alpha\rangle\langle\sqrt{N}\alpha|)|\alpha^*\rangle$. We can show that the optimal fidelity is given by

$$\sup_{\mathcal{E}} F^*_{N,1,\lambda}(\mathcal{E}) = \frac{N+\lambda}{N+\lambda+1}, \quad (12)$$

and is achieved by the classical MP scheme

$$\mathcal{E}^*_{MP}(\rho) := \frac{1}{\pi} \int \langle\alpha|\rho|\alpha\rangle \left| \frac{\sqrt{N}\alpha^*}{N+\lambda} \right\rangle \left\langle \frac{\sqrt{N}\alpha^*}{N+\lambda} \right| d^2\alpha. \quad (13)$$

Proof. We start by defining $J^*_{\mathcal{E}}(s,\xi,\kappa) := \int d^2\alpha p_s(\alpha) \langle\alpha^*|_A \langle\kappa\alpha^*|_B \mathcal{E}_A \otimes I_B(|\psi_\xi\rangle\langle\psi_\xi|) |\kappa\alpha^*\rangle_B |\alpha^*\rangle_A$ similarly to Eq. (4). Here, different from the previous case, we assume a weaker constraint of $\kappa \geqslant 0$. This suggests the phase-conjugation task with either attenuation or amplification. Similar to Eq. (5) we can confirm the following relation with the help of Eqs. (6) and (7):

$$J^*_{\mathcal{E}}(s,\kappa,\xi) = \frac{s(1-\xi^2)}{\lambda} F^*_{N,1,\lambda}(\mathcal{E}). \quad (14)$$

An upper bound of $J^*_{\mathcal{E}}(s,\xi,\kappa)$ is given by the optimization of the density operator $\rho = \mathcal{E} \otimes I(|\psi_\xi\rangle\langle\psi_\xi|)$ over the physically possible states, namely, we have

$$\sup_{\mathcal{E}} J^*_{\mathcal{E}}(s,\kappa,\xi) = \max_{\rho} \mathrm{Tr}[\rho M^*] \leqslant \|M^*\|, \quad (15)$$

where we define

$$M^* = \int p_s(\alpha) |\alpha\rangle\langle\alpha| \otimes |\kappa\alpha\rangle\langle\kappa\alpha| d^2\alpha. \quad (16)$$

This operator is also a two-mode Gaussian state, and its covariance matrix is calculated to be

$$\gamma_{M^*} = \mathbb{1}_4 + \frac{2}{s} \begin{pmatrix} \mathbb{1}_2 & \kappa\mathbb{1}_2 \\ \kappa\mathbb{1}_2 & \kappa^2\mathbb{1}_2 \end{pmatrix}.$$

This covariance matrix can be diagonalized by a beamsplitter transformation and the symplectic eigenvalues are determined to be $(\nu_+, \nu_-) = [1, 1 + 2(1+\kappa^2)/s]$.

Hence, we have

$$\|M^*\| = 4/[(\nu_+ + 1)(\nu_- + 1)] = \frac{s}{s+1+\kappa^2}. \quad (17)$$

Equations (15) and (17) lead to

$$\sup_{\mathcal{E}} J^*_{\mathcal{E}}(s,\kappa,\xi) \leqslant \frac{s}{s+1+\kappa^2}.$$

Using this relation and Eqs. (6), (7), (8), and, (14) we obtain the upper bound of the fidelity for the phase-conjugation task

$$\sup_{\mathcal{E}} F^*_{N,1,\lambda}(\mathcal{E}) \leqslant \frac{\lambda}{(1-\xi^2)} \frac{1}{N+\lambda+1} \leqslant \frac{N+\lambda}{N+\lambda+1}.$$

On the other hand, this bound is achieved by the MP scheme of Eq. (13) [i.e., $F_{N,1,\lambda}(\mathcal{E}^*_{MP}) = \frac{N+\lambda}{N+\lambda+1}$ holds]. We thus obtain Eq. (12). ■

The value of the optimal fidelity for the covariant approach [12,13] is reproduced when we set $N = 1$ and take the limit $\lambda \to 0$. Our result shows that the optimality of the classical device for the phase-conjugation task occurs beyond the case of the uniform distribution. The optimality of the classical device suggests the coincidence of the quantum limit and classical limit. Such a coincidence also occurs when the target states are orthogonal to each other [17]. Note that when the optimization of the state $\rho_{\mathcal{E}}$ in Eq. (9) is limited over the positive-partial-transpose states [26], the value of the optimal fidelity corresponds to the value of the optimal fidelity for the phase-conjugation task. Hence, for many of the tasks whose target states are given by the transpose of the input states, it is likely that the gap between the quantum limit and classical limit disappears.

In conclusion, we have presented quantum limitations on the phase-insensitive linear amplification and phase conjugation in terms of the average fidelity by assuming transformation tasks on a Gaussian distributed set of coherent states. Thereby, an experimental test can be done by using coherent states with a finite amount of phase-space displacement on the same footing as the success criterion for continuous-variable quantum teleportation and quantum memory. It was also shown that both of the fidelity limits can be achieved by the known Gaussian machines and that the known results for the case of the uniform distribution are safely reproduced. The present results give a solid foundation to experimentally observe how well the real device approximates the quantum-limited device in a legitimate manner.

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