Erratum: Multicopy programmable discrimination of general qubit states [Phys. Rev. A 82, 042312 (2010)]

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(Received 15 March 2011; published 30 March 2011)

DOI: 10.1103/PhysRevA.83.039909 PACS number(s): 03.67.Hk, 03.65.Ta, 99.10.Cd

The sentences starting at the third line after Eq. (23), including Eq. (24), should read:

The minimal inconclusive probability for these two states can be obtained with a positive operator-values measure (POVM), whose elements are $E_1 = [\psi^{\otimes n}] \otimes [\psi^{\otimes n}]^{\perp}$, $E_2 = [\psi^{\otimes n}]^{\perp} \otimes [\psi^{\otimes n}]$, both representing conclusive answers, and $E_{\rm inc} = \mathbb{1} \otimes \mathbb{1} - E_1 - E_2$, which represents the inconclusive one. In these expressions $[\psi^{\otimes n}]^{\perp} = \mathbb{1}_n - [\psi^{\otimes n}]$. Note that this POVM checks whether the state in each register is $|\psi\rangle$ or not. The probability of obtaining the inconclusive answer reads

$$P^{\rm UA}(\psi) = \frac{1}{2}({\rm tr}E_{\rm inc}\,\sigma_1 + {\rm tr}E_{\rm inc}\,\sigma_2) = \frac{1}{n+1} \tag{24}$$

independently of the state $|\psi\rangle$.

Equation (A1) in Appendix A should read

$$P^{\rm UA} = \frac{1}{2}\left(\frac{1}{\sqrt{d_1}} - \frac{1}{\sqrt{d_2}}\right)^2 d_{ABC} + \frac{1}{\sqrt{d_1 d_2}} \sum_{k=0}^{n_C} (n_A + n_B - n_C + 2k + 1) \sqrt{\frac{\binom{n_A+n_B-n_C+k}{n_B}\binom{n_B+k}{n_B}}{\binom{n_A+n_B}{n_B}\binom{n_C+n_B}{n_B}}}, \tag{A1}$$

where $d_1 = (n_A + n_B + 1)(n_C + 1)$, $d_2 = (n_A + 1)(n_B + n_C + 1)$, and $d_{ABC} = n_A + n_B + n_C + 1$. Notice that in our paper we missed the term proportional to d_{ABC}, which vanishes if $d_1 = d_2$.

We next outline the derivation of this equation. We stick to the notation in our paper and assume that σ_1 and σ_2 both occur with prior probability 1/2. Without loss of generality we can also assume that $n_A > n_C$. Then, $(1/2)\,\sigma_1 = \sum_{J=J^1_{\rm min}}^{J_{\rm max}} \sum_{M=-J}^{J} p_J\, \pi^1_J [j_{AB}; JM]$ and $(1/2)\,\sigma_2 = \sum_{J=J^2_{\rm min}}^{J_{\rm max}} \sum_{M=-J}^{J} p_J\, \pi^2_J [j_{BC}; JM]$, where $p_J = (1/d_1 + 1/d_2)/2$, $\pi^1_J = 1/(2p_J\, d_1)$, $\pi^2_J = 1/(2p_J\, d_2)$ for $j_B + j_A - j_C \equiv J^1_{\rm min} \leqslant J \leqslant J_{\rm max} \equiv j_A + j_B + j_C$, whereas $p_J = 1/(2d_2)$, $\pi^1_J = 0$, $\pi^2_J = 1$ for $|j_B + j_C - j_A| \equiv J^2_{\rm min} \leqslant J < J^1_{\rm min}$. We view p_J as the probability of obtaining the outcome (M) J in a measurement of the (z component of the) total angular momentum on the unknown state. Likewise, we view $\pi^1_J, \pi^2_J = 1 - \pi^1_J$ as the probabilities that the unknown state be $[j_{AB}; JM]$ or $[j_{BC}; JM]$ for that specific pair of outcomes J and M (note that these probabilities are actually independent of M). If the condition $c_J^2/(1+c_J^2) \leqslant \pi^1_J \leqslant 1/(1+c_J^2)$, where $c_J = |\langle j_{AB}; JM | j_{BC}; JM\rangle|$, holds, then the probability of obtaining the inconclusive answer when we finally discriminate between $[j_{AB}; JM]$ and $[j_{BC}; JM]$ is [1] $P^{\rm UA}_J = 2\sqrt{\pi^1_J \pi^2_J} c_J$. One can prove that the condition above holds for $J^1_{\rm min} \leqslant J < J_{\rm max}$, whereas $P^{\rm UA}_{J_{\rm max}} = 1$, and $P^{\rm UA}_J = 0$ for $J^2_{\rm min} \leqslant J < J^1_{\rm min}$. By adding up the contributions from the different values of J one finally obtains Eq. (A1).

Proceeding along similar lines and recalling that $P^{\rm ME}_J = (1 - \sqrt{1 - 4\pi^1_J \pi^2_J c_J^2})/2$ for the minimal error [1], one can prove that Eq. (A2) in Appendix A should read

$$P^{\rm ME} = \frac{1}{4}\left\{1 + \frac{d_1}{d_2} - \frac{d_1 + d_2}{d_1 d_2} \sum_{k=0}^{n_C} (n_A + n_B - n_C + 2k + 1) \sqrt{1 - 4\frac{d_1 d_2}{(d_1 + d_2)^2} \frac{\binom{n_A+n_B-n_C+k}{n_B}\binom{n_B+k}{n_B}}{\binom{n_A+n_B}{n_B}\binom{n_C+n_B}{n_B}}}\right\}. \tag{A2}$$

We thank M. Hayashi for bringing our attention to the discrepancy with the wrong Eq. (A2) in our paper.

[1] J. A. Bergou, U. Herzog, and M. Hillery, in *Quantum State Estimation*, Lecture Notes in Physics, edited by M. Paris and J. Rehacek, Vol. 649 (Springer, Berlin, 2004), pp. 417–465.

