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Engineering squeezed states of microwave radiation with circuit quantum electrodynamics

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We introduce a squeezed state source for microwave radiation with tunable parameters in circuit quantum electrodynamics. We show that when a superconducting artificial multilevel atom interacting with a transmission line resonator is suitably driven by external classical fields, two-mode squeezed states of the cavity modes can be engineered in a controllable fashion from the vacuum state via adiabatic following of the ground state of the system. This scheme appears to be robust against decoherence and is realizable with present techniques in circuit quantum electrodynamics.

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Solid-state superconducting quantum circuits [1–4] have both fundamental and practical implications in condensed-matter physics, quantum optics, and quantum information. Over the past years, we have witnessed a tremendous development in this field. These circuits can be designed and constructed on demand and possess the advantages of integration and scaling on a chip. Among numerous developments, the field of circuit quantum electrodynamics (QED) [1,5–7], an on-the-chip counterpart of cavity QED systems [8,9], has attracted great interest. This system provides an unprecedented level of tunability and flexibility in the implementation of the strong-coupling limit [10] and the most promising platform for studying quantum optics on a chip [11–19]. With the development of the study of quantum optics in solid-state superconducting quantum circuits, recently the phenomenon of squeezing has been investigated in the context of circuit QED [20–24]. Squeezed states have both fundamental and practical implications in quantum optics and quantum information processing. The goal of this work is to design a two-mode squeezed field source utilizing a single superconducting artificial atom interacting with a transmission line resonator with today's techniques in circuit QED.

In this Brief Report, motivated by the recent experimental advance in designing superconducting artificial atoms [25,26], we present a controllable two-mode squeezed field source of microwave radiation with tunable parameters in circuit QED. We demonstrate that, using a superconducting artificial atom embedded in a transmission line resonator and suitably driven by external fields, we can find a nontrivial ground state (dark state) of the effective Hamiltonian controlling the dispersive interaction dynamics. Adiabatically following this ground state can transfer the resonator modes from the vacuum state to the two-mode squeezed state in a controllable way, while the superconducting artificial atom remains in its lowest energy level. The degree of squeezing and preparation time can be directly controlled by tuning the external driving fields. Compared to previous studies on squeezing in circuit QED, this scheme is insensitive to the decay of the excited states of the artificial atom due to adiabatic following of the ground state. The experimental implementation of this two-mode squeezed state source is accessible with currently available experimental techniques in circuit QED.

The system we consider consists of a three-level superconducting artificial atom embedded in a transmission line resonator, as sketched in Fig. 1. The atomic levels are labeled $|g\rangle,|e_1\rangle$, and $|e_2\rangle$, where $|g\rangle$ is the ground state, and $|e_1\rangle$ and $|e_2\rangle$ are the first and second excited states. It has been experimentally demonstrated that, in superconducting artificial atoms, the transitions among the three states can be cyclic under certain conditions. We apply three classical fields of frequencies ω_0, ω_1, and ω_2 dispersively driving the transitions $|g\rangle \leftrightarrow |e_1\rangle, |g\rangle \leftrightarrow |e_2\rangle$, and $|e_1\rangle \leftrightarrow |e_2\rangle$, with Rabi frequencies Ω_0, Ω_1, and Ω_2. Two cavity modes with frequencies ν_1 and ν_2 dispersively couple the transitions $|e_1\rangle \leftrightarrow |e_2\rangle$ and $|g\rangle \leftrightarrow |e_2\rangle$ with coupling constants g_1 and g_2. The detunings for these transitions are $\Delta_0 = \omega_{01} - \omega_0$, $\Delta_1 = \omega_{02} - \omega_1 = \omega_{12} - \nu_1$, and $\Delta_2 = \omega_{12} - \omega_2 = \omega_{02} - \nu_2$. The Hamiltonian of the whole system in the interaction picture with respect to $H_0 = \hbar\sum_{i=1,2}[\nu_i \hat{a}_i^\dagger \hat{a}_i + \omega_{0i}|e_i\rangle\langle e_i|]$, where the energy of the ground state $|g\rangle$ has been taken to be zero, takes the form

$$\begin{aligned} H_I = {}& \hbar\Omega_0|e_1\rangle\langle g|e^{i\Delta_0 t} + \hbar\Omega_1|e_2\rangle\langle g|e^{i\Delta_1 t} + \hbar\Omega_2|e_2\rangle\langle e_1|e^{i\Delta_2 t} \\ & + \hbar g_1\hat{a}_1|e_2\rangle\langle e_1|e^{i\Delta_1 t} + \hbar g_2\hat{a}_2|e_2\rangle\langle g|e^{i\Delta_2 t} + \text{H.c.}, \quad (1) \end{aligned}$$

where $\hat{a}_i$ is the annihilation operator for the cavity mode with frequency ν_i $(i = 1,2)$. The following conditions are assumed: (i) $|\Delta_0|,|\Delta_1|,|\Delta_2|,|\Delta_0 - \Delta_1|,|\Delta_0 - \Delta_2|,|\Delta_1 - \Delta_2| \gg |\Omega_0|,|\Omega_1|,|g_1|,|\Omega_2|,|g_2|$; (ii) $|\Omega_0|,|\Omega_1|,|\Omega_2| \gg g_1,g_2$; and (iii) $-|\Omega_1|^2/|\Delta_1| \simeq |\Omega_2|^2/|\Delta_2| \simeq |\Omega_0|^2/|\Delta_0|$. Condition (i) guarantees that the dominant process is the two-photon Raman transition between states $|g\rangle$ and $|e_1\rangle$ via the excited state $|e_2\rangle$. Condition (ii) ensures that the terms proportional to $|g_1|^2$ and $|g_2|^2$ can be neglected. Condition (iii) can completely cancel the energy shifts (Stark shifts) and the related terms. Thus we obtain the effective Hamiltonian of two-color Jaynes-Cummings form,

$$\begin{aligned} H_{\text{eff}} = {}& \hbar[\Theta_2(t)\hat{a}_2 - \Theta_1(t)\hat{a}_1^\dagger]|e_1\rangle\langle g| \\ & + \hbar[\Theta_2(t)\hat{a}_2^\dagger - \Theta_1(t)\hat{a}_1]|g\rangle\langle e_1|, \quad (2) \end{aligned}$$

where $\Theta_i(t) = |\frac{\Omega_i g_i}{\Delta_i}|$, and $\Theta_2(t) > \Theta_1(t)$ is assumed in the following.

We now try to find the eigenstate of Hamiltonian H_{eff} with zero eigenenergy (dark state) and zero population in the excited

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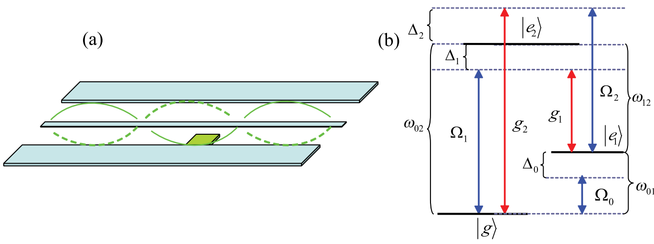

FIG. 1. (Color online) (a) A superconducting multilevel artificial atom (fluxonium) embedded in a transmission line resonator. (b) Energy level structure with couplings to the cavity modes and microwave driving fields.

state $|e_1\rangle$. We denote the state as $|\psi_D\rangle = |g\rangle|\phi\rangle_c$, where $|\phi\rangle_c$ is a to-be-determined state vector for the cavity modes. We define the quantity $N_- = \hat{a}_2^\dagger\hat{a}_2 - \hat{a}_1^\dagger\hat{a}_1 + |e_1\rangle\langle e_1|$. It is easy to show that the quantity N_- remains constant under the action of the Hamiltonian $H_{\rm eff}$. Thus, if the system initially stays in $|0_1,0_2\rangle_c|g\rangle$ (i.e., $N_- = 0$), then at any time it will be zero. The state $|n_1,n_2\rangle_c$ is the two-mode Fock state for the cavity modes. The conservation of the quantity N_- is very significant in the following discussions for obtaining the dark state of the system. From the eigenvalue equation $H_{\rm eff}|\psi_D\rangle = 0$, we have

$$[\Theta_2(t)\hat{a}_2 - \Theta_1(t)\hat{a}_1^\dagger]|\phi\rangle_c = 0. \tag{3}$$

From the Baker-Hausdoff formula $e^{\mathbf{A}}\mathbf{B}e^{-\mathbf{A}} = \mathbf{B} + [\mathbf{A},\mathbf{B}] + \frac{1}{2!}[\mathbf{A},[\mathbf{A},\mathbf{B}]] + \cdots$, it is easy to obtain

$$\begin{aligned}[\Theta_2(t)\hat{a}_2 - \Theta_1(t)\hat{a}_1^\dagger] &= \sqrt{\Theta_2^2 - \Theta_1^2}(\cosh\zeta_1\hat{a}_2 - \sinh\zeta_1\hat{a}_1^\dagger)\\ &= \sqrt{\Theta_2^2 - \Theta_1^2}e^{\zeta_1\hat{a}_1^\dagger\hat{a}_2^\dagger - \zeta_1\hat{a}_1\hat{a}_2}\hat{a}_2e^{\zeta_1\hat{a}_1\hat{a}_2 - \zeta_1\hat{a}_1^\dagger\hat{a}_2^\dagger},\end{aligned}$$

with $\cosh\zeta_1 = \frac{\Theta_2}{\sqrt{\Theta_2^2-\Theta_1^2}}$, $\sinh\zeta_1 = \frac{\Theta_1}{\sqrt{\Theta_2^2-\Theta_1^2}}$. Inserting the above relation into $[\Theta_2(t)\hat{a}_2 - \Theta_1(t)\hat{a}_1^\dagger]|\phi\rangle_c = 0$, we have $\hat{a}_2e^{\zeta_1\hat{a}_1\hat{a}_2 - \zeta_1\hat{a}_1^\dagger\hat{a}_2^\dagger}|\phi\rangle_c = 0$. This means that the state $e^{\zeta_1\hat{a}_1\hat{a}_2 - \zeta_1\hat{a}_1^\dagger\hat{a}_2^\dagger}|\phi\rangle_c$ is a vacuum state for cavity mode 2, or $e^{\zeta_1\hat{a}_1\hat{a}_2 - \zeta_1\hat{a}_1^\dagger\hat{a}_2^\dagger}|\phi\rangle_c = |0_2,\phi_1\rangle_c$, where $|\phi_1\rangle$ is an arbitrary state for cavity mode 1 and we choose $|\phi_1\rangle = |0_1\rangle$ due to the conservation of the quantity N_-. So at present we obtain the dark state of the system,

$$|\psi_D(t)\rangle = e^{\zeta_1(t)\hat{a}_1^\dagger\hat{a}_2^\dagger - \zeta_1(t)\hat{a}_1\hat{a}_2}|0_1,0_2\rangle_c|g\rangle. \tag{4}$$

Obviously, the state $e^{\zeta_1\hat{a}_1^\dagger\hat{a}_2^\dagger - \zeta_1\hat{a}_1\hat{a}_2}|0_1,0_2\rangle_c = \sum_{n=0}^\infty(\tanh\zeta_1)^n/\cosh\zeta_1|0_1,0_2\rangle_c$ is a two-mode squeezed state [24]. The squeezing parameter $\zeta_1(t) = \tanh^{-1}\epsilon(t)$, where $\epsilon(t) = \frac{\Theta_1(t)}{\Theta_2(t)}$, which can be controlled through tuning the Rabi frequencies $\Omega_1(t),\Omega_2(t)$. The physics that results in the dark state (4) is a quantum interference phenomenon: the destructive interference between two different pathways $|i_1,j_2\rangle_c|g\rangle \leftrightarrow |(i+1)_1,j_2\rangle_c|e_1\rangle$ and $|(i+1)_1,(j+1)_2\rangle_c|g\rangle \leftrightarrow |(i+1)_1,j_2\rangle_c|e_1\rangle$, where i,j refer to the cavity excitations.

Equation (4) represents the key result of this work, based on which we can generate the squeezed state $e^{\zeta_1\hat{a}_1^\dagger\hat{a}_2^\dagger - \zeta_1\hat{a}_1\hat{a}_2}|0_1,0_2\rangle_c$ from the initial vacuum state $|0_1,0_2\rangle_c$. By varying the parameters slowly, this allows the system to adiabatically follow the dark state (4). If we require that

$$\lim_{t\to t_0}\frac{\Theta_1(t)}{\Theta_2(t)} = 0, \quad \lim_{t\to t_f}\frac{\Theta_1(t)}{\Theta_2(t)} = \epsilon_0, \tag{5}$$

then the dark state (4) consequently has the limits

$$|\psi_D(t_0)\rangle = |0_1,0_2\rangle_c|g\rangle,\ \ |\psi_D(t_f)\rangle = e^{\zeta^0\hat{a}_1^\dagger\hat{a}_2^\dagger - \zeta^0\hat{a}_1\hat{a}_2}|0_1,0_2\rangle_c|g\rangle, \tag{6}$$

where $\zeta^0 = \tanh^{-1}\epsilon_0$. So we can produce two-mode field squeezing from the vacuum state of the cavity modes through adiabatic passage while the superconducting artificial atom remains in its lowest energy state during this process. This adiabatic process is somewhat like the fractional stimulated Raman adiabatic passage method [27], which can transfer the population from a single state to a superposition state.

To confirm the above analysis, we perform numerical simulations to see whether the system adiabatically follows the dark state of Eq. (4) and two-mode squeezing can be produced. The evolution of the system is governed by the following master equation:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_I,\rho] + \sum_{i=1,2}\kappa_i\mathcal{D}(\hat{a}_i)\rho + \sum_i\gamma_i\mathcal{D}(|g\rangle\langle e_i|)\rho, \tag{7}$$

where $\mathcal{D}[A]B \equiv ABA^\dagger - \{A^\dagger A,B\}$, and κ_i and γ_i denote the cavity photon decay rate and the decay rate from state $|e_i\rangle \to |g\rangle$, respectively. The initial state of the global system is $\rho(0) = |0_1,0_2\rangle_c\langle 0_2,0_1| \otimes |g\rangle\langle g|$. As mentioned above, to implement this protocol requires that the two classical fields have a counterintuitive pulse sequence initially, but they terminate simultaneously while maintaining a constant ratio of amplitudes. Here we choose the following pulse shape for them [27]: $\Omega_1(t) = \tilde{\Omega}_0\sin\alpha e^{-(t-\tau)^2/T^2}$ and $\Omega_2(t) = \tilde{\Omega}_0e^{-(t+\tau)^2/T^2} + \tilde{\Omega}_0\cos\alpha e^{-(t-\tau)^2/T^2}$. This pulse sequence is believed to make the fractional stimulated Raman adiabatic passage as robust as the general stimulated Raman adiabatic process [27]. Through numerical solution of the master equation, we plot the overlap between the density matrix operator ρ and the instantaneous dark state $|\psi_D(t)\rangle$ [i.e., $F = \langle\psi_D(t)|\rho|\psi(t)_D\rangle$], the average photon number per mode $n = \langle\hat{a}_i^\dagger\hat{a}_i\rangle (i = 1,2)$, and the occupation of the excited state $|e_i\rangle$ [i.e., $p = \langle|e_i\rangle\langle e_i|\rangle$ $(i = 1,2)$]. Figure 2(a) displays the time evolution of the Rabi frequencies $\Omega_1(t),\Omega_2(t)$. Figure 2(b) shows the numerical results for the overlap F, the occupation of the state $|e_1\rangle$ p, and the average photon number n. As predicted, the transfer to the target state is very efficient. That the overlap is always close to 1 and the occupation of the excited state $|e_1\rangle$ or $|e_2\rangle$ is nearly zero signify that the system always remains in the instantaneous dark state in the adiabatic limit. We also notice that the average photon number per mode evaluated from the master equation coincides with that in the case of an ideal two-mode squeezed state. At the end of the process, we can prepare the cavity modes in a two-mode squeezed state with about 12 photons per mode from the given parameters, with a preparation time of about $2T(< 1/\kappa \simeq 50T)$. With typical experimental parameters in circuit QED, the characteristic time T is about 100 ns, which

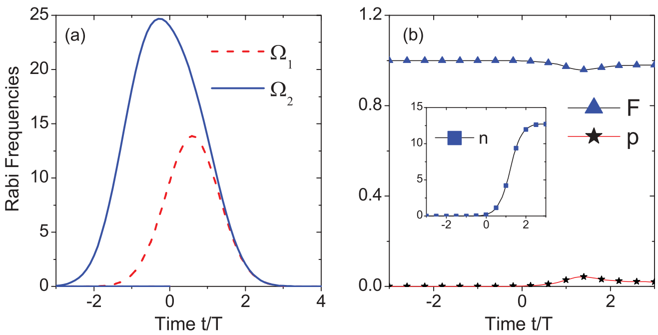

FIG. 2. (Color online) (a) Time evolution of the Rabi frequencies $\Omega_1(t)$ and $\Omega_2(t)$. The parameters are $\alpha = \pi/4.1$, $\tau = 0.6T$, and $\tilde{\Omega}_0 = 20T^{-1}$. (b) Numerical results of the overlap F, and the occupation of the excited state $|e_1\rangle$ p. In the inset the average photon number per mode, n, is shown. The parameters are chosen as $|\Delta_0| \simeq |\Delta_2| = 10\tilde{\Omega}_0$, $|\Delta_1| = 5\tilde{\Omega}_0, \Omega_0 = \tilde{\Omega}_0$, $g_1 \simeq g_2 = 0.2\tilde{\Omega}_0$, $k_1 \simeq k_2 = 0.005 g_1$, and $\gamma_1 \simeq \gamma_2 = 0.002 g_1$.

means that $g_1 \simeq g_2 \sim 50$ MHz, and $\kappa_1 \simeq \kappa_2 \sim 0.3$ MHz, with a quality factor of $Q \sim 10^5$ for the cavity. The photon numbers and preparation time can be further controlled through tuning the external driving fields.

In principle, the above generic proposal can be experimentally demonstrated with various Josephson-junction-based artificial atoms coupling to a transmission line resonator (e.g., the flux qubit); when the bias magnetic flux is slightly away from the flux degeneracy point, it possesses only a few bound states among which the transitions can be cyclic [28]. Here we propose a convenient demonstration with a new superconducting artificial atom design dubbed fluxonium that has recently been introduced in experiments [25,26]. The fluxonium has the same closed-loop topology as the flux qubit, but the loop contains a Josephson junction series array, giving an inductance much larger than the simple geometric inductance. Experimental results show that this new qubit design is totally insensitive to offset charges and exhibits very long phase coherence times reaching several microseconds [25,26]. This artificial atom can be described by the Hamiltonian $H_a = 4E_C\hat{N}^2 + \frac{E_L}{2}(\hat{\varphi} + 2\pi\frac{\Phi_{\rm ext}}{\Phi_0})^2 - E_J\cos\hat{\varphi}$, where $\hat{N}$ is the charge on the junction capacitance and $\hat{\varphi}$ is the reduced flux. The externally imposed, static magnetic flux $\Phi_{\rm ext}$ threading the loop Φ_0 modulates the spacings of energy levels of the artificial atom. When $\Phi_{\rm ext}/\Phi_0 \neq 0, 0.5$, the transitions among the lowest three bound states $|g\rangle$, $|e_1\rangle$, and $|e_2\rangle$ can be cyclic; that is, bound states of this artificial atom lose their well-defined parities. Then we can write the energies of the fluxonium atom as $H_a = \sum_{j=1,2}\hbar\omega_{0j}|e_j\rangle\langle e_j|$, where $\hbar\omega_{0j}$ is the energy difference between the two states $|e_j\rangle$ and $|g\rangle$. The spacings of energy levels can be modulated by the external flux bias $\Phi_{\rm ext}$. For example, when the external flux $\Phi_{\rm ext}$ is near the half-flux-quantum $\Phi_0/2$, the transition frequency $\omega_{01}/2\pi$ is about 350 MHz, while $\omega_{02}/2\pi$ is about 14 GHz [26]. So the two levels $|g\rangle$ and $|e_1\rangle$ are nearly degenerate, and they can be coupled via stimulated Raman pulses through the third level $|e_2\rangle$, forming a Λ energy level structure. In our proposal, we need to properly tune the external magnetic flux to allow the transition frequencies ω_{12} and ω_{02} to nearly match the first and second harmonic modes, respectively, of a transmission line cavity.

It is noted that several proposals for two-mode field squeezing have been presented in the context of cavity QED [29–32]. The dark-state method used in this work is quite different from that in Ref. [32]. Cheng *et al.* [32] have assumed that the atomic variables decay much more rapidly than the cavity fields, and they have eliminated the atomic variables adiabatically to get the dissipative evolution of the cavity modes only. In the present work, the dark state involves the atomic state and the field state. We do not have to assume the strong dissipation condition. In fact, in a typical circuit QED experiment, the characteristic decay rates for the cavity mode and superconducting qubit are on the same order of magnitude. Therefore, the above condition cannot be satisfied in general.

In conclusion, we have presented an efficient controllable two-mode squeezed state source of microwave radiation with an artificial atom capacitively coupled to a superconducting transmission line resonator. The dark-state approach presented in this work is generic and differs fundamentally from the existing methods to produce microwave field squeezing. Because of an unprecedented level of tunability and flexibility in this setup, the degree of squeezing, the number of photons, and the preparation time can be directly controlled. This scheme is robust against decay of the superconducting atom and is realizable with present techniques in circuit quantum electrodynamics.

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