

Randomly poled nonlinear crystals as a source of photon pairs

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The generation of photon pairs from randomly poled nonlinear crystals is investigated using an analytically soluble model and numerical calculations. Randomly poled crystals are discovered as sources of entangled ultra broadband signal and idler fields. Their photon-pair generation rates scale linearly with the number of domains. Entanglement times as short as several fs can be reached. A comparison with chirped periodically poled structures is given and reveals close similarity.

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I. INTRODUCTION

The first experimentally observed nonlinear optical effect, second-harmonic generation, was investigated more than 40 years ago by Franken [1]. Since then, many other nonlinear effects have been revealed and understood even at quantum level. Among them, spontaneous parametric down-conversion (SPDC) with its production of photon pairs is the most fascinating. The reason is that two photons comprising a photon pair generated in one quantum event of this process are mutually strongly correlated (entangled), as was discovered by Hong, Ou, and Mandel in the 1980's [2]. In their experiments they used nonlinear bulk crystals that later became the most common sources of photon pairs [3]. In order to observe spontaneous parametric down-conversion, phase-matching conditions of the interacting three optical fields have to be fulfilled. Unfortunately, they cannot be naturally fulfilled in many highly nonlinear crystals. However, Armstrong [4] presented the concept of additional periodic modulation of nonlinear susceptibility that has been practically developed in periodical poling of nonlinear crystals [5]. In this concept, the wave vector of the additionally introduced nonlinear modulation is added to the natural phase-matching condition and the so-called quasi-phase matching of the interacting optical fields is reached in this way. Since then, highly nonlinear materials can be exploited. We note that shortening of a nonlinear medium such that phase-matching conditions lose their importance is the only alternative way to periodical poling. This approach has been applied in photonic-band-gap structures in which optical interference is crucial for reaching an efficient nonlinear interaction [6–11].

Periodical poling has proven to be extraordinarily useful. It has provided not only compensation for the natural phase mismatch, but has given the ability to tailor the properties of emitted optical fields. It is based on using ordered nonlinear domains with variable lengths (chirped periodical poling). The presence of domains of different lengths in an ordered structure allows an efficient nonlinear interaction of fields in a broad spectral range. For example, signal and idler fields with spectra several hundreds of nm wide can be generated

in chirped LiNbO₃ crystals. On the other hand, domains with different lengths can also be ordered randomly. A bit surprisingly, the nonlinear interaction can be efficient even in this case, sometimes referred as stochastic quasi-phase matching (SQPM). Similarly, as ordered poled structures, the randomly poled structures (RPSs) allow spectrally broadband nonlinear interaction. It is not surprising that the efficiency of random structures is worse as compared to ordered ones. However, they usually put smaller requirements to polarization properties of the interacting optical fields as well as orientation of the nonlinear medium [12]. Also fabrication of RPSs is much easier because high precision is required in the production of ordered periodically poled structures.

The role of SQPM in one dimension (1D) has been already addressed for the process of second-harmonic generation [13–17] and the process of difference-frequency generation [18]. Moreover, full domain random structures allowing SQPM for transversal second-harmonic generation have been studied in Ref. [19].

Here, we focus our attention to the generation of photon pairs in randomly poled 1D nonlinear crystals [20]. It is shown that spectral properties of photon pairs and photon-pair generation rates are comparable in RPSs and chirped periodically poled structures (CPPSs) [21–24]. This is very promising for easy production of broadband and efficient sources of photon pairs. These broadband sources are important, e.g., in metrology (quantum optical coherence tomography [25]) or quantum-information processing [26,27]. We note that broadband photon-pair sources can also be constructed by using zero group-velocity dispersion conditions [28]. However, such conditions can be met only for certain pump frequencies considering a given material.

The paper is organized as follows. In Sec. II, a general theory of SPDC modified to random structures is presented. Photon-pair generation rates and intensity spectra are studied both for random and chirped structures in Sec. III. Section IV is devoted to temporal properties of the generated photon pairs. Spatial properties of photon pairs are addressed in Sec. V. The temperature behavior of the quantities characterizing photon pairs is analyzed in Sec. IV. Section VI concerns the analysis of fabrication errors. The role of ordering in chirped structures is studied in Sec. VIII. Finally, conclusions are drawn in Sec. IX.

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II. SPONTANEOUS PARAMETRIC DOWN CONVERSION IN POLED NONLINEAR CRYSTALS

The process of SPDC in a nonlinear crystal can be conveniently described by the following interaction Hamiltonian \hat{H}_{int} [29,30]:

$$\hat{H}_{\text{int}}(t) = \varepsilon_0 \mathcal{B} \int_{-L}^0 dz \chi^{(2)}(z) E_p^{(+)}(z, t) \hat{E}_s^{(-)}(z, t) \hat{E}_i^{(-)}(z, t) + \text{H.c.} \quad (1)$$

L denotes the crystal length. In Eq. (1), the positive-frequency part of the pump electric-field amplitude is denoted as $E_p^{(+)}$ and $E_s^{(-)}$ ($E_i^{(-)}$) stands for the negative-frequency part of the signal (idler) electric-field amplitude operator. The z -dependent second-order susceptibility tensor $\chi^{(2)}$ describes poling of the nonlinear material. Vacuum permittivity is denoted as ε_0 and \mathcal{B} means the transverse area of the optical fields. H.c. replaces the Hermitian-conjugated term.

Electric-field amplitudes occurring in Eq. (1) can be conveniently decomposed into harmonic plane waves with frequencies ω_a and wave vectors k_a :

$$\hat{E}_a^{(-)}(z, t) = \frac{1}{2\pi} \int d\omega_a \hat{E}_a^{(-)}(\omega_a) \exp(-ik_a z + i\omega_a t), \quad (2)$$

$$a = s, i.$$

The quantum spectral amplitudes $\hat{E}_a^{(-)}(\omega_a)$ in Eq. (2) can then be expressed using photon creation operators $\hat{a}_a^\dagger(\omega_a)$:

$$\hat{E}_a^{(-)}(\omega_a) = -i \sqrt{\frac{\hbar \omega_a}{2\varepsilon_0 c n_a(\omega_a) \mathcal{B}}} \hat{a}_a^\dagger(\omega_a); \quad (3)$$

c is speed of light in vacuum, \hbar is the reduced Planck constant, and n_a stands for the index of refraction of field a .

A first-order perturbation solution of the Schrödinger equation with the initial vacuum state $|\text{vac}\rangle$ in the signal and idler fields results in the following two-photon quantum state $|\psi\rangle$:

$$|\psi\rangle = \int d\omega_s \int d\omega_i \Phi(\omega_s, \omega_i) \hat{a}_s^\dagger(\omega_s) \hat{a}_i^\dagger(\omega_i) |\text{vac}\rangle. \quad (4)$$

The two-photon spectral amplitude Φ introduced in Eq. (4) is given as follows:

$$\Phi(\omega_s, \omega_i) = g(\omega_s, \omega_i) E_p^{(+)}(\omega_s + \omega_i) F(\Delta k(\omega_s, \omega_i)), \quad (5)$$

where g denotes a coupling constant, $g(\omega_s, \omega_i) = i \sqrt{\omega_s \omega_i} / [2c\pi \sqrt{n_s(\omega_s) n_i(\omega_i)}] \chi^{(2)}(0)$, and $E_p^{(+)}(\omega_p)$ stands for the pump-field amplitude spectrum. The stochastic function F introduced in Eq. (5) describes SQPM and is derived in the form [20]

$$F(\Delta k) = \int_{-L}^0 dz \frac{\chi^{(2)}(z)}{\chi^{(2)}(0)} \exp(i\Delta k z). \quad (6)$$

Δk describes the natural phase mismatch for the interacting fields, $\Delta k = k_p - k_s - k_i$.

In a periodically poled structure, neighbor domains differ in signs of $\chi^{(2)}$ nonlinearity, and function F defined in Eq. (6) can be recast into the form

$$F(\Delta k) = \sum_{n=1}^{N_L} (-1)^{n-1} \int_{z_{n-1}}^{z_n} dz \exp(i\Delta k z). \quad (7)$$

N_L denotes the number of domains and the n th domain extends from $z = z_{n-1}$ to $z = z_n$. Positions z_n of domain boundaries are random and can be expressed as $z_n = z_{n-1} + l_0 + \delta l_n$ ($n = 1, \dots, N_L$, $z_0 = -L$) in our model using stochastic Gaussian declinations δl_n . The basic layer length l_0 is determined such that quasi-phase matching is reached, i.e., $l_0 = \pi / \Delta k_0$, $\Delta k_0 \equiv \Delta k(\omega_s^0, \omega_i^0)$, and ω_a^0 means the central frequency of field a . The random declinations δl_n are mutually independent and can be described by the joint Gaussian probability distribution P :

$$P(\delta \mathbf{L}) = \frac{1}{(\sqrt{\pi} \sigma)^{N_L}} \exp(-\delta \mathbf{L}^T \mathbf{B} \delta \mathbf{L}). \quad (8)$$

Covariance matrix \mathbf{B} is assumed to be diagonal and its nonzero elements equal $1/\sigma^2$. Stochastic vector $\delta \mathbf{L}$ is composed of declinations δl_n ; the symbol T stands for transposition. The characteristic function G of the distribution P in Eq. (8) takes the form

$$G(\delta \mathbf{K}) \equiv \langle \exp(i\delta \mathbf{K} \cdot \delta \mathbf{L}) \rangle_{\text{av}} = \prod_{j=1}^N G(\delta k_j); \quad (9)$$

the symbol \cdot means the scalar product. Vector $\delta \mathbf{K}$ of parameters of the characteristic function G is composed of elements δk_j . The 1D characteristic function $G(\delta k)$ in Eq. (9) equals $\exp(-\sigma^2 \delta k^2 / 4)$.

In order to obtain analytical formulas, we integrate the expression for function F in Eq. (7) domain by domain and modify the contributions of the first and last domains in such a way that the following simple formula is reached:

$$F(\Delta k) = \frac{2i}{\Delta k} \sum_{n=0}^{N_L} (-1)^n \exp(i\Delta k z_n). \quad (10)$$

As a typical structure contains hundreds of domains, incorrect inclusion of fields from the first and the last domains leads to negligible declinations. The formula in Eq. (10) can be interpreted such that SPDC occurs only in domains with positive susceptibility $\chi^{(2)}$ at doubled amplitude and domains with negative susceptibility $\chi^{(2)}$ play only the role of a ‘‘linear’’ filler. This interpretation elucidates why a pair of domains having one positive and one negative sign of susceptibility $\chi^{(2)}$ forms an elementary unit for understanding the properties of photon pairs.

III. PHOTON-PAIR GENERATION RATES AND INTENSITY SPECTRA

The photon-pair generation rate as well as intensity spectra can be derived easily from the mean spectral density of the number of generated photon pairs $n(\omega_s, \omega_i)$. The mean spectral density n corresponding to the quantum state $|\psi\rangle$ is defined by

the formula

$$n(\omega_s, \omega_i) = \langle \langle \psi | \hat{a}_s^\dagger(\omega_s) \hat{a}_s(\omega_s) \hat{a}_i^\dagger(\omega_i) \hat{a}_i(\omega_i) | \psi \rangle \rangle_{\text{av}}, \quad (11)$$

where the symbol $\langle \rangle_{\text{av}}$ means stochastic averaging over an ensemble of geometric configurations of a RPS. Assuming the quantum state $|\psi\rangle$ written in Eq. (4), we arrive at the formula

$$n(\omega_s, \omega_i) = |g(\omega_s, \omega_i)|^2 |E_p^{(+)}(\omega_s + \omega_i)|^2 \langle |F(\Delta k(\omega_s, \omega_i))|^2 \rangle_{\text{av}}. \quad (12)$$

Spectrum S_s of, e.g., the signal field and photon-pair generation rate N , can then be derived using the expressions

$$S_s = \hbar \omega_s \int d\omega_i n(\omega_s, \omega_i), \quad (13)$$

$$N = \int d\omega_s \int d\omega_i n(\omega_s, \omega_i). \quad (14)$$

The averaged squared modulus of the phase-matching function F as determined by the formula in Eq. (10) can be written in the form:

$$\langle |F(\Delta k)|^2 \rangle_{\text{av}} = \frac{4}{\Delta k^2} \left\{ (N_L + 1) \frac{1 - |H(\delta k)|^2}{|1 - H(\delta k)|^2} - \left[\frac{H(\delta k)[1 - H(\delta k)^{N_L+1}]}{|1 - H(\delta k)|^2} + \text{c.c.} \right] \right\}, \quad (15)$$

$\delta k(\omega_s, \omega_i) = \Delta k(\omega_s, \omega_i) - \Delta k_0$. The symbol c.c. replaces the complex-conjugated term. Function $H(\delta k)$ occurring in

Eq. (15) is defined as

$$H(\delta k) = \exp[i\delta k l_0] G(\Delta k_0 + \delta k), \quad (16)$$

$$G(\Delta k) = \exp\left(-\frac{\sigma^2 \Delta k^2}{4}\right).$$

The averaged squared modulus $\langle |F(\Delta k)|^2 \rangle_{\text{av}}$ of the phase-matching function given in Eq. (15) determines the averaged spectral density n and behaves as follows. It holds that $|H| \leq 1$ and $|H| = 1$ for a fully ordered structure. If $\delta k = 0$ in a fully ordered structure, H is real ($H = 1$) and the averaged squared phase-matching function $\langle |F(\Delta k)|^2 \rangle_{\text{av}}$ reaches its maximum value $4(N_L + 1)^2$. The nonzero phase mismatch δk shifts H into the complex plane that results in lower values of the mean value $\langle |F(\Delta k)|^2 \rangle_{\text{av}}$. The larger the δk , the smaller the mean value $\langle |F(\Delta k)|^2 \rangle_{\text{av}}$. Inspection of the formula for H in Eq. (16) also shows that the larger the value of the basic layer length l_0 , the faster the decrease of mean values $\langle |F(\Delta k)|^2 \rangle_{\text{av}}$ for given δk . According to the formula in Eq. (16), the larger the standard deviation σ of a random structure, the smaller the value of $|H|$. The decrease of values of $|H|$ results in an increase of the range of values of the phase mismatch δk in which the averaged squared modulus $\langle |F(\Delta k)|^2 \rangle_{\text{av}}$ of the phase-matching function attains non-negligible values [see the formula in Eq. (15)].

The formula for averaged squared modulus $\langle |F(\Delta k)|^2 \rangle_{\text{av}}$ of phase-matching function in Eq. (15) can be substantially simplified under the assumption $\sigma^2(\Delta k_0)^2 N_L/2 \gg 1$:

$$\langle |F(\Delta k)|^2 \rangle_{\text{av}} = \frac{2N_L}{(\Delta k_0 + \delta k)^2} \frac{1 - G(\Delta k_0 + \delta k)}{1 - 2G(\Delta k_0 + \delta k) \cos(\delta k l_0) + G(\Delta k_0 + \delta k)^2}. \quad (17)$$

Increasing values of phase mismatch δk lead to greater values of the denominator in the fraction on the right-hand side (r.h.s.) of Eq. (17) that result in the decrease of values of the averaged squared modulus $\langle |F(\Delta k)|^2 \rangle_{\text{av}}$ of the phase-matching function. On the other hand, increasing values of deviation σ weaken this behavior.

For comparison, we consider another type of RPS defined such that $z_n = -L + nl_0 + \delta l_n$, where δl_n is a random declination of the n th boundary. These “weakly-random” structures are more ordered as compared to those considered earlier because the change of length of an n th domain is compensated by the change in length of an $(n + 1)$ th domain. The averaged squared modulus $\langle |F^{\text{w-r}}(\Delta k)|^2 \rangle_{\text{av}}$ of the phase-matching function can be derived in this case as follows:

$$\langle |F^{\text{w-r}}(\Delta k)|^2 \rangle_{\text{av}} = \frac{4}{(\Delta k)^2} \left\{ N_L + 1 + |G(\Delta k_0 + \delta k)|^2 \left[\frac{\exp(i\delta k l_0)}{1 - \exp(i\delta k l_0)} \left(N_L - \exp(i\delta k l_0) \frac{1 - \exp[i\delta k l_0 N_L]}{1 - \exp(i\delta k l_0)} \right) + \text{c.c.} \right] \right\}. \quad (18)$$

Disorder of the boundary positions manifests itself as a filter for the averaged squared modulus $\langle |F^{\text{w-r}}(\Delta k)|^2 \rangle_{\text{av}}$ of the phase-matching function, as evident from the expression in Eq. (18). This leads to spectral filtering of the spectral density n . This behavior is qualitatively different from that observed in RPS as described by the formula in Eq. (15), indicating broadening of the spectral density n with increasing values of the deviation σ .

Spectral broadening is the most interesting feature of ordered CPPSs that we consider here for comparison. Positions of boundaries in these structures are described by the formula $z_n = -L + nl_0 + \zeta'(n - N_L/2)^2 l_0^2$, $\zeta' = \zeta/\Delta k_0$, and ζ denotes chirping parameter. The phase-matching function $F^{\text{chirp}}(\Delta k)$ then takes the form [22]

$$F^{\text{chirp}}(\Delta k) = \frac{2\sqrt{\pi}}{\sqrt{i\Delta k^3 \zeta' l_0}} \exp(i\Delta k N_L l_0/2) \exp\left(-\frac{i\delta k^2}{4\Delta k \zeta'}\right) \{\text{erf}[f(N_L/2)] - \text{erf}[f(-N_L/2)]\}, \quad (19)$$

$$f(x) = \frac{\sqrt{-i}}{2} \left[\sqrt{\zeta'(\Delta k_0 + \delta k)x l_0} + \frac{\delta k}{\sqrt{\zeta'(\Delta k_0 + \delta k)}} \right].$$

The error function erf is defined as $\text{erf}(x) = 2/\sqrt{\pi} \int_0^x \exp(-y^2) dy$. A detailed inspection of the formula in Eq. (19) reveals that the larger the value of chirping parameter ζ' , the broader the phase-matching function $F^{\text{chirp}}(\Delta k)$.

As an example, we consider spectrally degenerate (nearly) collinear down-conversion from periodically poled LiNbO₃ pumped at the wavelength $\lambda_p^0 = 775$ nm by a cw laser beam. The signal and idler photons thus occur at the fiber-optics communication wavelength $\lambda_s = \lambda_i = 1.55$ μm . The crystal optical axis is perpendicular to the fields' propagation direction and is parallel to the vertical direction. All fields are vertically polarized and so the largest element $\chi_{33}^{(2)}$ of the susceptibility tensor is used. The natural phase mismatch for this configuration is compensated by the basic domain length l_0 equal to 9.51535 μm . A structure composed of $N_L = 700$ layers is ~ 6.5 mm long and typically delivers 4×10^6 photon pairs per 100 mW of pumping in case of weakly random positions of boundaries (small values of variance σ).

The most striking feature of RPSs is that the photon-pair generation rate N increases linearly with the number N_L of domains, independently of the standard deviation σ of the random positions of boundaries [see Fig. 1(a)]. The standard deviation σ plays the central role in the determination of

spectral widths ΔS_s and ΔS_i of the signal and idler fields. The larger the value of deviation σ , the broader the signal- and idler-field spectra S_s and S_i , as documented in Fig. 1(b). This behavior is easily understandable because structures generated with larger values of σ have statistically a broader spatial spectrum of the $\chi^{(2)}(z)$ modulation, which gives more freedom for the fulfillment of quasi-phase-matching conditions. It holds that the broader the signal- and idler-field spectra S_s and S_i , the smaller the photon-pair generation rate N [compare Figs. 1(a) and 1(b)]. It reflects the fact that constructive interference of fields from different domains is enhanced in the area outside the central frequencies ω_s^0 and ω_i^0 , whereas this interference is weakened in the area around the central frequencies.

The photon-pair generation rate N increases roughly linearly with the number N_L of domains, also in the case of “weakly random” structures described by the averaged squared modulus $\langle |F^{\text{w-r}}(\Delta k)|^2 \rangle_{\text{av}}$ of the phase-matching function in Eq. (18), as shown in Fig. 2. The greater the standard deviation σ , the smaller the photon-pair generation rate N . As for the signal-field spectral width ΔS_s , its values do not practically depend on the variance σ .

The behavior of photon-pair generation as observed in RPSs can also be found in ordered CPPSs. Also here the photon-pair generation rate N is linearly proportional to the number N_L of domains, and spectral widths ΔS_s and ΔS_i increase with increasing chirping parameter ζ . The main result of our analysis is that this similarity is both qualitative and quantitative. For any value of the chirping parameter ζ there exists a value of the standard deviation σ such that spectral widths ΔS_s and ΔS_i of the generated signal and idler fields are the same. Moreover (and a bit surprisingly), also photon-pair generation rates N are comparable. This is illustrated in Fig. 3 for a chirped structure with $N_L = 700$ domains. Its signal-field spectrum S_s is extraordinarily broad (larger than 1 μm) for sufficiently large values of the chirping parameter ζ [see Fig. 3(a)]. The signal-field spectra S_s of the same width can also be generated from RPSs with sufficiently large randomness (i.e., having large values of the deviation σ). Values of the standard deviation σ corresponding to the values

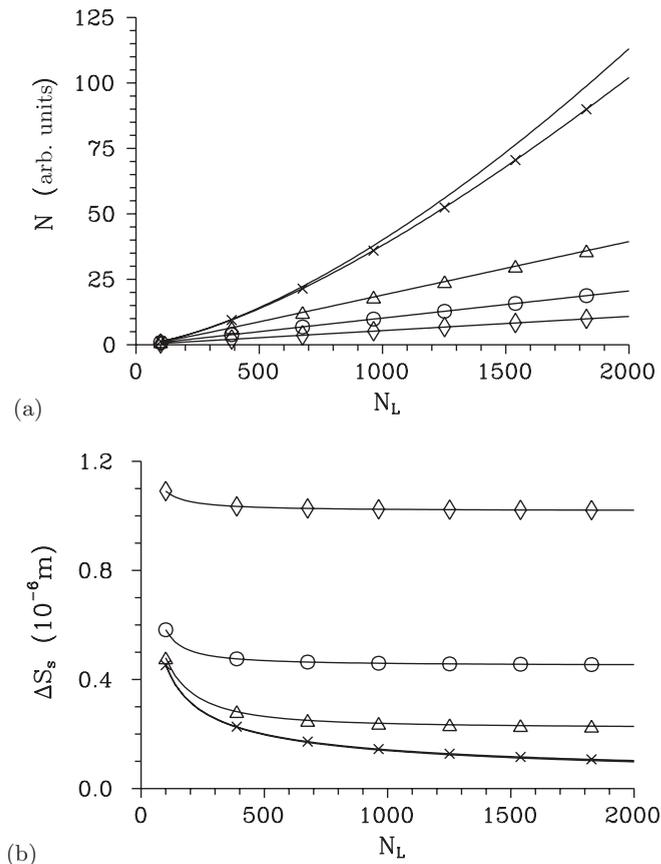


FIG. 1. (a) Photon-pair generation rate N and (b) signal-field spectral width ΔS_s [full width at half maximum (FWHM)] as functions of the number N_L of domains for an ensemble of RPSs with standard deviation σ equal to 0 m (solid curve), 0.1×10^{-6} m (solid curve with \times), 0.5×10^{-6} m (solid curve with Δ), 1×10^{-6} m (solid curve with \circ), and 2×10^{-6} m (solid curve with \diamond); “arb. units” stands for arbitrary units.

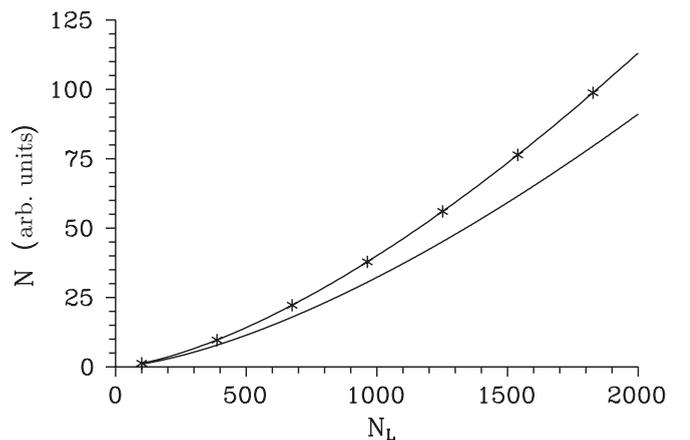


FIG. 2. Photon-pair generation rate N as it depends on the number N_L of domains for an ensemble of weakly-random structures with standard deviation σ equal to 0 m (solid curve with $*$) and 2×10^{-6} m (solid curve).

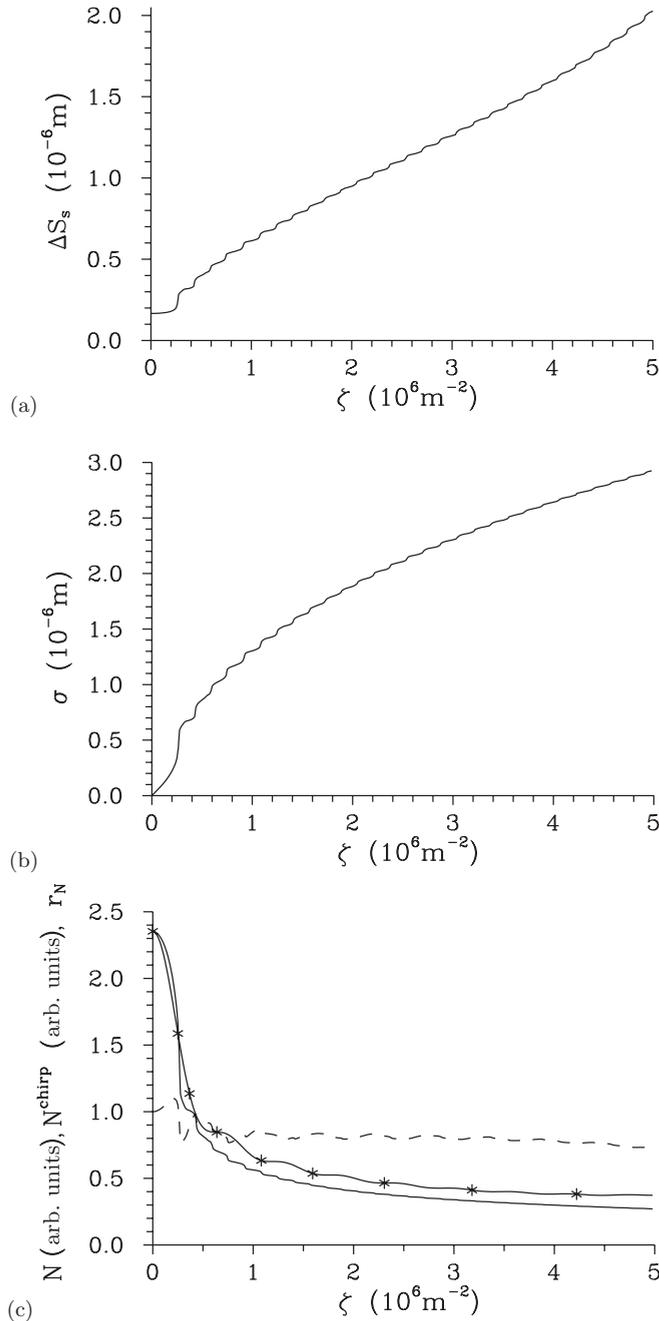


FIG. 3. (a) Signal-field spectral width ΔS_s (FWHM) as a function of chirping parameter ζ . (b) Transformation curve between the standard deviation σ and chirping parameter ζ assuming the same spectral widths ΔS_s . (c) Photon-pair generation rate for chirped (N^{chirp} , solid curve with $*$) and random (N , solid curve) structures and their ratio r_N ($r_N = N/N^{\text{chirp}}$, dashed curve) as functions of chirping parameter ζ ; $N_L = 700$.

of chirping parameter ζ are plotted in Fig. 3(b). Photon-pair generation rates N for RPSs and CPPSs are drawn for comparison in Fig. 3(c) in this case. Whereas CPPSs give better photon-pair generation rates N for larger values of chirping parameter ζ , RPSs even slightly overcome on average CPPSs for smaller values of ζ . Moreover, the signal-field spectra S_s of individual realizations may be even broader, which results in sharper temporal features. On the other hand, these spectra

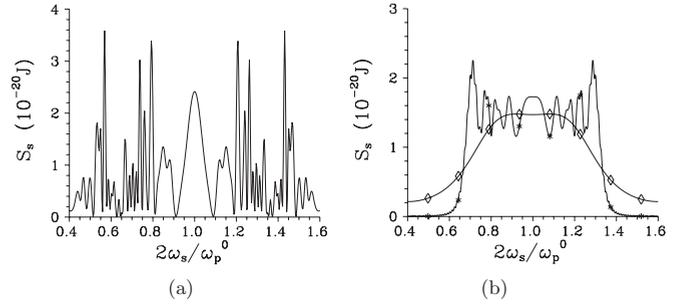


FIG. 4. Signal-field spectrum S_s for (a) one typical realization of RPS (solid curve) and (b) CPPS (solid curve with $*$) and an ensemble of RPSs (solid curve with \diamond). Spectra S_s are normalized such that one photon is emitted; $\sigma = 2.1 \times 10^{-6} \text{m}$, $\zeta = 2.5 \times 10^6 \text{m}^{-2}$, $N_L = 700$.

are typically composed of many local peaks (see Fig. 4). RPSs thus represent an alternative broadband and efficient source of photon pairs with properties comparable to CPPSs. We note that histograms of domain lengths corresponding to RPSs are broader compared to those characterizing CPPSs.

Alternatively, RPSs and CPPSs can be compared under the requirement of equal photon-pair generation rates N . Values of the photon-pair generation rate N decrease with the increasing values of chirping parameter ζ [see Fig. 3(c)]. The transformation curve between standard deviation σ and chirping parameter ζ stemming from the requirement of equal generation rates N is monotonous and is plotted in Fig. 5(a) in the considered case. The corresponding signal-field widths ΔS_s plotted in Fig. 5(b) reveal that CPPSs provide broader spectra for the most of values of chirping parameter ζ . However, the difference in spectral widths in CPPSs and RPSs is not dramatic.

The above presented results for RPSs represent an ensemble average. In practical applications, properties of individual realizations of a given RPS are naturally important. In Fig. 6, we show generation rates N and signal-field widths ΔS_s for 10 000 realizations of the RPS. Histograms of the generation rates N and signal-field widths ΔS_s plotted in Figs. 6(b) and 6(c) are close to Gaussian distributions, in accordance with the central limiting theorem.

It holds that the larger the deviation σ , the smaller the relative quadratic fluctuations δN and $\delta \Delta S_s$ of photon-pair generation rates N and signal-field spectral widths ΔS_s , respectively [$\delta x = \sqrt{\langle (\Delta x)^2 \rangle_{\text{av}}} / \langle x \rangle_{\text{av}}$, $\Delta x = x - \langle x \rangle_{\text{av}}$] (see Fig. 7). However, we should note that these relative fluctuations are quite large and can even approach 40%.

IV. TEMPORAL CORRELATIONS AND ENTANGLEMENT TIME

There occurs a strong correlation between possible detection times of a signal photon and its twin from one photon pair because both photons are generated inside the nonlinear medium at one instant. A finite distance between the detection times of both photons is a consequence of dispersion properties of the nonlinear medium through which both photons at different frequencies propagate before they leave the crystal. Temporal correlations of the signal and idler photons can be conveniently described using a two-photon

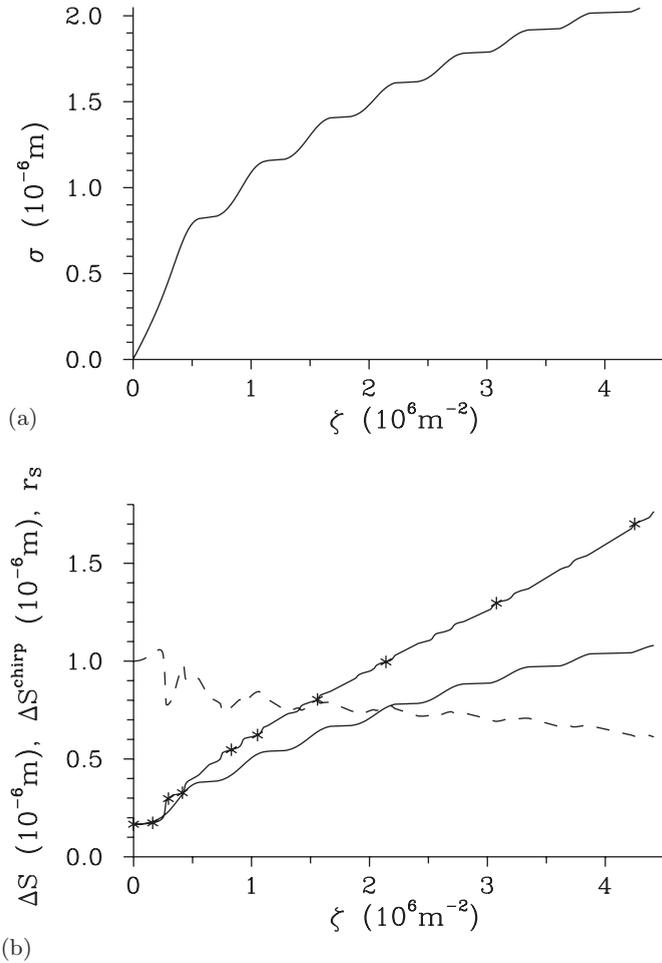


FIG. 5. (a) Transformation curve between the standard deviation σ of RPSs and chirping parameter ζ assuming the same photon-pair generation rates N . (b) Signal-field spectral widths (FWHM) for random (ΔS_s , solid curve) and chirped ($\Delta S_s^{\text{chirp}}$, solid curve with *) structures and their ratio r_S ($r_S = S_s/S_s^{\text{chirp}}$, dashed curve) as functions of chirping parameter ζ ; $N_L = 700$.

temporal amplitude \mathcal{A} defined as

$$\mathcal{A}(t_s, t_i) = \langle \text{vac} | \hat{E}_s^{(+)}(t_s) \hat{E}_i^{(+)}(t_i) | \psi \rangle. \quad (20)$$

This amplitude $\mathcal{A}(t_s, t_i)$ gives the probability amplitude of detecting a signal photon at time t_s and an idler photon at time t_i .

The simplest experimental method for the estimation of a typical constant characterizing the temporal width of the two-photon detection window (entanglement time) uses a Hong-Ou-Mandel interferometer. In this interferometer, the signal and idler fields mutually interfere on a beam-splitter and photons leaving the beam splitter at different output ports are subsequently detected in a coincidence-count measurement. The coincidence-count rate R_n depends on a mutual time delay τ introduced between the signal and idler photons. It holds that the measured temporal extension of the interference part in the coincidence-count rate R_n is shorter or equal to the entanglement time. This behavior originates in the phase modulation along the two-photon spectral amplitude. This modulation naturally broadens the temporal two-photon

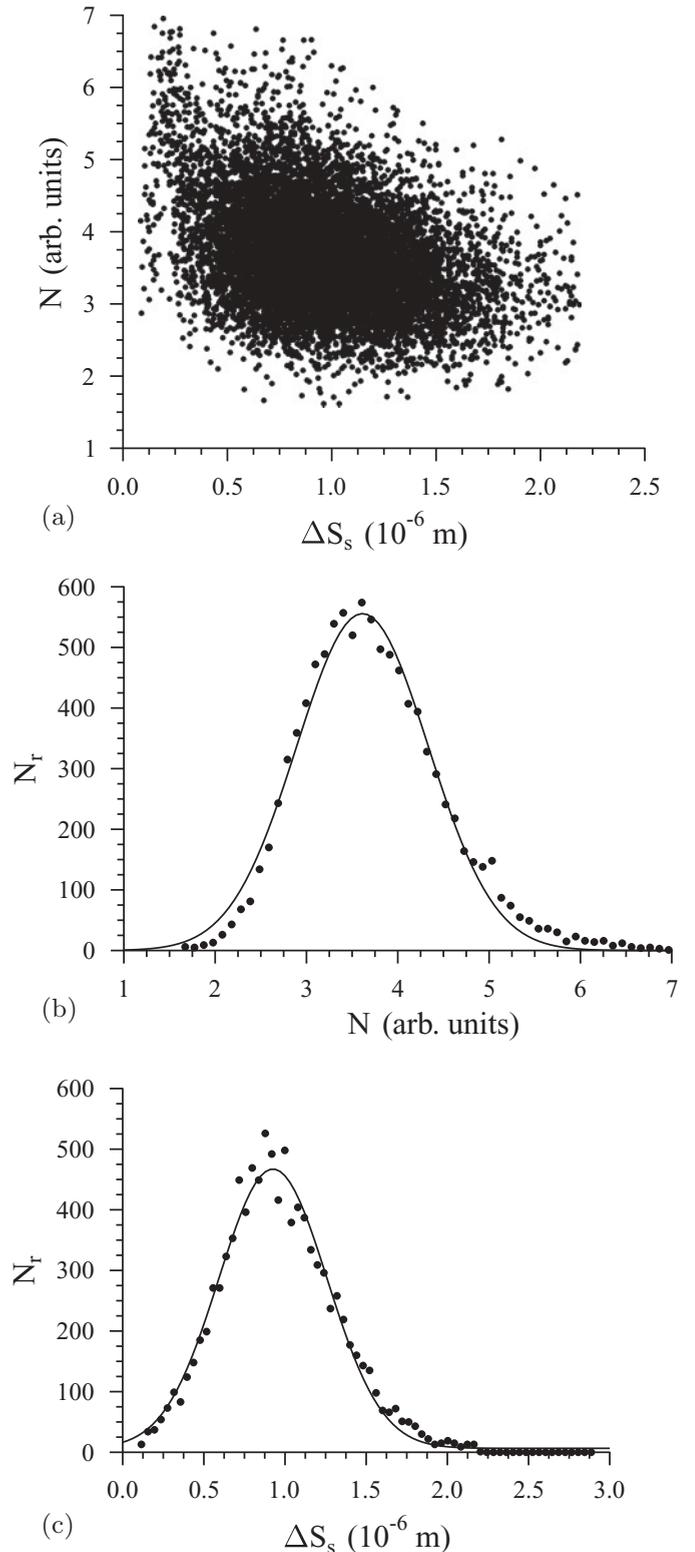


FIG. 6. (a) Photon-pair generation rates N and signal-field spectral widths ΔS_s for 10 000 realizations of RPSs (each realization is depicted as a point). Histograms of rates N (b) and widths ΔS_s (c); N_r gives the number of samples with given properties. Solid curves in (b) and (c) are best-fit Gaussian curves; $\sigma = 2.1 \times 10^{-6}$ m, $N_L = 700$.

amplitude. However, the coincidence-count rate R_n is not sensitive to the spectral modulation in the cw regime

[21,31,32]. Only if the spectral modulation is not present, the temporal width measured in the Hong-Ou-Mandel interferometer gives the correct entanglement time. The coincidence-count rate R_n as a function of relative time delay τ is described

by the following formula:

$$R_n(\tau) = 1 - \varrho(\tau), \quad (21)$$

where

$$\rho(\tau) = \frac{1}{2R_0} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \text{Re}[\langle \mathcal{A}(t_1, t_2 - \tau) \mathcal{A}^*(t_2, t_1 - \tau) \rangle_{\text{av}}], \quad (22)$$

$$R_0 = \frac{1}{2} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \langle |\mathcal{A}(t_1, t_2)|^2 \rangle_{\text{av}}. \quad (23)$$

Inserting Eqs. (20) and (4) for the two-photon temporal amplitude \mathcal{A} and quantum state $|\psi\rangle$, respectively, into Eqs. (22) and (23), we arrive at the expressions

$$\rho(\tau) = \frac{\pi \hbar^2}{4\epsilon_0^2 c^2 \mathcal{B}^2} \frac{1}{R_0} \text{Re} \left[\int_0^{\infty} d\omega_s \int_0^{\infty} d\omega_i \omega_s \omega_i \langle \Phi(\omega_s, \omega_i) \Phi^*(\omega_i, \omega_s) \rangle_{\text{av}} \exp(i\omega_i \tau) \exp(-i\omega_s \tau) \right], \quad (24)$$

$$R_0 = \frac{\pi \hbar^2}{4\epsilon_0^2 c^2 \mathcal{B}^2} \int_0^{\infty} d\omega_s \int_0^{\infty} d\omega_i \omega_s \omega_i \langle |\Phi(\omega_s, \omega_i)|^2 \rangle_{\text{av}}. \quad (25)$$

For simplicity, we further assume cw pumping with amplitude ξ_p at frequency ω_p^0 , i.e., $E_p^{(+)}(\omega_p) = \xi_p \delta(\omega_p - \omega_p^0)$. Formulas in Eqs. (24) and (25) can be simplified in this case and recast into the following form:

$$\rho(\tau) = \frac{\hbar^2}{8\epsilon_0^2 c^2 \mathcal{B}^2} \frac{|\xi_p|^2}{R_0} \text{Re} \left[\exp(i\omega_p^0 \tau) \int_0^{\infty} d\omega_s \omega_s (\omega_p^0 - \omega_s) |g(\omega_s, \omega_p^0 - \omega_s)|^2 \times \exp(-2i\omega_s \tau) \mathcal{F}(\Delta k(\omega_s, \omega_p^0 - \omega_s), \Delta k(\omega_p^0 - \omega_s, \omega_s)) \right], \quad (26)$$

$$R_0 = \frac{\hbar^2 |\xi_p|^2}{8\epsilon_0^2 c^2 \mathcal{B}^2} \int_0^{\infty} d\omega_s \omega_s (\omega_p^0 - \omega_s) |g(\omega_s, \omega_p^0 - \omega_s)|^2 |\mathcal{F}(\Delta k(\omega_s, \omega_p^0 - \omega_s), \Delta k(\omega_s, \omega_p^0 - \omega_s))|^2. \quad (27)$$

Function \mathcal{F} introduced in Eqs. (26) and (27) incorporates phase-matching conditions into the description of temporal properties of photon pairs and is defined according to the formula

$$\mathcal{F}(\Delta k, \Delta k') = \langle F(\Delta k) F^*(\Delta k') \rangle_{\text{av}}; \quad (28)$$

phase-matching function F has been introduced in Eq. (10).

Considering RPSs with fluctuations of boundaries described by a Gaussian distribution in Eq. (8), function \mathcal{F} in Eq. (28) takes the form

$$\mathcal{F}(\Delta k, \Delta k') = \frac{4}{\Delta k \Delta k'} \tilde{\mathcal{F}}(\Delta k, \Delta k') \exp[-i(\Delta k - \Delta k') N_L l_0], \quad (29)$$

$$\tilde{\mathcal{F}}(\Delta k, \Delta k') = \frac{1 - H(Dk)^{N_L+1}}{1 - H(Dk)} \left[\frac{H(\Delta k)}{H(\Delta k) + H(Dk)} \left(-H(\Delta k) \frac{1 - [-H(\Delta k)]^{N_L}}{1 + H(\Delta k)} - H(Dk) \frac{1 - [H(Dk)]^{N_L}}{1 - H(Dk)} \right) + (\Delta k \longleftrightarrow -\Delta k') \right]; \quad (30)$$

$Dk = \Delta k - \Delta k'$. Function H occurring in Eq. (30) has been defined in Eq. (16). The symbol $(\Delta k \longleftrightarrow -\Delta k')$ in Eq. (30) replaces the term that is obtained by the indicated exchange applied to the preceded term inside the brackets.

Considering weakly random structures, the following form of the function $\tilde{\mathcal{F}}$ can be derived:

$$\tilde{\mathcal{F}}^{\text{w-r}}(\Delta k, \Delta k') = G(Dk) \frac{1 - \exp[iDk l_0 (N_L + 1)]}{1 - \exp(iDk l_0)} + \left[\frac{G(\Delta k) G(\Delta k')}{1 - \exp(-i\Delta k' l_0)} \left(\frac{1 - \exp(i\Delta k N_L l_0)}{1 - \exp(-i\Delta k l_0)} + \frac{1 - \exp(-iDk N_L l_0)}{1 - \exp(-iDk l_0)} \right) + (\Delta k \longleftrightarrow -\Delta k') \right]. \quad (31)$$

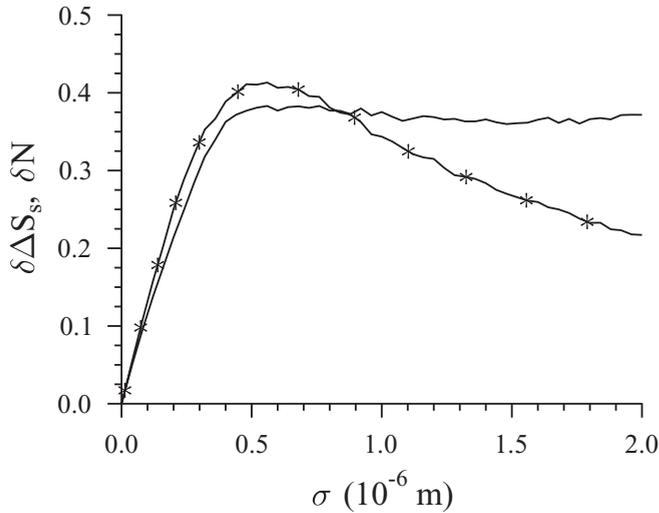


FIG. 7. Relative quadratic fluctuations $\delta\Delta S_s$ of the signal-field spectral width (solid curve) and relative quadratic fluctuations δN of the photon-pair generation rate (solid curve with $*$) as they depend on standard deviation ζ ; $N_L = 700$.

$$I^{\text{sum}}(\tau) = \frac{\eta^{\text{sum}} \hbar^2}{4\epsilon_0^2 c^2 \mathcal{B}^2} \int_0^\infty d\omega_p |E_p^{(+)}(\omega_p)|^2 \int_0^\infty d\omega_s \sqrt{\omega_s(\omega_p - \omega_s)} \int_0^\infty d\omega'_s \sqrt{\omega'_s(\omega_p - \omega'_s)} g(\omega_s, \omega_p - \omega_s) g^*(\omega'_s, \omega_p - \omega'_s) \times \exp[-i(\omega_s - \omega'_s)\tau] \mathcal{F}(\Delta k(\omega_s, \omega_p - \omega_s), \Delta k(\omega'_s, \omega_p - \omega'_s)). \quad (34)$$

When deriving Eqs. (33) and (34) we have assumed that the nonlinear medium in which sum-frequency generation occurs is ideally phase matched for frequencies present in the signal and idler fields.

A detailed analysis has shown that the time $\Delta\tau^{\text{HOM}}$ measured in a Hong-Ou-Mandel interferometer is inversely proportional to the spectral widths ΔS_s and ΔS_i of the signal and idler fields despite their complex profiles. This behavior originates in the insensitivity of the coincidence-count rate R_n to phase variations along the signal- and idler-field spectra in the cw regime. This property is manifested in nonlocal dispersion cancellation [31,33]. The time $\Delta\tau^{\text{HOM}}$ determined by a temporal extension [full width at half maximum (FWHM)] of the coincidence-count interference pattern in the rate $R_n(\tau)$ thus shortens with increasing values of the standard deviation σ for RPSs. The dependence of time $\Delta\tau^{\text{HOM}}$ on the deviation σ for an ensemble of RPSs composed of 700 domains is shown in Fig. 8. We can see in Fig. 8 that the times $\Delta\tau^{\text{HOM}}$ can be as short as several fs for sufficiently large values of the deviation σ . This indicates that temporal quantum correlations can be confined into an interval characterizing one optical cycle provided that the spectral phase variations in the signal and idler fields are compensated. The times $\Delta\tau^{\text{HOM}}$ of CPPSs with the same signal-field spectral widths ΔS_s are plotted in Fig. 8 for comparison, which reveals a nearly identical behavior of both types of structures. Typical coincidence-count interference patterns given by the rate R_n for photon pairs generated in both types

On the other hand, function $\tilde{\mathcal{F}}^{\text{chirp}}$ attains a simple form in the case of CPPSs:

$$\tilde{\mathcal{F}}^{\text{chirp}}(\Delta k, \Delta k') = F^{\text{chirp}}(\Delta k) F^{\text{chirp}*}(\Delta k'), \quad (32)$$

where the formula for F^{chirp} is written in Eq. (19).

The true characteristics of temporal correlations (entanglement time) between the signal and idler fields can be reached in the measurement of sum-frequency intensity in a nonlinear medium combining the signal and idler photons and having a sufficiently high value of $\chi^{(2)\text{sum}}$ nonlinearity. This process allows us to determine the temporal correlation function I^{sum} of the intensities of the signal and idler fields. The intensity I^{sum} of the sum-frequency field is given along the formula

$$I^{\text{sum}}(\tau) = \eta^{\text{sum}} \int_{-\infty}^{\infty} dt |\langle \text{vac} | \hat{E}_s^{(+)}(t) \hat{E}_i^{(+)}(t - \tau) | \psi \rangle|^2, \quad (33)$$

where constant η^{sum} incorporates the value of $\chi^{(2)\text{sum}}$ nonlinearity and quantum detection efficiency.

The general formula in Eq. (33) can be recast into the following form using the expression for function \mathcal{F} in Eq. (28):

of structures are compared in Fig. 9. They demonstrate a close similarity of photon-pair behavior in a Hong-Ou-Mandel interferometer. There occur typical oscillations at the shoulders of the interference dips. Whereas regular oscillations characterize CPPSs, irregular oscillations with larger amplitudes occur for

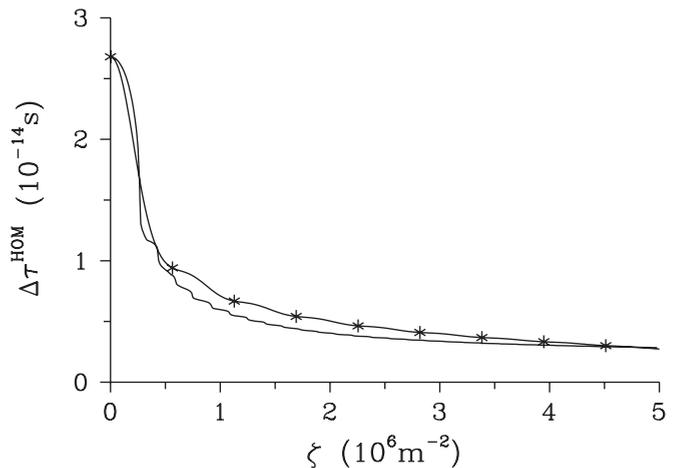


FIG. 8. Time $\Delta\tau^{\text{HOM}}$ (FWHM) characterizing correlations in a Hong-Ou-Mandel interferometer as a function of chirping parameter ζ for an ensemble of RPSs with standard deviations σ derived from the curve in Fig. 2(b) (solid curve) and CPPSs (solid curve with $*$); $N_L = 700$.

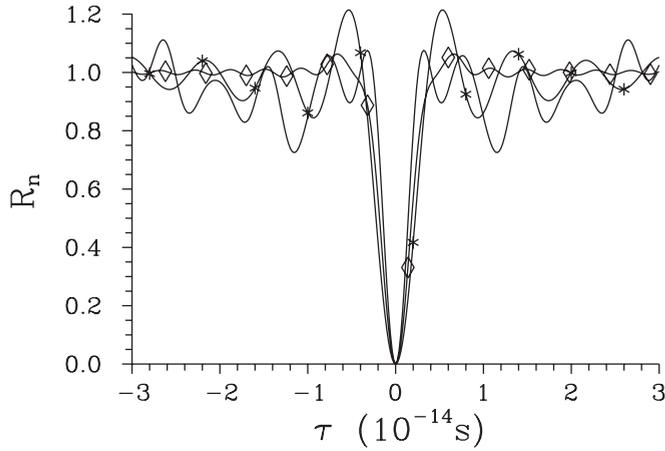


FIG. 9. Coincidence-count rate R_n as it depends on relative time delay τ for one realization of RPS (solid curve), CPPS (solid curve with *), and an ensemble of RPSs (solid curve with \diamond); $\sigma = 2.1 \times 10^{-6}$ m, $\zeta = 2.5 \times 10^6$ m $^{-2}$, $N_L = 700$.

individual realizations of RPSs. However, the widths of the interference dips remain practically unchanged for different realizations of RPSs.

Correlation times $\Delta\tau^{\text{sum}}$ emerging from sum-frequency generation are in general longer than times $\Delta\tau^{\text{HOM}}$ observed in a Hong-Ou-Mandel interferometer because of a strong phase modulation along the wide signal- and idler-field spectra S_s and S_i (see Fig. 10). Correlation times $\Delta\tau^{\text{sum}}$ can be even an order of magnitude greater as compared to times $\Delta\tau^{\text{HOM}}$ measured in a Hong-Ou-Mandel interferometer for structures with ultrabroadband spectra. However, the phase modulation along the spectrum can be compensated to certain extent, which gives shorter correlation times $\Delta\tau^{\text{sum}}$. CPPSs have a more regular spectral phase behavior (as demonstrated in Fig. 10) and the quadratic phase compensation is usually sufficient to provide wave packets several fs long. As for individual realizations of RPSs, the quadratic compensation is less efficient because

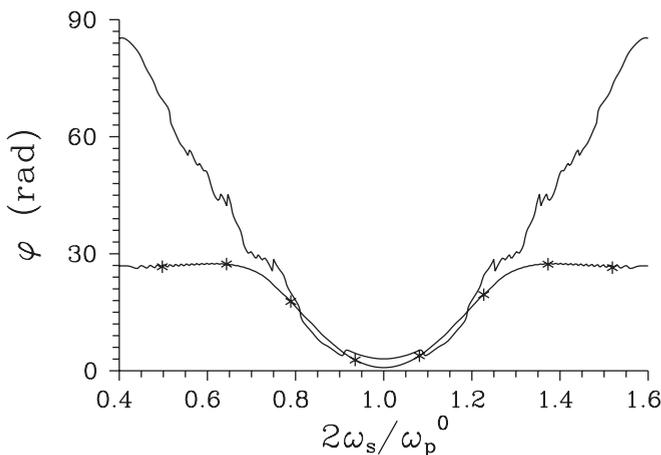


FIG. 10. Phase φ of the two-photon spectral amplitude $\Phi(\omega_s, \omega_p^0 - \omega_s)$ as it depends on normalized signal-field frequency $2\omega_s/\omega_p^0$ for one realization of RPS (solid curve) and CPPS (solid curve with *); $\sigma = 2.1 \times 10^{-6}$ m, $\zeta = 2.5 \times 10^6$ m $^{-2}$, $N_L = 700$.

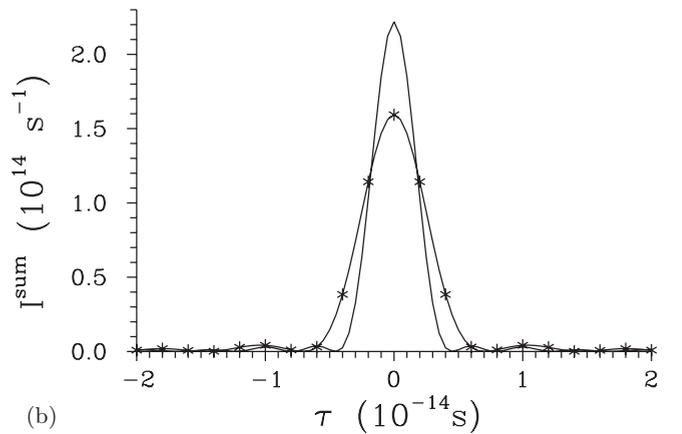
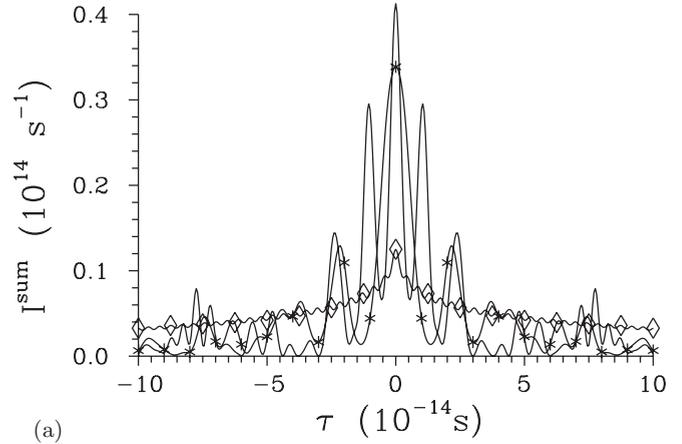


FIG. 11. Sum-frequency field intensity I^{sum} as a function of relative time delay τ for one realization of RPS (solid curve), CPPS (solid curve with *), and an ensemble of RPSs (solid curve with \diamond). In (a) the quadratic chirp in the signal-field amplitude spectrum is compensated for one realization of RPS and CPPS; in (b) a complete spectral phase compensation is assumed. The curves are normalized such that $\int_{-\infty}^{\infty} d\tau I^{\text{sum}}(\tau) = 1$; $\sigma = 2.1 \times 10^{-6}$ m, $\zeta = 2.5 \times 10^6$ m $^{-2}$, $N_L = 700$.

of more irregular phase spectral behavior. Despite this, the values of temporal constants typical for chirped structures can be approached [see Fig. 11(a)]. Provided that an ideal phase compensation is reached, both types of structures give comparable results [see Fig. 11(b)] and are capable of generating photon pairs with wave packets extending over the duration of one optical cycle. Experimentally, pulse shapers have been developed for this task and their capabilities in the area of photon pairs have already been demonstrated [34]. Comparison of the results obtained with the quadratic and ideal compensations reveals that correlation times $\Delta\tau^{\text{sum}}$ are approximately two times larger if we restrict ourselves to the quadratic compensation. Also, the value of quadratic chirp that needs compensation differs for individual realizations of a RPS. This requires an adaptive phase compensator. On the other hand, the phase compensation in case of a CPPS can be reached in a simpler way, e.g., by inserting a piece of suitable material of defined length [35,36]. Despite this, RPSs are challenging both for basic physical experiments as well as metrology applications.

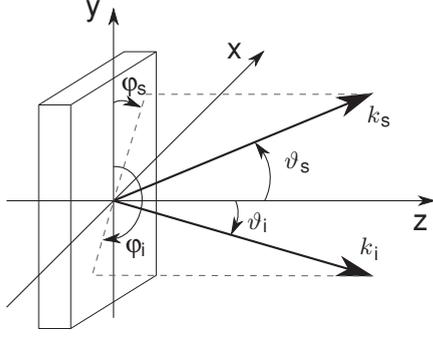


FIG. 12. Geometric scheme for the description of spatial properties. Direction of the signal-(idler)-field wave vector k_s (k_i) is given by radial ϑ_s (ϑ_i) and azimuthal φ_s (φ_i) emission angles.

V. CORRELATIONS IN THE TRANSVERSE PLANE

In order to describe the spatial properties of the signal and idler beams (in the transverse plane), a simple generalization of the model presented in Sec. II has to be developed. The inclusion of phase-matching conditions also in the directions along the x and y axes and the assumption of a spectrally flat transverse pump-beam profile $E_{p\perp}(x, y)$ result in the following separable form of a two-photon spectral amplitude Φ that additionally depends on radial (ϑ_s , ϑ_i) and azimuthal (φ_s , φ_i) signal- and idler-field emission angles (see Fig. 12):

$$\begin{aligned} \Phi(\omega_s, \omega_i, \vartheta_s, \varphi_s, \vartheta_i, \varphi_i) \\ = \Phi_z(\omega_s, \omega_i, \vartheta_s, \varphi_s, \vartheta_i, \varphi_i) \Phi_{xy}(\omega_s, \omega_i, \vartheta_s, \varphi_s, \vartheta_i, \varphi_i), \end{aligned} \quad (35)$$

where function Φ_z arises from phase-matching conditions in the z direction and function Φ_{xy} originates in phase-matching conditions in the transverse xy plane (see Fig. 12). Function Φ_z can be derived in analogy with the formula in Eq. (5):

$$\begin{aligned} \Phi_z(\omega_s, \omega_i, \vartheta_s, \varphi_s, \vartheta_i, \varphi_i) \\ = g(\omega_s, \omega_i) E_p^{(+)}(\omega_s + \omega_i) F(\Delta k(\omega_s, \omega_i, \vartheta_s, \varphi_s, \vartheta_i, \varphi_i)), \end{aligned} \quad (36)$$

where the stochastic function F has been introduced in Eq. (6). Phase-matching conditions in the xy plane give the function Φ_{xy} in the following form:

$$\begin{aligned} \Phi_{xy}(\omega_s, \omega_i, \vartheta_s, \varphi_s, \vartheta_i, \varphi_i) \\ = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy E_{p\perp}(x, y) \exp(i\Delta k_x x + i\Delta k_y y), \end{aligned} \quad (37)$$

which includes a pump-beam amplitude profile $E_{p\perp}(x, y)$ in the transverse plane. Assuming normal incidence of the pump beam, the Cartesian components of nonlinear phase mismatch in Eqs. (36) and (37) can be written as

$$\begin{aligned} \Delta k_x &= k_s(\omega_s) \sin(\vartheta_s) \sin(\varphi_s) + k_i(\omega_i) \sin(\vartheta_i) \sin(\varphi_i), \\ \Delta k_y &= k_s(\omega_s) \sin(\vartheta_s) \cos(\varphi_s) + k_i(\omega_i) \sin(\vartheta_i) \cos(\varphi_i), \\ \Delta k_z &= k_p(\omega_s + \omega_i) - k_s(\omega_s) \cos(\vartheta_s) - k_i(\omega_i) \cos(\vartheta_i). \end{aligned} \quad (38)$$

We assume a Gaussian pump-beam transverse profile in numerical calculations: $E_{p\perp}(x, y) = 1/(\pi\Delta x_p\Delta y_p)$

$\exp[-(x/\Delta x_p)^2 - (y/\Delta y_p)^2]$ and Δx_p (Δy_p) stands for the pump-beam width along the x (y) direction.

We first pay attention to transverse properties of the signal beam only. Its spectral density s_s , defined as

$$\begin{aligned} s_s(\omega_s, \vartheta_s, \varphi_s) &= \sin(\vartheta_s) \int d\omega_i \int d\vartheta_i \sin(\vartheta_i) \\ &\times \int d\varphi_i |\Phi(\omega_s, \omega_i, \vartheta_s, \varphi_s, \vartheta_i, \varphi_i)|^2, \end{aligned} \quad (39)$$

depends on the signal-field radial (ϑ_s) and azimuthal (φ_s) emission angles. As we study photon-pair emission near the collinear geometry, the signal beam (as well as the idler beam) has rotational symmetry along the z axis. The dependence of spectral density s_s on signal-field radial emission angle ϑ_s is shown in Fig. 13. Investigating one realization of RPS, we observe a typical ‘‘striplike’’ behavior depicted in Fig. 13(a). Fixing the value of radial emission angle ϑ_s , spectrum $s_s(\omega_s)$ is composed of many peaks occurring at positions specific for the studied realization [compare also with Fig. 4(a)]. Each peak changes its central frequency continuously as the radial emission angle ϑ_s moves. We note that this is typical also for layered structures that form band gaps [6]. Averaging over many realizations of RPSs smoothes this striplike behavior [see Fig. 13(b)] and leads to that resembling CPPSs [compare Figs. 13(b) and 13(c)]. In these cases spectral splitting is observed [37]. This behavior originates in phase-matching conditions along the z direction.

Integration of spectral densities s_s over the signal-field frequency ω_s gives us densities n_s of photon-pair numbers that are plotted in Fig. 14 for the structures studied in Fig. 13. Whereas the profile of density $n_s(\vartheta_s)$ is complex for one realization of RPS, typical shapes with one maximum around a nonzero value of ϑ_s characterize the profiles of density $n_s(\vartheta_s)$ for an ensemble of RPSs and CPPS.

The correlated area g_i of an (idler) photon in a pair represents spatial analogy to entanglement time and characterizes correlations of photon twins in the transverse plane. By definition, it gives the probability of emitting an idler photon into radial emission angle ϑ_i and azimuthal emission angle φ_i provided that its signal twin has been emitted in a fixed radial emission angle ϑ_s and azimuthal emission angle φ_s , i.e.,

$$\begin{aligned} g_i(\vartheta_i, \varphi_i; \vartheta_s, \varphi_s) \\ = \sin(\vartheta_s) \sin(\vartheta_i) \int d\omega_s \int d\omega_i |\Phi(\omega_s, \omega_i, \vartheta_s, \varphi_s, \vartheta_i, \varphi_i)|^2. \end{aligned} \quad (40)$$

Because we mainly pay attention to beams propagating in the vicinity of the z axis, we assume that the signal photon is emitted along the z axis ($\vartheta_s = \varphi_s = 0^\circ$). The correlated area as described by function g_i in Eq. (40) then has a rotational symmetry and its profiles along the radial emission angle ϑ_i for CPPS and an ensemble of RPSs nearly coincide, as documented in Fig. 15(a). On the other hand, broader profiles are typical for individual realizations of RPSs. These individual realizations form compact correlated areas without large local peaks (compare with Fig. 4 where spectrum S_s for one realization of RPS is plotted). The width $\Delta\vartheta_i$ of the

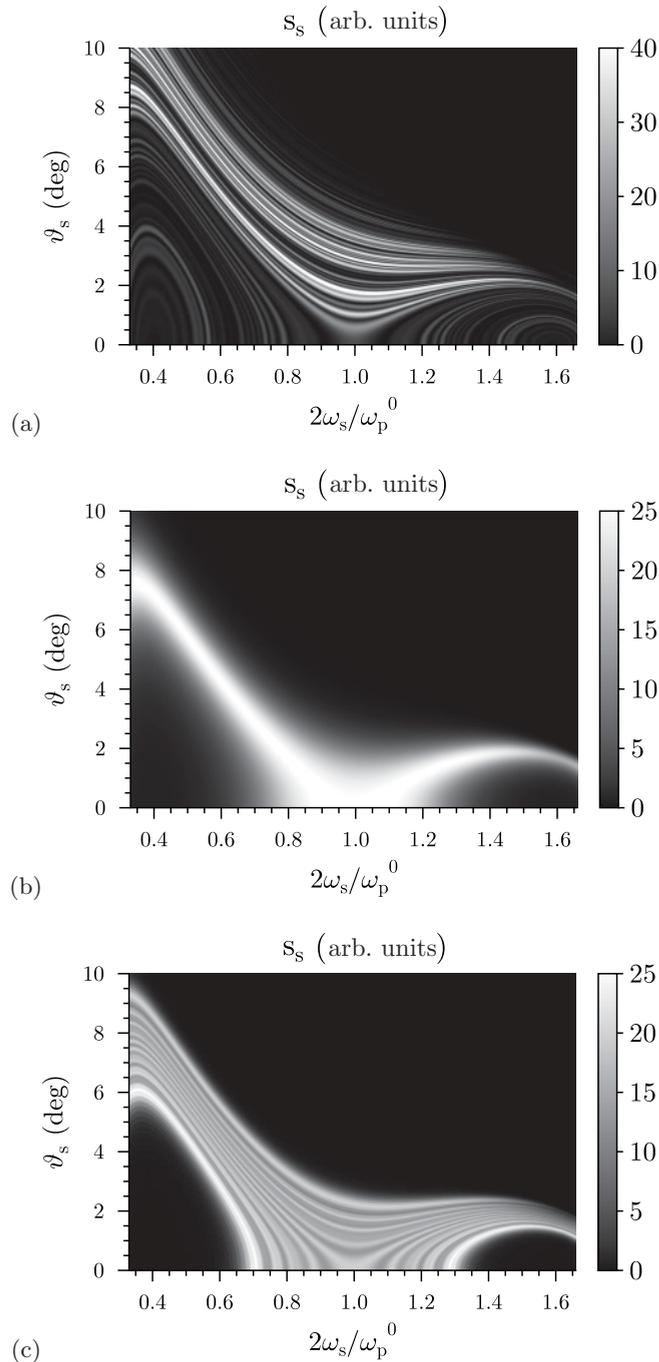


FIG. 13. Map of signal-field spectral density s_s as it depends on signal-field radial emission angle ϑ_s for (a) one realization of RPS, (b) an ensemble of RPSs, and (c) CPPS; $\varphi_s = 0^\circ$, $\sigma = 2.1 \times 10^{-6}$ m, $\zeta = 2.5 \times 10^6$ m $^{-2}$, $N_L = 700$.

correlated area along the radial angle ϑ_i depends in general on phase-matching conditions along the z and ϑ_i axes. Thus the length of the structure, the pump-field (temporal) spectral width, as well as the width of the pump-beam waist determine together the width $\Delta\vartheta_i$ (for more details, see Ref. [37]). For example, focusing the pump beam, the values of the radial width $\Delta\vartheta_i$ can be varied nearly by one order of magnitude [see Fig. 15(b)]. This behavior can be easily explained by the fact that the more the pump beam is focused, the wider its

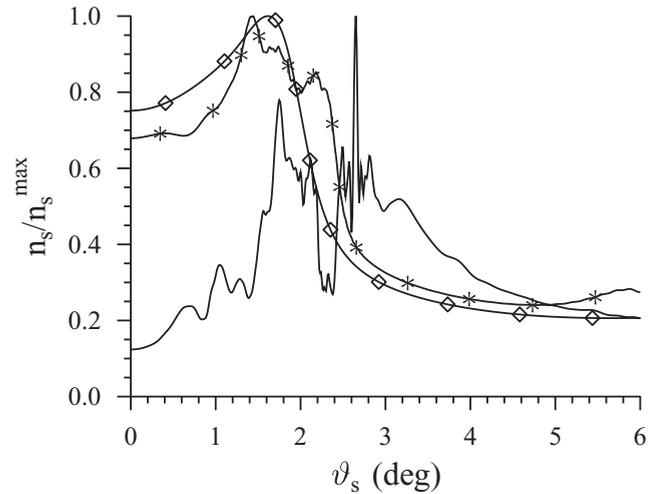


FIG. 14. Profile of density n_s of signal-field photon numbers as a function of signal-field radial emission angle ϑ_s for one realization of RPS (solid curve), CPPS (solid curve with *), and an ensemble of RPSs (solid curve with \diamond); $n_s^{\max} = \max_{\vartheta_s} [n_s(\vartheta_s)]$. Plane-wave pumping is assumed; $n_s(\vartheta_s, \varphi_s) = \int d\omega_s s_s(\omega_s, \vartheta_s, \varphi_s)$; $\varphi_s = 0^\circ$, $\sigma = 2.1 \times 10^{-6}$ m, $\zeta = 2.5 \times 10^6$ m $^{-2}$, $N_L = 700$.

spatial spectrum in the transverse plane, and so the weaker the phase-matching conditions in this plane.

VI. THE ROLE OF TEMPERATURE

We have seen that an ensemble of RPSs and a CPPS have similar properties. This close similarity is preserved also when studying temperature dependences [21] that are in general weak. On the other hand, behavior of individual realizations of RPSs manifests a stronger temperature dependence. However, the influence of temperature varies from realization to realization. Whereas properties of the realization of RPS studied above do not considerably change with temperature (see Fig. 16 for the signal-field spectral width ΔS_s in the temperature range from 284 to 300 K), other realizations are more prone to a change in temperature. This can be conveniently used for efficient temperature modifications of properties of photon pairs. We note that these effects have their origin in temperature dependence of indexes of refraction [21].

VII. THE ROLE OF SMALL RANDOM (FABRICATION) ERRORS

In the fabrication process, a small random error necessarily occurs [38]. This error is sometimes called a duty cycle error and, in general, leads to lowering of photon-pair emission rates [18]. Considering spectral widths, they are resistant against this error in uniformly periodically poled crystals [38] [see also Eq. (18), valid for weakly-random structures]. On the other hand, spectral widths are slightly reduced in CPPSs as documented in Fig. 17. We can see in Fig. 17 that a (large) fabrication error with variance $\sigma_{er} = 5 \times 10^{-7}$ m results in the reduction of signal-field spectral width ΔS_s only $\sim 10\%$. Individual realizations of RPSs are much more sensitive to the fabrication error. The observed spectral changes depend on

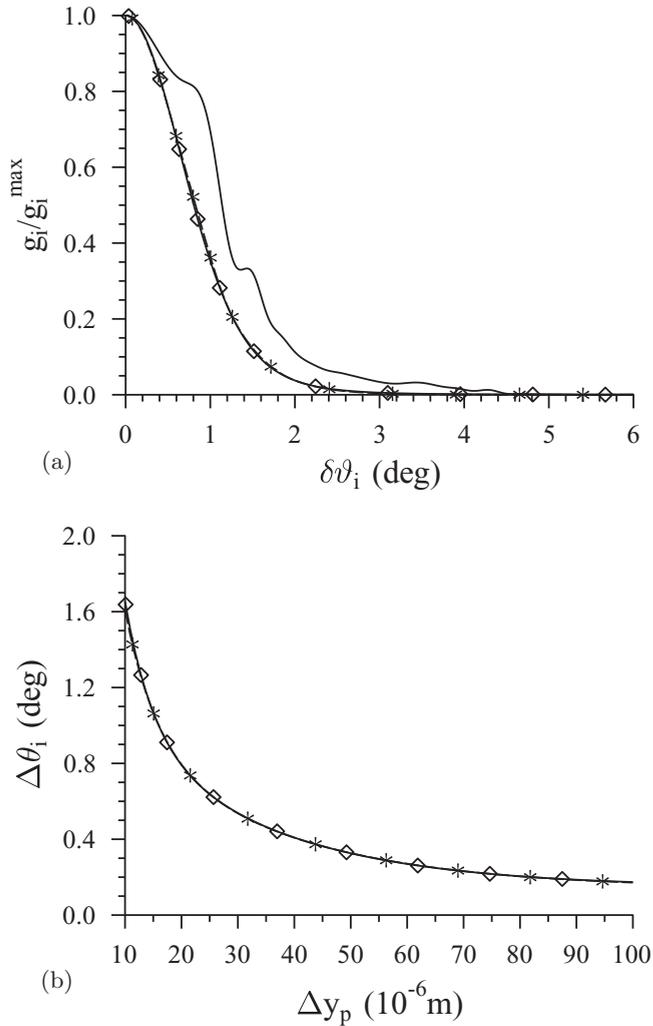


FIG. 15. (a) Radial profile $g_i(\vartheta_i)$ of the correlated area for $\Delta y_p = 1 \times 10^{-5}$ m and (b) radial width $\Delta\vartheta_i$ of the correlated area as it depends on pump-beam width Δy_p for one realization of RPS (solid curve), CPPS (solid curve with *), and an ensemble of RPSs (solid curve with \diamond); $\vartheta_i = \vartheta_i^0 + \delta\vartheta_i$; $g_i^{\max} = \max_{\vartheta_i} [g_i(\vartheta_i)]$. A radially symmetric pump beam is assumed, i.e., $\Delta x_p = \Delta y_p$; $\varphi_s = \vartheta_s = 0^\circ$, $\varphi_i = 180^\circ$, $\vartheta_i^0 = 0^\circ$; $\sigma = 2.1 \times 10^{-6}$ m, $\zeta = 2.5 \times 10^6$ m $^{-2}$, $N_L = 700$.

individual realizations. As an example, the signal-field spectral width ΔS_s of the sample analyzed above decreases with the increasing variance σ_{er} of the fabrication error. This is natural, because the spectrum of this realization is broader as compared to the ensemble mean value. We note that it holds also here that the narrower the signal-field spectrum, the greater the photon-pair generation rate N and vice versa.

VIII. THE ROLE OF ORDERING IN CHIRPED PERIODICALLY POLED STRUCTURES

The benefit of ordering of individual domains by their lengths in CPPS can be quantified as follows. We take an ordered structure and divide it into segments containing d domains. We then randomly position these segments in a new artificial structure and finally obtain mean values of physical quantities after averaging over random positions. In

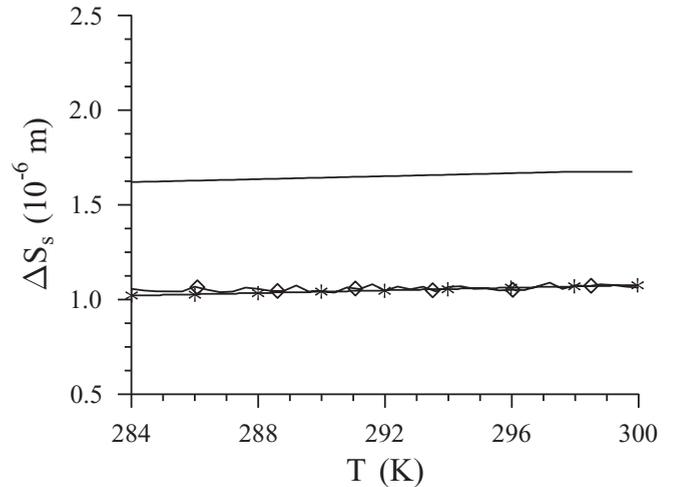


FIG. 16. Signal-field spectral width ΔS_s as it depends on temperature T for one realization of RPS (solid curve), CPPS (solid curve with *), and an ensemble of RPSs (solid curve with \diamond); $\sigma = 2.1 \times 10^{-6}$ m, $\zeta = 2.5 \times 10^6$ m $^{-2}$, $N_L = 700$.

the limiting case of $d = 1$, we have a completely random structure similar to those studied above. It can be shown that the signal-field spectral width ΔS_s decreases with the decreasing segment length d (see Fig. 18). This is accompanied by an increase of photon-pair generation rate N . This behavior reflects the fact that spectra of the fields coming from individual domains are combined in a more constructive way in the central spectral area with the increasing randomness (decreasing value of segment length d). The graph in Fig. 18 also demonstrates that the requirement for the same spectral widths ΔS_s of RPSs and CPPS inevitably implies that the histogram of the domains' lengths for RPSs is broader than that obtained for CPPS.

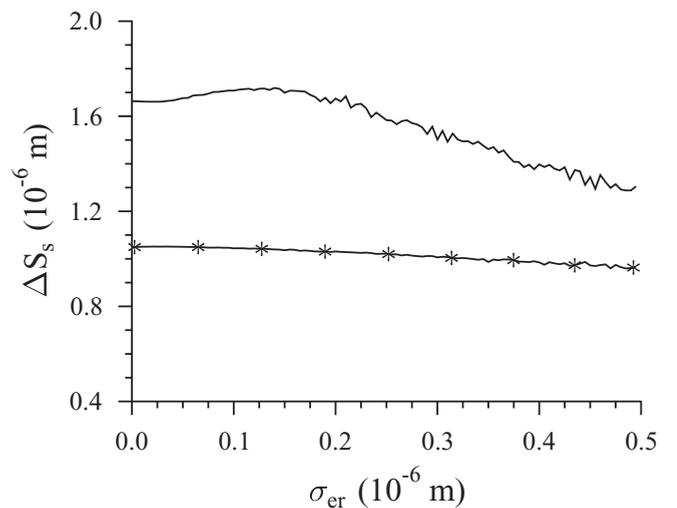


FIG. 17. Signal-field spectral width ΔS_s as a function of variance σ_{er} of the fabrication error for one realization of RPS (solid curve) and CPPS (solid curve with *). Averaging over the fabrication error was done in 1000 randomly chosen positions; $\sigma = 2.1 \times 10^{-6}$ m, $\zeta = 2.5 \times 10^6$ m $^{-2}$, $N_L = 700$.

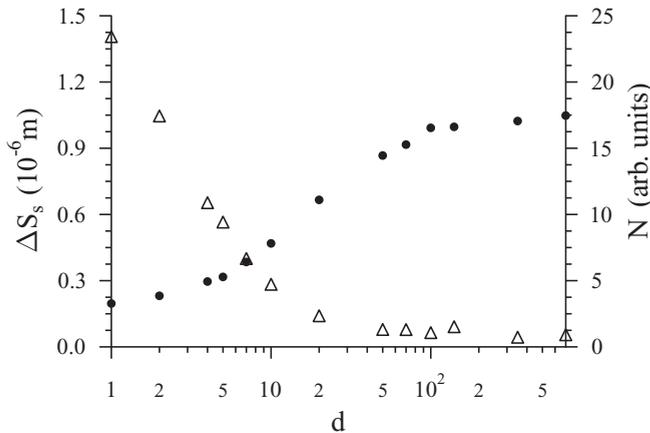


FIG. 18. Signal-field spectral width ΔS_s (solid curve with ●) and photon-pair generation rate N (solid curve with Δ) as functions of segment length d . Averaging over 1000 random positions of segments was used in calculations. $\zeta = 2.5 \times 10^6 \text{ m}^{-2}$, $N_L = 700$.

IX. CONCLUSIONS

Properties of photon pairs generated in randomly poled structures have been found to be quantitatively similar to those characterizing chirped periodically poled structures. Especially, ultrabroadband signal and idler fields can be

emitted in randomly poled structures. The accompanying sharp mutual temporal correlations of the signal and idler fields can even reach the duration of a single-photon cycle (several fs) provided that the spectral phase modulation is compensated. The phase compensation has to be tailored individually for each random structure. Photon-pair generation rates depending linearly on the number of domains are specific to random structures. Stronger temperature dependences of parameters characterizing photon pairs in random structures (their individual realizations) compared to those found in chirped periodically poled crystals have been observed. In general, the application potential of randomly poled structures similar to that of chirped periodically poled structures has been revealed. Contrary to chirped periodically poled structures, the randomly poled structures do not require high precision in their fabrication. This shows great promise for the use of randomly poled structures.

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