Strongly interacting photons in hollow-core waveguides

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Hollow-core photonic-crystal waveguides filled with cold atoms can support giant optical nonlinearities through nondispersive propagation of light tightly confined in the transverse direction. Here we explore electromagnetically induced transparency is such structures, considering a pair of counterpropagating weak quantum fields in the medium of coherently driven atoms in the ladder configuration. Strong dipole-dipole interactions between optically excited, polarized Rydberg states of the atoms translate into a large dispersive interaction between the two fields. This can be used to attain a spatially homogeneous conditional phase shift of π for two single-photon pulses, realizing a deterministic photonic phase gate, or to implement a quantum nondemolition measurement of the photon number in the signal pulse by a coherent probe, thereby achieving a heralded source of single- or few-photon pulses.

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I. INTRODUCTION

Photons are ideal carriers of information in terms of transfer rates and distances. Yet, scalable and efficient quantum-information processing [1] with photons would require implementing deterministic quantum logic between single-photon qubits [2], which is hindered by the weakness of optical nonlinearities in conventional media. Highly enhanced nonlinear interactions in atomic vapors [3] in the regime of electromagnetically induced transparency (EIT) [4–6] have emerged as a promising route to circumvent these difficulties and to achieve large conditional phase shifts ϕ for pairs of slowly propagating photons. Attaining the phase shift of $\phi = \pi$ would amount to realizing the universal CPHASE gate for photonic qubits [1].

Among the many relevant proposals [7–13], one of the most promising schemes is based on employing EIT in a ladder configuration [11], wherein the photon-photon interaction is mediated by strong dipole-dipole interactions (DDIs) between optically excited Rydberg states of the atoms [14–16]. An important advantage of this scheme is that the long-range nature of the DDI relaxes the need for tight focusing of the quantum fields to the atomic absorption cross section $\varsigma \sim \lambda^2$, which is close to the diffraction limit.

In Ref. [11] we presented an effective one-dimensional (1D) treatment of the dynamics of two slowly counterpropagating, weakly focused single-photon pulses. We did so by considering the electric fields only on the propagation axis, and showed that, for a pair of photons passing through each other, the accumulated conditional phase shift ϕ can be both large and uniform in the longitudinal direction. In free space, however, the 1D treatment of interacting quantum fields is incomplete, because it does not capture the diffraction effects and the fact that, in the transverse direction, the resulting phase shift is inhomogeneous due to the relative coordinate dependence of the DDI potential [17]. To remedy these problems and achieve nondiffracting, uniform transverse phase

fronts, here we propose to impose onto the quantum fields only a single transverse mode by confining them into a hollow-core photonic-crystal waveguide [18,19] filled with an ensemble of cold alkali atoms [20]. In what follows, we present a rigorous derivation of 1D propagation equations for two interacting quantum fields. We extend our earlier scheme by considering the atomic level configuration involving different Rydberg states. We then discuss the conditional phase shift for two single-photon pulses. Furthermore, we analyze a quantum nondemolition measurement (QND) of the photon number in the signal pulse inducing a phase shift of the coherent probe pulse. This can serve as a heralded source of single- or few-photon pulses.

II. EQUATIONS OF MOTION

We begin by assuming that the transverse intensity profile of the counterpropagating fields $\hat{E}_1$ and $\hat{E}_2$ in the cylindrically symmetric waveguide is described by a Gaussian $e^{-r_\perp^2/w_f^2}$ of width w_f, where $r_\perp = |\mathbf{r}_\perp|$ is the distance from the propagation z axis. The corresponding electric field can then be expressed as $\hat{E}_l(\mathbf{r}) = \varepsilon_l e^{-r_\perp^2/2w_f^2} \hat{\mathcal{E}}_l(z)$ ($l = 1,2$), where $\varepsilon_l = \sqrt{\hbar\omega_l/2\epsilon_0 V}$ is the field per photon of frequency ω_l within the quantization volume $V = \pi w_f^2 L$, with L the waveguide length, while $\hat{\mathcal{E}}_l(z) = \sum_k a_l^k e^{ikz}$ is the traveling-wave field operator, given by a superposition of bosonic operators a_l^k for the longitudinal field modes k, yielding the commutation relations $[\hat{\mathcal{E}}_l(z), \hat{\mathcal{E}}_{l'}^\dagger(z')] = L\delta_{ll'}\delta(z - z')$. An ensemble of N cold atoms is trapped in the hollow core of the waveguide [20]; the corresponding atomic density is then $\rho(\mathbf{r}) = (\pi w_a^2)^{-1} e^{-r_\perp^2/w_a^2}(N/L)$, where $w_a (\lesssim w_f)$ is the width of the transverse Gaussian distribution. The level configuration of the atoms, all of which are initially prepared in the ground state $|g\rangle$, is schematically shown in Fig. 1(a). The quantum fields $\hat{E}_{1,2}$ resonantly interact with the atoms on the transitions $|g\rangle \rightarrow |e_{1,2}\rangle$, respectively. The intermediate states $|e_{1,2}\rangle$ are resonantly coupled by two strong (classical) driving fields with Rabi frequencies $\Omega_{1,2}$ to the Rydberg states $|d_{1,2}\rangle$. In a static electric field $E_{\text{st}}\mathbf{e}_z$, these Rydberg states possess
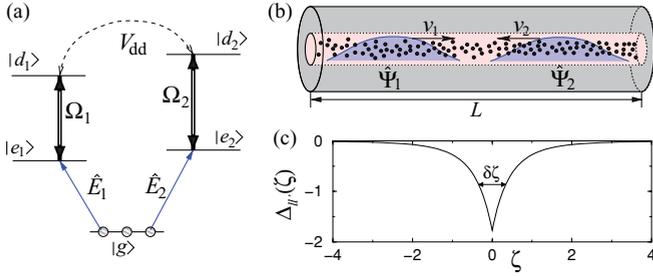
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FIG. 1. (Color online) (a) Level scheme of atoms resonantly interacting with quantum fields $\hat{E}_{1,2}$ and classical driving fields $\Omega_{1,2}$ on the corresponding transitions. $V_{\rm dd}$ denotes the DDI between atoms in Rydberg states $|d\rangle$. (b) The quantum fields transversely confined in a hollow-core waveguide of length L filled with the atoms, counterpropagate as dark-state polaritons $\hat{\Psi}_{1,2}$ having slow group velocities $v_{1,2}$ and interacting via long-range potential $\Delta_{12}(z_1 - z_2)$ mediated by $V_{\rm dd}$. (c) The potential $\Delta_{ll'}(\zeta)$ of Eq. (5), as a function of dimensionless distance ζ, in units of $2C_{ll'}/\hbar(\sqrt{2}w)^3$ Hz.

permanent dipole moments $\mathbf{p} = \frac{3}{2}nqea_0\mathbf{e}_z$, where n and q are the (effective) principal and parabolic quantum numbers, e is the electron charge, and a_0 is the Bohr radius [21]. A pair of atoms at positions $\mathbf{r}$ and $\mathbf{r}'$ excited to states $|d_l\rangle$ and $|d_{l'}\rangle$ interact with each other via a DDI potential $V_{\rm dd}$ resulting in an energy shift

$$\hbar\Delta_{ll'}(\mathbf{r} - \mathbf{r}') = C_{ll'}\frac{1 - 3\cos^2\vartheta}{|\mathbf{r} - \mathbf{r}'|^3}, \qquad (1)$$

where ϑ is the angle between vectors $\mathbf{e}_z$ and $\mathbf{r}' - \mathbf{r}$, and $C_{ll'} = \wp_{d_l}\wp_{d_{l'}}/(4\pi\epsilon_0)$ is proportional to the product of atomic dipole moments $\wp_{d_l} = \langle d_l|\mathbf{p}|d_l\rangle$. We assume that state mixing within the same n manifold is suppressed by a proper choice of parabolic q and magnetic m quantum numbers [21].

We use collective atomic transition operators $\hat{\sigma}_{\mu\nu}(\mathbf{r}) = 1/N_\mathbf{r}\sum_{j=1}^{N_\mathbf{r}}|\mu\rangle_{jj}\langle\nu|$ averaged over the volume element ΔV containing $N_\mathbf{r} = \rho(\mathbf{r})\Delta V \gg 1$ atoms around position $\mathbf{r}$. In the frame rotating with the frequencies of the optical fields, the interaction Hamiltonian $H = V_{\rm af} + V_{\rm dd}$ contains the atom-field and DDI terms

$$V_{\rm af} = -\hbar\int d^3r\,\rho(\mathbf{r})\sum_{l=1,2}\big[g_l e^{-r_\perp^2/2w_f^2}\hat{\mathcal{E}}_l(z)\hat{\sigma}_{e_l g}(\mathbf{r})$$
$$+ \Omega_l\hat{\sigma}_{d_l e_l}(\mathbf{r})\big] + {\rm H.c.}, \qquad (2a)$$

$$V_{\rm dd} = \hbar\int d^3r\,\rho(\mathbf{r})\int d^3r'\rho(\mathbf{r}')$$
$$\times \frac{1}{2}\sum_{l,l'=1,2}\hat{\sigma}_{d_l d_l}(\mathbf{r})\Delta_{ll'}(\mathbf{r} - \mathbf{r}')\hat{\sigma}_{d_{l'} d_{l'}}(\mathbf{r}'), \qquad (2b)$$

where $g_l = (\wp_{ge_l}/\hbar)\varepsilon_l$ is the corresponding atom-field coupling constant, with $\wp_{ge_l}$ being the dipole matrix element on the transition $|g\rangle \rightarrow |e_l\rangle$.

Using Hamiltonian H, we derive the Heisenberg-Langevin equations for the atomic operators $\hat{\sigma}_{ge_l}(\mathbf{r})$, $\hat{\sigma}_{gd_l}(\mathbf{r})$ and the propagation equations for the slowly varying quantum fields $\hat{\mathcal{E}}_l(z)$. Solving for the atomic operators perturbatively in the small parameters $g_l\hat{\mathcal{E}}_l/\Omega_l$ and in the adiabatic approximation [4,6,11], and after substituting into the equations for the

fields, we obtain the following propagation equations for the dark-state polaritons $\hat{\Psi}_l = \sqrt{c/v_l}\,\hat{\mathcal{E}}_l$ [4]:

$$\left(\frac{\partial}{\partial t} \pm v_l\frac{\partial}{\partial z}\right)\hat{\Psi}_l(z,t) = -i\sin^2\theta_l\hat{S}_l(z,t)\hat{\Psi}_l(z,t). \qquad (3)$$

The sign "+" or "−" corresponds to $l = 1$ or 2, respectively, $v_l = c\cos^2\theta_l$ is the group velocity of the corresponding field in the EIT medium, and the mixing angles θ_l are defined through $\tan^2\theta_l = (g_l^2N/|\Omega_l|^2)(w/w_a)^2$, with $w = w_a w_f (w_a^2 + w_f^2)^{-1/2}$. Operators $\hat{S}_l(z,t)$ are responsible for the self- and cross-phase modulation between the fields,

$$\hat{S}_l(z,t) = \frac{1}{L}\int_0^L dz'[\Delta_{ll}(z - z')\sin^2\theta_l\hat{\mathcal{I}}_l(z',t)$$
$$+ \Delta_{ll'}(z - z')\sin^2\theta_{l'}\hat{\mathcal{I}}_{l'}(z',t)], \qquad (4)$$

where $\hat{\mathcal{I}}_l \equiv \hat{\Psi}_l^\dagger\hat{\Psi}_l = (c/v_l)\hat{\mathcal{E}}_l^\dagger\hat{\mathcal{E}}_l$ are the polariton intensity (excitation number) operators in the EIT medium, which correspond to the photon number operators outside the medium ($v_l = c$) [4], while the effective one-dimensional DDI potentials $\Delta_{ll'}(z - z')$ result from $\Delta_{ll'}(\mathbf{r} - \mathbf{r}')$ upon double integration over the transverse coordinates,

$$\Delta_{ll'}(z - z') = \frac{1}{(\pi w^2)^2}\int d^2r_\perp\int d^2r'_\perp e^{-(r_\perp^2 + r'^2_\perp)/w^2}\Delta_{ll'}(\mathbf{r} - \mathbf{r}')$$
$$= \frac{2C_{ll'}}{\hbar(\sqrt{2}w)^3}[2|\zeta| - \sqrt{\pi}(1 + 2\zeta^2)e^{\zeta^2}{\rm erfc}(|\zeta|)],$$
$$\zeta \equiv (z - z')/\sqrt{2}w. \qquad (5)$$

As seen in Fig. 1(c), $\Delta_{ll'}(\zeta)$ is sharply peaked around $\zeta = 0$ with the range (full width at half maximum) of $\delta\zeta \simeq 0.65$.

It follows from Eq. (3) that the intensity operators $\hat{\mathcal{I}}_l$ are constants of motion: $\hat{\mathcal{I}}_l(z,t) = \hat{\mathcal{I}}_l(z \mp v_l t, 0)$, the upper (lower) sign corresponding to $l = 1$ ($l = 2$). The solution for the field operators then reads

$$\hat{\Psi}_l(z,t) = \exp\left[-i\sin^2\theta_l\int_0^t dt'\hat{S}_l(z \mp v_l(t - t'),t')\right]$$
$$\times \hat{\Psi}_l(z \mp v_l t, 0). \qquad (6)$$

The validity of this dissipation-free solution hinges on the following assumptions: (i) The duration T_l of each pulse exceeds the inverse of the corresponding EIT bandwidth $\delta\omega_l = |\Omega_l|^2/(\gamma_{ge_l}\sqrt{\kappa_l L})$, where γ_{ge_l} is the transversal relaxation rate and $\kappa_l \simeq \varsigma_l\bar{\rho}$ is the resonant absorption coefficient, with $\varsigma_l = 3\lambda_l^2/(2\pi)$ the absorption cross section on the transition $|g\rangle \rightarrow |e_l\rangle$ and $\bar{\rho} = N/[\pi(w_a^2 + w_f^2)L]$ the effective atomic density. With $v_l = 2|\Omega_l|^2/(\kappa_l\gamma_{ge_l})$, this yields the condition $(\kappa_l L)^{-1/2} \ll T_l v_l/L < 1$ which requires a medium with large optical depth $\kappa_l L \gg 1$ [5,6]. (ii) The DDI-induced frequency shifts lie within the EIT bandwidths, $\sin^2\theta_l\langle\hat{S}_l(z)\rangle < \delta\omega_l$, $\forall z \in [0,L]$. (iii) The propagation/interaction time of each pulse $t_{\rm out} = L/v_l$ is limited by the relaxation rate γ_{gd_l} of the $\hat{\sigma}_{gd_l}$ coherence via $t_{\rm out}\gamma_{gd_l} \ll 1$.

In what follows, we employ Eq. (6) to demonstrate the quantum phase gate between two single-photon pulses $\hat{E}_{1,2}$, and to realize a quantum nondemolition measurement of photon number in the signal pulse $\hat{E}_2$ by a coherent probe pulse $\hat{E}_1$. For simplicity of notation, we set $\theta_{1,2} = \theta$, i.e., $g_1^2N/|\Omega_1|^2 = g_2^2N/|\Omega_2|^2$.

III. PHOTONIC PHASE GATE

We are concerned with the evolution of input state $|\Phi_{\text{in}}\rangle = |1_1\rangle|1_2\rangle$ composed of two single-excitation wave packets $|1_l\rangle = [\frac{1}{L}\int dz f_l(z)\hat{\Psi}_l^\dagger(z)]|0\rangle$ whose spatial envelopes inside the medium $f_l(z) = \langle 0|\hat{\Psi}_l(z,0)|1_l\rangle$ are normalized as $\frac{1}{L}\int dz |f_l(z)|^2 = 1$. With the operator solution (6), for the (equal-time) correlation amplitude or the "two-photon wave function" $F_{12}(z_1,z_2,t) = \langle 0|\hat{\Psi}_1(z_1,t)\hat{\Psi}_2(z_2,t)|\Phi_{\text{in}}\rangle$ [2,7], we obtain

$$F_{12}(z_1,z_2,t) = f_1(z_1 - vt)f_2(z_2 + vt)\exp[i\phi_{12}(z_1,z_2,t)], \tag{7}$$

$$\phi_{12}(z_1,z_2,t) = -\sin^4\theta \int_0^t dt' \Delta_{12}(z_1 - z_2 - 2v(t-t')). \tag{8}$$

Hence, the two polaritons counterpropagate in a shape-preserving manner with group velocities $\pm v$. Since $\hat{\mathcal{I}}_l\hat{\Psi}_l|1_l\rangle = 0$, the self-interaction within each pulse is absent, while the cross interaction between the pulses results in the phase shift (8). Assume that at $t = 0$ the first pulse is centered at $z_1 = 0$ and the second pulse at $z_2 = L$, while after the interaction, $t_{\text{out}} = L/v$, the coordinates of the two pulses are $z_1 = L$ and $z_2 = 0$, respectively. The accumulated phase shift is then $\phi_{12}(L,0,L/v) = -\sin^4\theta/v \int_0^L dz' \Delta_{12}(2z'-L)$. To evaluate the integral, we replace the variable $(2z'-L)/\sqrt{2}w \to \zeta'$ and extend the integration limits to $L/(\sqrt{2}w) \to \infty$, obtaining

$$\phi_{12} = \frac{C_{12}\sin^4\theta}{\hbar w^2 v}, \tag{9}$$

which is *spatially uniform* and the state of the system at t_{out} is $|\Phi_{\text{out}}\rangle = e^{\phi_{12}}|\Phi_{\text{in}}\rangle$. Since for input states $|m_1\rangle|n_2\rangle$ ($m,n = 0,1$) there is no phase shift when $m + n < 2$, the conditional two-photon phase shift $\phi_{12} = \pi$ is equivalent to the CPHASE gate $|\Phi_{\text{out}}\rangle = (-1)^{mn}|m_1\rangle|n_2\rangle$ [1].

IV. QND MEASUREMENT OF PHOTON NUMBER

We next consider the probe pulse in a multimode coherent state $|\alpha_1\rangle \equiv \Pi_k|\alpha_1^k\rangle$, which is an eigenstate of the field operator $\hat{\Psi}_1(z)$ with eigenvalue $\alpha_1(z) = \sum_k \alpha_1^k e^{ikz}$, that is, $\hat{\Psi}_1(z)|\alpha_1\rangle = \alpha_1(z)|\alpha_1\rangle$. The signal pulse can be in any superposition or mixture of the n-photon number states $|n_2\rangle = \frac{1}{\sqrt{n!}}[\frac{1}{L}\int dz f_2(z)\hat{\Psi}_2^\dagger(z)]^n|0\rangle$. Given an input state $|\Phi_{\text{in}}\rangle = |\alpha_1\rangle|n_2\rangle$ and neglecting the self-interaction, for the expectation value of the probe field we have

$$\langle\hat{\Psi}_1(z,t)\rangle = \alpha_1(z - vt)$$
$$\times \langle n_2|\exp\left[-i\frac{\sin^4\theta}{L}\int_0^t dt'\int_0^L dz'\right.$$
$$\left. \times \Delta_{12}(z - z' - v(t-t'))\hat{\mathcal{I}}_2(z' + vt',0)\right]|n_2\rangle. \tag{10}$$

As before, we assume that at $t = 0$ the probe and signal pulses are centered, respectively, at $z = 0$ and $z = L$. The output

probe field at $t_{\text{out}} = L/v$ and $z = L$ is then

$$\langle\hat{\Psi}_1(L,L/v)\rangle$$
$$= \alpha_1(0)\langle n_2|\exp\left[-i\frac{\sin^4\theta}{L}\int_0^{L/v} dt'\int_0^L dz'\right.$$
$$\left. \times \Delta_{12}(z' - vt')\hat{\mathcal{I}}_2(z' + vt',0)\right]|n_2\rangle. \tag{11}$$

Recall that the DDI potential $\Delta_{ll'}(z)$ is sharply peaked around $z = 0$ with the range $\delta z \lesssim w \ll L$ [Fig. 1(c)], while $\int_{-\infty}^\infty dz\Delta_{ll'}(z) = -2C_{ll'}/(\hbar w^2)$. On the other hand, in the EIT medium, $\langle n_l|\hat{\mathcal{I}}_l(z)|n_l\rangle = n|f_l(z)|^2$ are smooth pulses of length $T_l v \lesssim L$. To evaluate the integral in the exponential of Eq. (11), we may therefore replace the DDI potential as $\Delta_{12}(z) \to -2C_{12}/(\hbar w^2)\delta(z)$. We then obtain $\langle\hat{\Psi}_1(L,L/v)\rangle = \alpha_1(0)\exp(i\phi_{12}n_2)$, with ϕ_{12} given by Eq. (9). This indicates that at the output from the medium $[\hat{\Psi}_1(L+0) = \hat{\mathcal{E}}_1(L+0)]$, the coherent probe field has acquired a phase proportional to the number of photons n_2 in the signal field. This phase can be measured by, e.g., a single-port homodyne detection using another coherent field of the same amplitude $|\alpha_1|$. The average detector signal is then $s(n_2) = 4|\alpha_1|^2\sin^2(\phi_{12}n_2/2)$ with the corresponding uncertainty $\delta s(n_2) = \sqrt{2s(n_2)}$. Our aim is to distinguish with high probability the photon number states with $n_2 \in [0,n_{\text{max}}]$. This requires that $\phi_{12}n_{\text{max}} \leqslant \pi$, while the measurement uncertainty constraint yields $s(n_2) - s(n_2 - 1) > \frac{1}{2}[\delta s(n_2) + \delta s(n_2 - 1)]$.

Above we have neglected the self-interaction within the probe pulse, which would otherwise dephase the coherent state. We can estimate its effect as follows: $-\sin^4\theta/L\int_0^{L/v} dt'\int_0^L dz'\Delta_{11}(z'-vt')|\alpha_1(z'-vt')|^2 \simeq 2C_{11}\sin^4\theta/(\hbar w^2 v)|\alpha_1(0)|^2$, which should be small compared to ϕ_{12}. This leads to the condition $2C_{11}|\alpha_1(0)|^2 < C_{12}$. (Note that, as long as we are concerned with determining the photon number in the signal field, its self-interaction is immaterial.) Thus, for the QND measurement of the signal photon number by a coherent probe, the Rydberg states $|d_{1,2}\rangle$ should be chosen such that $\wp_{d_1} < \wp_{d_2}/(2|\alpha_1(0)|^2)$, and therefore the self-interaction of the probe, $C_{11} \propto \wp_{d_1}^2$, is small compared to the cross interaction $C_{12} \propto \wp_{d_1}\wp_{d_2}$. On the other hand, to realize the CPHASE gate between two single-photon pulses, $\phi_{12} = \pi$, both states $|d_{1,2}\rangle$ should have large and comparable dipole moments $\wp_{d_{1,2}}$ so that C_{12} is large.

Of course, in all cases we need to satisfy condition (ii), since otherwise the DDI frequency shifts beyond the EIT transparency window would induce strong self- and/or cross absorption of the fields [20]. We therefore require that

$$\frac{2C_{ll'}\sin^4\theta}{\hbar w^2 L}\max\langle\hat{\mathcal{I}}_{l'}(z)\rangle < \delta\omega_l \quad (l,l' = 1,2). \tag{12}$$

In terms of the phase shift per photon $\phi_{ll'}$, Eq. (9), and assuming smooth n_l-photon pulses of lengths $T_l v \lesssim L$, we then have $2\phi_{ll'}n_{l'} < T_{l'}\delta\omega_l$ for cross interaction and $2\phi_{ll}(n_l - 1) < T_l\delta\omega_l$ for self-interaction. In turn, the product of the pulse duration and EIT bandwidth is restricted by the optical depth as $T_{l'}\delta\omega_l \lesssim \frac{1}{2}\sqrt{\kappa_l L}$. We thus obtain that the maximal cross and self phase shifts are limited by

$$\phi_{ll'}n_{l'}, \phi_{ll}(n_l - 1) < \frac{\sqrt{\kappa_l L}}{4}. \tag{13}$$

Alternatively, the photon number in each pulse is limited by $n_l < \sqrt{\kappa_{l'} L}/(4\phi_{ll'})$, $\sqrt{\kappa_l L}/(4\phi_{ll}) + 1$.

V. EXPERIMENTAL CONSIDERATIONS

To relate the foregoing discussion to a realistic experiment, we assume a hollow-core waveguide of length $L \sim 1$ cm with the lowest transverse mode of width $w_f \simeq 2\ \mu$m [18–20]. The waveguide is filled with $N \simeq 5 \times 10^4$ cold Rb atoms tightly confined by a guided dipole trap to $w_a \simeq 2\ \mu$m, leading to the effective density $\bar{\rho} \simeq 2 \times 10^{11}$ cm^{-3}. For the two quantum fields tuned to the D1 and D2 transitions $|g\rangle \rightarrow |e_{1,2}\rangle$ ($\lambda_1 = 795$ nm, $\lambda_2 = 780$ nm), the corresponding optical depths are $\kappa_1 L \simeq 600$ and $\kappa_2 L \simeq 580$. With $\gamma_{ge_1} \simeq 1.8 \times 10^7$ s^{-1}, $\gamma_{ge_2} \simeq 1.9 \times 10^7$ s^{-1}, and taking $\Omega_1 \simeq 7.35 \times 10^6$ rad/s, $\Omega_2 \simeq 7.43 \times 10^6$ rad/s, the group velocities are $v_{1,2} = 100$ m/s. The bandwidth of the pulses $T_l^{-1} \gtrsim v/L = 10^4$ s^{-1} is smaller than the EIT bandwidth $\delta\omega_l \simeq 1.2 \times 10^5$ rad/s. To realize the CPHASE gate, we choose the Rydberg states $|d_{1,2}\rangle$ with $\wp_{d_1} = \wp_{d_2} = 315ea_0$ (quantum numbers $n = 15$ and $q = n - 1$), leading to the conditional phase shift $\phi_{12} = \pi$. For the QND measurement of photon number $n_2 \leqslant 2$ in the signal field with a weak coherent probe $|\alpha_1|^2 \simeq 4$, the corresponding dipole moments for the Rydberg states are $\wp_{d_1} = 50ea_0$ and $\wp_{d_2} = 450ea_0$, leading to the cross phase shift per photon of $\phi_{12} = 0.7$. We have verified that in both cases the DDI frequency shifts are within the EIT window $\delta\omega_l$ [cf. Eq. (12)].

Hence, the present scheme enables a realization of deterministic quantum gates with photonic qubits and is capable of distinguishing with high probability the photon number states via QND measurement, which can serve as a heralded source of single- or few-photon pulses. In closing, we note that all the necessary ingredients of our proposal, including EIT via Rydberg states [15,16] and in hollow-core waveguides [18–20], have already been demonstrated experimentally.

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