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Light-shift-induced quantum phase transitions of a Bose-Einstein condensate in an optical cavity

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In this paper we reveal the rich ground-state properties induced by the strong nonlinear atom-photon interaction which has been found in the recent experiment about a Bose-Einstein condensate coupling with a high-finesse cavity [Nature (London) **464**, 1301 (2010)]. Two detuning-dependent phase diagrams are revealed by investigating the experimentally measurable atomic population. In particular, two quantum phase transitions from the superradiant phase or the normal phase to a dynamically unstable phase are predicted in the blue detuning. Moreover, the three-phase coexistence points are found. It is also demonstrated that these predicted quantum phase transitions are the intrinsic transitions governed only by the second-order derivative of the ground-state energy. Finally, we also point out that the region involving coexistence of the superradiant phase and the normal phase previously predicted cannot happen in the ground state.

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Quantum phase transition, which describes a structural change in the properties of the ground-state energy spectrum, is not only a fundamental way to explore many-body physics [1], but also a good resource for processing quantum information [2]. In the Dicke model all two-level atoms are coupled with a cavity field [3]. An interesting quantum phase transition from the normal phase to the superradiant phase, which was predicted more than 30 years ago [4,5], occurs. However, in order to capture this quantum phase transition, the collective atom-photon coupling strength needs to be of the same order as the energy separation between the two atomic levels, which was thought to be impossible to satisfy. Recently, by introducing two optical Raman transitions in a four-level atomic ensemble, this challenging transition condition can be accessible by controlling the pump laser power [6]. More importantly, in a Bose-Einstein condensate (BEC) with a high-finesse optical cavity, which has been regarded as a promising platform to explore the exotic many-body phenomena from atomic physics to quantum optics in a well-controlled way [7–24], this superradiant-normal quantum phase transition has been observed successfully in experiment [25].

It is worth noting that in the above BEC-cavity experiment, a generalized Dicke model with a nonlinear atom-photon interaction $S_z a^\dagger a$, where S_z is the collective spin operator and $a^\dagger$ (a) are the creation (annihilation) operator of the photon, is first realized [25]. This nonlinear interaction arises from the dispersive shift of cavity frequency. For a weak nonlinear interaction, the onset of self-organization for the ultracold atoms can be used to detect the superradiant-normal quantum phase transition in the blue detuning of cavity frequency [26]. However, the magnitude of this nonlinear interaction can arrive at the same order as those of the detuning of cavity frequency and the collective coupling strength. Moreover, the rich dynamical properties including the multiphase coexistence regions and the persistent optomechanical oscillations have been predicted theoretically in the strong nonlinear interaction [27].

In the present paper we will reveal the rich ground-state properties induced by the strong nonlinear atom-photon interaction under the current experimental setup of the BEC-cavity system [25]. Two detuning-dependent phase diagrams will be predicted by investigating the experimentally measurable atomic population. In the blue detuning, the superradiant-normal quantum phase transition can be well driven by this nonlinear interaction. However, in the red detuning, two new quantum phase transitions from the superradiant phase or the normal phase to a dynamically unstable phase will be found, and the three-phase (the superradiant, normal, and dynamically unstable phases) coexistence points will be given. Moreover, it will be demonstrated that our preicted quantum phase transitions are not depicted by considering the conventional Landau's theory with the breaking of symmetry, but belong to the intrinsic transitions governed only by the second-order derivative of the ground-state energy. Finally, we will point out that the region involving coexistence of the superradiant phase and the normal phase, which was predicted in Ref. [27], cannot happen in the ground state.

Figure 1 shows the recent experimental setup that a ^{87}Rb BEC with the 10^5 ultracold atoms interacts with a high-finesse optical cavity with a transverse pump laser [25]. In particular, in Fig. 1(b), an effective four-level atomic ensemble, whose levels are respectively labeled by the zero momentum state $|0,0\rangle$, the excited states $|p_x,p_z\rangle = |\pm k,0\rangle$ and $|p_x,p_z\rangle = |0,\pm k\rangle$, and the symmetric superposition of states $|p_x,p_z\rangle = |\pm k,\pm k\rangle$ with p_x and p_z being the momenta in the x and z directions, is constructed by means of two balanced Raman channels from light scattering the pump laser and the cavity mode. In the dispersive limit, the excited atoms re-emit virtual photons either along (x) or transverse (z) to the cavity. As a result, the excited-state levels $|p_x,p_z\rangle = |\pm k,0\rangle$ and $|p_x,p_z\rangle = |0,\pm k\rangle$ can be eliminated adiabatically, and an effective two-level system with the zero momentum state $|p_x,p_z\rangle = |0,0\rangle$ and the symmetric superposition of states $|p_x,p_z\rangle = |\pm k,\pm k\rangle$ is formed [6]. This physical realization has

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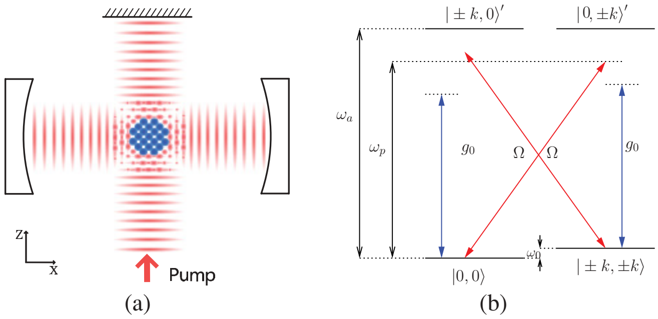

FIG. 1. (Color online) (a) A ^{87}Rb BEC with the ultracold atoms interacts strongly with a high-finesse optical cavity driven by a transverse pump laser. (b) The effective two-level system with the atomic zero momentum state $|p_x,p_z\rangle = |0,0\rangle$ and the symmetric superposition of states $|p_x,p_z\rangle = |\pm k,\pm k\rangle$ is constructed by means of two balanced Raman channels from light scattering the pump laser and the cavity mode [25].

a good advantage in that the smaller energy scale $E_k = k^2/2m$ of the effective two levels is derived. When all ultracold atoms with these different momentum states are coupled identically with the single-mode cavity field, a generalized Dicke model is obtained by [25,26]

$$H = \omega a^\dagger a + \omega_0 S_z + \frac{U}{N} S_z a^\dagger a + \frac{g_0}{\sqrt{N}}(aS_+ + a^\dagger S_-) + \frac{g_1}{\sqrt{N}}(a^\dagger S_+ + aS_-). \tag{1}$$

In Hamiltonian (1), the collective atomic operators are expressed as $S_+ = S_-^\dagger = \sum_i |\pm k,\pm k\rangle_{ii}\langle 0,0|$ and $S_z = \sum_i (|\pm k,\pm k\rangle_{ii}\langle \pm k,\pm k| - |0,0\rangle_{ii}\langle 0,0|)$, where the index i labels the ultracold atom. The nonlinear atom-photon interaction, resulting from the dispersive shift of cavity frequency ω_c, is given by $U = N\Xi U_0 = N\Xi(g')^2/(\omega_p - \omega_a)$, where g' is the coupling strength between the single atom and the photon, ω_a is the atomic transition frequency, ω_p is the pump laser frequency, and Ξ is a constant. The effective cavity frequency becomes $\omega = \omega_c - \omega_p + NU_0(1+\Xi)/2$. The effective atomic frequency is twice the atomic recoil energy $\omega_r = k^2/2m$, namely $\omega_0 = 2\omega_r$. The collective coupling strength is given by $g_0 = g_1 = g'\Omega\sqrt{N}/2(\omega_p - \omega_a)$ with Ω being the pump Rabi frequency, which is tuned in experiment by varying the pump laser power [25].

To understand quantum phase transition, it is necessary to investigate the ground-state properties for a given many-body system [1]. For Hamiltonian (1) with the large atomic number ($N \sim 10^5$), its ground-state properties can be well determined by means of the Holstein-Primakoff transformation, which is defined as $S_+ = b^\dagger\sqrt{N - b^\dagger b}$, $S_- = \sqrt{N - b^\dagger b}\, b$, and $S_z = (b^\dagger b - N/2)$ with $[b, b^\dagger] = 1$ [28]. Under this transformation, Hamiltonian (1) becomes $H = \omega' a^\dagger a + \omega_0(b^\dagger b - \frac{N}{2}) + \frac{U}{N} b^\dagger b a^\dagger a + \frac{g_0}{\sqrt{N}}(ab^\dagger\sqrt{N - b^\dagger b} + a^\dagger\sqrt{N - b^\dagger b}\, b) + \frac{g_1}{\sqrt{N}}(a^\dagger b^\dagger \sqrt{N - b^\dagger b} + a\sqrt{N - b^\dagger b}\, b)$ with $\omega' = \omega_c - \omega_p + NU_0/2$. In order to describe the collective behaviors of both the ultracold atoms and the photon, we should introduce two shifting boson operators $c^\dagger$and $d^\dagger$ with properly scaled auxiliary parameters α and β such that $c^\dagger = a^\dagger + \sqrt{N}\alpha$ and $d^\dagger = b^\dagger - \sqrt{N}\beta$ [28]. By means of the boson expansion method, the scaled energy can be given by

$$\frac{E(\alpha,\beta)}{N} = \omega\alpha^2 + (\omega_0 + U\alpha^2)\left(\beta^2 - \frac{1}{2}\right) - 2\alpha\beta g\sqrt{1-\beta^2}, \tag{2}$$

where

$$g = g_0 + g_1. \tag{3}$$

With the equilibrium condition $\partial E(\alpha)/\partial\alpha = 0$, namely,

$$\alpha = \frac{g\beta\sqrt{1-\beta^2}}{\omega + U(\beta^2 - 1/2)}, \tag{4}$$

the scaled energy in Eq. (2) becomes $E(\beta)/N = \omega_0\beta^2 - \omega_0/2 - g^2\beta^2(1-\beta^2)/[\omega + U(\beta^2 - 1/2)]$. By using another equilibrium condition $\partial E(\beta)/\partial\beta = 0$, an equation governing the fundamental feature of quantum phase transitions for Hamiltonian (1) can be obtained by

$$\beta\left\{\frac{g^2[2\beta^2 - (1-\delta)]}{\omega + U(\beta^2 - 1/2)} + \frac{Ug^2\beta^2(1-\beta^2)}{[\omega + U(\beta^2 - 1/2)]^2}\right\} = 0, \tag{5}$$

where

$$\delta = \frac{\omega_0}{g^2}\left[\omega + U\left(\beta^2 - \frac{1}{2}\right)\right]. \tag{6}$$

For $U = 0$, Hamiltonian (1) reduces to the standard Dicke model $H_D = \Delta_c a^\dagger a + \omega_0 S_z + \frac{g}{\sqrt{N}}(a + a^\dagger)(S_+ + S_-)$ with $\Delta_c = \omega_c - \omega_p$ [3]. The corresponding auxiliary parameters α and β can be derived from $\alpha = g\beta\sqrt{1-\beta^2}/\Delta_c$ and $\beta[2\beta^2 - (1-\delta_D)] = 0$ with $\delta_D = \Delta_c\omega_0/g^2$. For the red detuning ($\Delta_c > 0$), it can be found that $\beta = \pm\sqrt{(1-\delta_D)/2}$ if $\delta_D < 1$ and $\beta = 0$ if $\delta_D > 1$. At the critical point $g_c = \sqrt{\Delta_c\omega_0}$ ($\delta_D = 1$), the well-known superradiant-normal quantum phase transition occurs [28]. Moreover, the nonzero parameters α and β show that the BEC-cavity system is in the superradiant phase with the macroscopic excitations for both the ultracold atoms (β) and the photon (α). However, for the blue detuning ($\Delta_c < 0$), whenever $\delta_D > -1$ or $\delta_D < -1$ resulting from $\beta^2 \leqslant 1, \beta \equiv 0$ since $\partial^2 E(\beta)/\partial\beta^2 < 0$ when $\beta^2 = (1-\delta_D)/2$. It means that only the normal phase ($\beta \equiv 0$) exists in the BEC-cavity system and no superradiant-normal quantum phase transition happens. Thus, in the usual considerations before, the case of the blue detuning has been almost omitted. However, we will demonstrate below that this Dicke-type quantum phase transition can be induced by the light-shift-induced interaction U in this blue detuning regime. More importantly, in the red detuning, two new quantum phase transitions will be predicted.

Having discussed the ground-state properties of Hamiltonian (1) with $U = 0$, we now focus on the case of $U \neq 0$. In terms of Eq. (5), the parameter β has the following solutions:

$$\beta = 0, \quad \beta_\pm^2 = -\frac{Q}{2P} \pm \frac{\sqrt{Q^2 - 4PR}}{2P} \tag{7}$$

with

$$P = U(g^2 + \omega_0 U), \tag{8}$$

$$Q = (g^2 + U\omega_0)(2\omega - U), \tag{9}$$

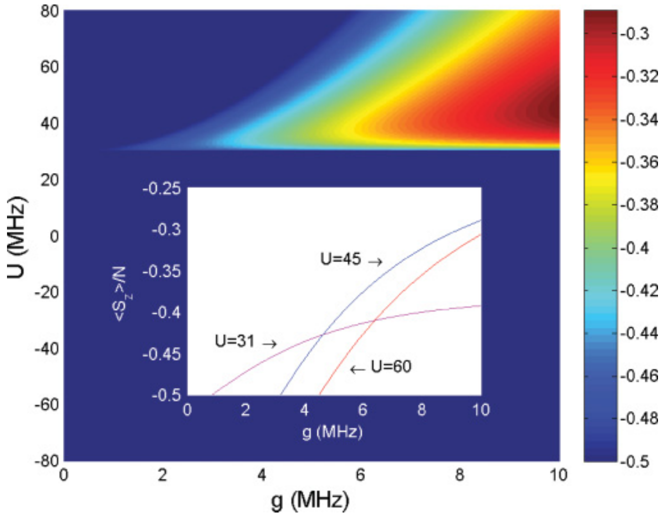

FIG. 2. (Color online) The scaled atomic population $\langle S_z\rangle/N$ as the function of the collective coupling strength g and the light-shift-induced interaction U. The plotted parameters are given by $\Delta_c = -20$ MHz, $\omega_0 = 1$ MHz, and $\Xi = 3/4$ [25]. Inset: $\langle S_z\rangle/N$ as a function of g for different U with the same parameters.

$$R = \left[\omega_0\left(\omega - \frac{U}{2}\right) - g^2\right]\left(\omega - \frac{U}{2}\right). \tag{10}$$

In the following, we will calculate numerically the experimentally measurable ground-state atomic population $\langle S_z\rangle/N = (\beta^2 - 1/2)$, based on Eq. (7) with the physical conditions $Q^2 - 4PR \geqslant 0$, $\beta_\pm^2 \leqslant 1$, and $\partial^2E(\beta)/\partial\beta^2 \geqslant 0$. We first address the case of the blue detuning ($\Delta_c < 0$). Figure 2 shows the scaled atomic population $\langle S_z\rangle/N$ as the function of the collective coupling strength g and the light-shift-induced interaction U. For $U = 0$, the BEC-cavity system is located at the normal phase without collective excitations, which agrees with the previous predictions [28]. With the increasing of both U and g, the quantum phase transition from the superradiant phase ($\langle S_z\rangle/N \neq -1/2$) to the normal phase ($\langle S_z\rangle/N = -1/2$) can be driven successfully by the light-shift-induced interaction U. Moreover, the critical point depends strongly on the magnitude of the light-shift-induced interaction U, as shown in the inset of Fig. 2.

For the red detuning ($\Delta_c > 0$), some surprising results that have not been obtained before are revealed. Figure 3(a) shows the numerical simulation of the scaled atomic population $\langle S_z\rangle/N$ as the function of the collective coupling strength g and the light-shift-induced interaction U. For a small U this BEC-cavity system exhibits the superradiant-normal quantum phase transition [(I)↔(II)], which has been predicted in the previous theory [9] and also demonstrated in the recent experiment [25]. With the decreasing of the negative U, the real parameter β in the normal and superradiant phases becomes complex since $Q^2 - 4PR < 0$ and the BEC-cavity system enters a dynamically unstable regime. Since the nonzero solutions $\beta_\pm^2$ match the $\kappa \to 0$ limit of Eq. (5) in Refs. [28] and [29], the dynamically unstable regime here is the same as that of Ref. [27]. However, in the present paper two regions exhibiting the superradiant-normal quantum phase transitions [(I)↔(III) and (III)↔(V)] occur, as shown in Fig. 3(a). More importantly, two new quantum phase transitions from the normal phase to the dynamically unstable phase [(I)↔(IV) and (V)↔(IV)] and from the superradiant phase to the dynamically unstable phase [(II)↔(IV) and (III)↔(IV)] happen. Moreover, three three-phase (the superradiant, normal, and dynamically unstable phases) coexistence points determined by $\partial^2E(\beta)/\partial\beta^2 = 0$ are found. As will be demonstrated below, the second-order derivative of the ground-state energy $\partial^2E(\beta)/\partial\beta^2$ is negative in this dynamically unstable phase. In Fig. 3(b), the scaled atomic population $\langle S_z\rangle/N$ as the function of the collective coupling strength g and the detuning Δ_c for a given light-shift-induced interaction $U = -20$ MHz is plotted. Figure 3(b) also shows the fundamental properties of our predicted quantum phase transitions.

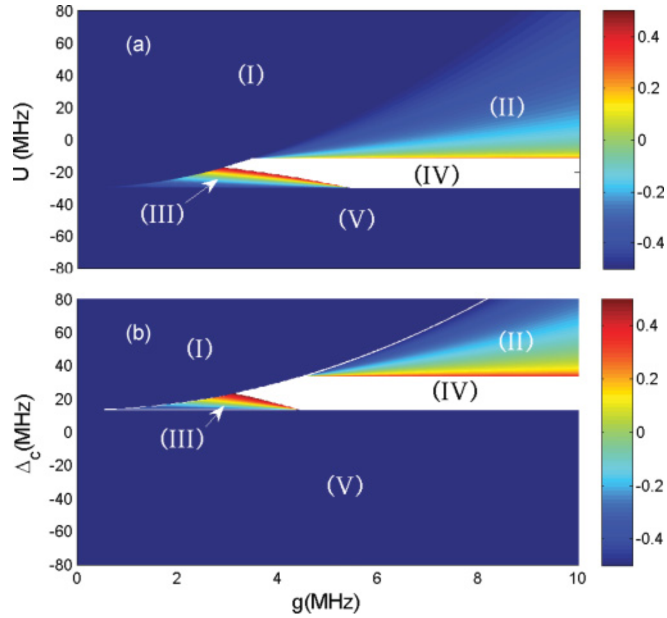

FIG. 3. (Color online) (a) The scaled atomic population $\langle S_z\rangle/N$ as the function of the collective coupling strength g and the light-shift-induced interaction U. The plotted parameters are given by $\Delta_c = 20$ MHz, $\omega_0 = 1$ MHz, and $\Xi = 3/4$ [25]. The superradiant regions (II) and (III) are determined by $\beta = \beta_+$ in Eq. (7). (b) $\langle S_z\rangle/N$ as the function of g and Δ_c with $U = -20$ MHz for $\omega_0 = 1$ MHz and $\Xi = 3/4$ [25].

On the other hand, in terms of Eq. (4), the mean-intracavity photon number $\langle a^\dagger a\rangle/N = \alpha^2$ becomes infinity and the ground-state energy is also infinitely negative for a finite $\beta = \pm\sqrt{1/2 - \omega/U}$. This is another important difference in that the dynamically unstable regime in this paper corresponds to a set of parameters for which Hamiltonian (1) is unbounded from below. In the open system with nonzero κ, this BEC-cavity system cannot acquire an infinite photon density, but can lead to a new superradiant solution given in Eq. (6) of Ref. [27]. Also, in region III, a solution with the infinite mean-intracavity photon number appears again, although the local energy minimum exists.

In the standard Dicke model with the normal phase, the system has the parity symmetry, which is broken in the superradiant phase. It implies that the well-known superradiant-normal quantum phase transition is governed by Landau's theory with

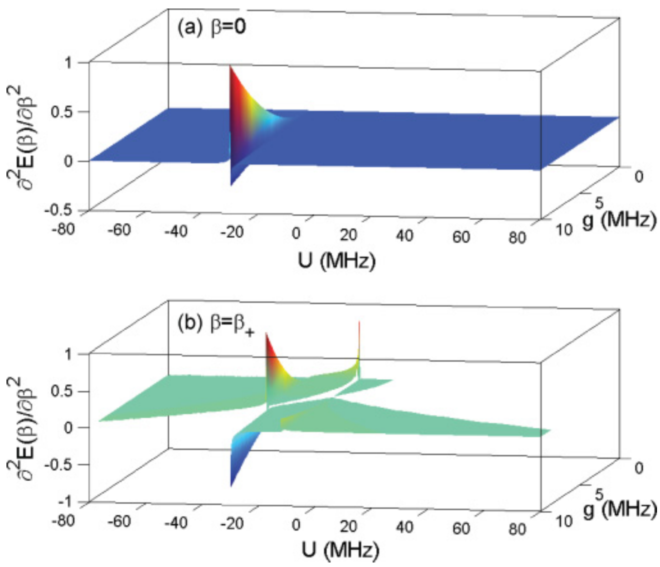

FIG. 4. (Color online) The second-order derivative of the scaled energy $\partial^2 E(\beta)/\partial\beta^2$ as the function of the collective coupling strength g and the light-shift-induced interaction U with (a) $\beta = 0$ and (b) $\beta = \beta_+$. The plotted parameters are given by $\Delta_c = 20$ MHz, $\omega_0 = 1$ MHz, and $\Xi = 3/4$ [25].

the breaking of symmetry. However, from this conventional Landau's theory, it seems that our predicted quantum phase transitions can not occur. What is the corresponding physical mechanism? In Fig. 4, the second-order derivative of the ground-state energy with respect to β as the function of the collective coupling strength g and the light-shift-induced interaction U is plotted. In the normal phase with $\alpha \equiv \beta \equiv 0$, this light-shift-induced nonlinear interaction governed by Hamiltonian $S_z a^\dagger a$ does not affect the ground-state properties since $\langle a^\dagger a\rangle = 0$. In this case, the second-order derivative of the ground-state energy $\partial^2 E(\beta)/\partial\beta^2$ is positive. However, when both the atoms and the photon are excited collectively, this nonlinear interaction plays an important role in the ground-state properties. In particular, this second-order derivative can be driven from the positive $[\partial^2 E(\beta)/\partial\beta^2 \geqslant 0]$ to the negative $[\partial^2 E(\beta)/\partial\beta^2 \leqslant 0]$ without the breaking of symmetry. Thus, we argue that these predicted quantum phase transitions are the intrinsic transitions governed only by the second-order derivative of the ground-state energy.

Finally, we make one remark about the multiphase coexistence behavior, which was predicted in Ref. [27]. In terms of the conditions $\partial E(\beta)/\partial\beta = 0$ and $\partial^2 E(\beta)/\partial\beta^2 \geqslant 0$, region (V) in Fig. 3(a) coexists with the superradiant phase ($\beta = \beta_+$) and the normal phase ($\beta = 0$). In fact, the energy for $\beta = \beta_+$ is higher than that for $\beta = 0$, namely, $E(\beta = \beta_+) > E(\beta = 0)$. For the ground state, only the case $\beta = 0$ is valid and the BEC-cavity system is in the metastable state for $\beta = \beta_+$. It implies that the region involving coexistence of the superradiant phase and the normal phase cannot happen in the ground state.

In summary, we have found two new quantum phase transitions from the superradiant phase or the normal phase to the dynamically unstable phase in the BEC-cavity system. Moreover, we have also pointed out that the multiphase coexistence regions predicted before cannot happen in the ground state. Based on the experimental developments, we believe that our predictions can be detected by measuring the atomic population or mean-intracavity photon number.

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