Entanglement detection for bipartite systems with continuous variables in non-Markovian baths

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By using the dynamics described with the quantum Langevin equation and the inseparability criterion for continuous-variable systems [L.-M. Duan, G. Giedke, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **84**, 2722 (2000).], we discuss a method to judge whether entanglement exists in the evolutions of bipartite systems with continuous variables in their baths. By using this method we investigate a nontrivial example, namely, we judge when the entanglement exists in the evolution of the two coupled anharmonic oscillators in their environments.

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I. INTRODUCTION

Quantum entanglement plays an essential role in all branches of quantum information theory, and it may be also important in other fields in which quantum physics should be used. For bipartite systems, measurements of entanglement have been introduced in many forms. The von Neumann entropy [1], negativity [2], etc. have been used to measure the degree of entanglement for pure states, while concurrence [3] has been applied to describe entanglement for not only pure states but also mixed states. These measures of entanglement have also been extended to describe the pure states of multipartite qubit systems in recent years [4,5].

Entanglement evolutions of bipartite systems [6], even multipartite systems with continuous variables [7,8], have recently been investigated in the Markovian approximation. The entanglement evolutions of discrete lower level quantum systems embedded in baths have also been studied in both Markovian [9,10] and non-Markovian approximations [11–13]. However, the entanglement evolution of quantum systems with continuous variables in non-Markovian approximation is still a challenge to theorists. The evolution process of entanglement is indeed a quantum dynamic process and it can be described by a system-plus-reservoir model [14]. There are several methods to deal with this model. For example, the Feynman path integral, stochastic dynamics, master equation of Redfield form, and quantum Langevin equation [15], etc. The former three schemes have been widely used to investigate the entanglement dynamics of discrete lower level quantum systems. In this paper we develop a model in which it is convenient to use a c-number quantum Langevin equation (QLE) [16–21] with the inseparability criterion of DGCZ (discovered by L. -M. Duan, G. Giedke, J. I. Cirac, and P. Zoller in Ref. [22]) for continuous variable systems to investigate whether there is entanglement for coupled bipartite quantum systems with continuous variables in their environments.

In the following, we first investigate the dynamics of two coupled anharmonic oscillators in their baths by using the QLE. Then, based on the dynamics, we use the DGCZ criterion to judge whether the coupled anharmonic systems in their environments are entangled in the process of evolution. It is shown that in the short evolution times the coupled anharmonic systems in baths are entangled. Although the method cannot be used to quantitatively measure the degree of entanglement, it may be a convenient tool for determining whether the systems with continuous variables are entangled.

II. MODEL AND DYNAMICS

As an example, we consider two coupled anharmonic oscillators being embedded in their respective baths. The Hamiltonian can be read as

$$H = H_1 + H_2 + H_{12}, \tag{1}$$

where

$$\begin{aligned} H_1 &= \frac{p_1^2}{2} + V_1(x_1) + \sum_{\alpha=1}^{N} \frac{p_\alpha^2}{2} + \frac{1}{2}\sum_{\alpha=1}^{N} k_\alpha (q_\alpha - x_1)^2, \\ H_2 &= \frac{p_2^2}{2} + V_2(x_2) + \sum_{\beta=1}^{N} \frac{p_\beta^2}{2} + \frac{1}{2}\sum_{\beta=1}^{N} k_\beta (q_\beta - x_2)^2, \end{aligned} \tag{2}$$

and

$$H_{12} = K x_1 x_2. \tag{3}$$

Here, $x_{1,2}$ and $p_{1,2}$ are the coordinate and momentum operators of the systems 1 and 2, and $\{q_\alpha, p_\alpha\}$, $\{q_\beta, p_\beta\}$ is the set of coordinate and momentum operators for the reservoir oscillators bilinearly coupled to their systems. K is the coupling coefficient between systems 1 and 2. The potentials $V_{1,2}(x)$ are due to the external force field for the Brownian particles. The coordinate and momentum operators follow the usual commutation relations $[x,p] = i\hbar$ and $[q_k, p_l] = i\hbar\delta_{kl}$. In our model the two anharmonic oscillators are respectively coupling to their baths. From Eq. (1), by eliminating the reservoir degrees of freedom of baths, we obtain the quantum Langevin equation for the two particles [23],

$$\begin{aligned} \ddot{x}_{1,2}(t) + V'(x_{1,2}) + \int_0^t dt' \gamma_{1,2}(t-t')\dot{x}_{1,2}(t') \\ = F_{1,2}(t) - K x_{2,1}, \end{aligned} \tag{4}$$

where the noise operator $F_{1,2}(t)$ and the memory kernel $\gamma_{1,2}(t)$ are given by

$$F_i(t) = \sum_j \left[\{q_j(t) - x_i(0)\} k_j \cos\omega_j t + p_j(0) k_j^{1/2} \sin\omega_j t \right] \tag{5}$$

and

$$\gamma_1(t) = \sum_\alpha k_\alpha \cos\omega_\alpha t, \qquad \gamma_2(t) = \sum_\beta k_\beta \cos\omega_\beta t, \tag{6}$$

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with $k_{a,\beta} = \omega_{\alpha,\beta}^2$. For convenience, in this paper we set $i = 1$ and 2, corresponding to $j = \alpha$ and β, unless specifically stated otherwise. As the bath oscillators are canonically distributed with respect to the bath Hamiltonian at $t = 0$, we have

$$\begin{aligned}\langle F_i(t)\rangle_{QS} &= 0,\\ \frac{1}{2}\{\langle F_i(t)F_i(t')\rangle_{QS} &+ \langle F_i(t')F_i(t)\rangle_{QS}\}\\ &= \frac{1}{2}\sum_j k_j\hbar\omega_j\left(\coth\frac{\hbar\omega_j}{2k_BT}\right)\cos\omega_j(t-t'). \end{aligned}\tag{7}$$

Here, $\langle\cdots\rangle_{QS}$ refers to quantum statistical average of bath degrees of freedom defined as

$$\langle O\rangle_{QS} = \frac{\mathrm{Tr}\,O\exp(-H_{\mathrm{bath}}/k_BT)}{\mathrm{Tr}\exp(-H_{\mathrm{bath}}/k_BT)}.\tag{8}$$

Here,

$$\begin{aligned} H_{\mathrm{bath}} &= H_{\mathrm{bath}}^1 + H_{\mathrm{bath}}^2,\\ H_{\mathrm{bath}}^1 &= \sum_{\alpha=1}^N\frac{p_\alpha^2}{2} + \frac{1}{2}\sum_{\alpha=1}^N k_\alpha(q_\alpha - x_1)^2,\\ H_{\mathrm{bath}}^2 &= \sum_{\beta=1}^N\frac{p_\beta^2}{2} + \frac{1}{2}\sum_{\beta=1}^N k_\beta(q_\beta - x_2)^2. \end{aligned}$$

To construct two c-number Langevin equations, we first carry out the quantum-mechanical average on Eq. (4) as

$$\begin{aligned}\langle\ddot{x}_{1,2}(t)\rangle &+ \int_0^t dt'\gamma_{1,2}(t-t')\langle\dot{x}_{1,2}(t')\rangle + \langle V'(x_{1,2})\rangle\\ &= \langle F_{1,2}(t)\rangle - K\langle x_{2,1}\rangle,\end{aligned}\tag{9}$$

Let $f_i(t) = \langle F_i(t)\rangle$. We then have

$$\begin{aligned} f_i(t) = \sum_j\big[&\{\langle q_j(t)\rangle - \langle x_i(0)\rangle\}k_j\cos\omega_j t\\ &+\langle p_j(0)\rangle k_j^{1/2}\sin\omega_j t\big].\end{aligned}\tag{10}$$

Thus, we can obtain the c-number equations from Eq. (9) as

$$\begin{aligned}\langle\ddot{x}_{1,2}(t)\rangle &+ \int_0^t dt'\gamma_{1,2}(t-t')\langle\dot{x}_{1,2}(t')\rangle + \langle V'(x_{1,2})\rangle\\ &= f_{1,2}(t) - K\langle x_{2,1}\rangle,\end{aligned}\tag{11}$$

with

$$\begin{aligned}\langle f_i(t)\rangle_s &= 0,\\ \langle f_i(t)f_i(t')\rangle_s &= \frac{1}{2}\sum_j k_j\hbar\omega_j\coth\left(\frac{\hbar\omega_j}{2k_BT}\right)\cos\omega_j(t-t').\end{aligned}\tag{12}$$

Referring to the quantum operators of the system in the Heisenberg picture, one may write

$$x_i = X_i + \delta x_i,\quad p_i = P_i + \delta p_i.\tag{13}$$

Here, $\langle x_i\rangle = X_i$ and $\langle p_i\rangle = P_i$ are the quantum-mechanical averages, and δx_i, δp_i are the correction operators. $\langle\delta x_i\rangle$ and $\langle\delta p_i\rangle$ are zero, and $[\delta x_m,\delta p_n] = i\hbar\delta_{mn}$. Then we can write Eq. (11) in the form

$$\begin{aligned}\dot{X}_{1,2} &= P_{1,2},\\ \dot{P}_{1,2} &= -\int_0^t dt'\gamma_{1,2}(t-t')P_{1,2}(t') - V'_{1,2}(X_{1,2})\\ &\quad + f_{1,2}(t) + Q_{1,2}(t) - KX_{2,1},\end{aligned}\tag{14}$$

with

$$Q_i(t) = V_i'(\langle x_i\rangle) - \langle V_i'(x_i)\rangle.\tag{15}$$

Here, $V_i'(x) = dV_i'(x)/dx$. Using Eq. (13) in $\langle V_i'(x)\rangle$ and a Taylor series expansion around $\langle x_i\rangle$, we can express $Q_i(t)$ as

$$Q_i(t) = -\sum_{n\leqslant 2}\frac{1}{n!}V_i^{(n+1)}(X_i)\big\langle\delta x_i^n(t)\big\rangle.\tag{16}$$

Here, $V_i^{(n)}(x) = dV_i^n(x)/dx^n$. Equations (14) are the equations of motion regarding quantum-mechanical average quantities X_i and P_i. Note that they are c-number equations governed by c-number quantum noise $f_i(t)$ and a quantum fluctuation term $Q_i(t)$.

In the following, we first investigate the quantum correction from $Q_i(t)$. We return to the operator equation, Eq. (4), and insert Eq. (13) to obtain

$$\begin{aligned}\delta\dot{x}_{1,2} &= \delta p_{1,2},\\ \delta\dot{p}_{1,2} &+ \int_0^t\gamma_{1,2}(t-t')\delta p_{1,2}(t')dt' + V''_{1,2}(X_{1,2})\delta x_{1,2}\\ &+ \sum_{n\leqslant 2}\frac{1}{n!}V_{1,2}^{(n+1)}(X_{1,2})\delta x_{1,2}^n = F_{1,2}(t) - f_{1,2}(t) - K\delta x_{1,2}.\end{aligned}\tag{17}$$

Note that $\langle\delta x_i^n(t)\rangle$ in $Q_i(t)$ does not include any statistical averaging and is a dispersion term obtained by pure quantum-mechanical averaging, which should not be confused with $\langle\delta x_i(t)\rangle_s$, where the subscript s means the statistical average. Thus, to obtain $\langle\delta x_i(t)\rangle$ we need only to perform a quantum-mechanical averaging over the initial product separable bath states $\Pi_{i=1}^\infty\{|a_i(0)|\}$ to get rid of the term $F_i(t) - f_i(t)$. So, it is easy to obtain the first- and second-order correction equations from $Q_i(t)$. Second, we discuss how to deal with the quantum noise $f_i(t)$. In the continuous limit, the correlation function $c_i(t-t')$ [see Eq. (12)] becomes

$$\begin{aligned}c_i(t-t') &= \langle f_i(t)f_i(t')\rangle_s\\ &= \frac{1}{2}\int_0^\infty d\omega\kappa_i(\omega)\rho_i(\omega)\hbar\omega\coth\left(\frac{\hbar\omega}{2k_BT}\right)\\ &\quad\times\cos\omega(t-t').\end{aligned}\tag{18}$$

On the other hand, we can generate a set of exponentially correlated color noise variables η_m according to

$$\dot{\eta}_m = -\frac{\eta_m}{\tau_m} + \frac{1}{\tau_m}\xi_m(t),\tag{19}$$

$$\begin{aligned}\langle\xi_m(t)\rangle &= 0,\\ \langle\xi_m(0)\xi_n(0)\rangle &= 2D_m\delta_{mn}\delta(\tau),\ (m,n = 1,2,3,\ldots).\end{aligned}\tag{20}$$

The quantum noise η_m is similar to the Ornstein-Uhlenbeck (OU) [24] process with properties

$$\langle \eta_m(t)\rangle = 0, \quad \langle \eta_m(t)\eta_n(t')\rangle = \delta_{mn}\frac{D_m}{\tau_m}\exp\left(\frac{-|t-t'|}{\tau_m}\right). \tag{21}$$

Thus the correlation function can be fitted with a superposition of several exponentials, namely,

$$c_i(t-t') \approx \sum_{m=1,2\ldots} \frac{D_m}{\tau_m}\exp\left(\frac{-|t-t'|}{\tau_m}\right). \tag{22}$$

The c-number quantum noise $f_i(t)$ due to the heat bath is therefore given by

$$f_i(t) = \sum_{m=1,2\ldots} \eta_m, \quad \langle f_i(t)f_i(t')\rangle_s = \sum_{m=1,2\ldots}\langle \eta_m(t)\eta_m(t')\rangle. \tag{23}$$

Thus, the dynamics of the reduced coupled anharmonic oscillator system can be described with

$$\begin{aligned} \dot{X}_1 &= P_1,\\ \dot{P}_1 &= -V_1'(X_1) + Q_1(t) + \sum_{m=1,2,3\ldots}\eta_m(t) + Z_1 - KX_2, \\ \dot{Z}_1 &= -\Gamma_1\frac{P_1}{\tau_{c1}} - \frac{Z_1}{\tau_{c1}}, \quad \dot{\eta}_m = -\frac{\eta_m}{\tau_1} + \frac{1}{\tau_1}\xi_1(t), \end{aligned} \tag{24}$$

$$\begin{aligned} \dot{X}_2 &= P_2,\\ \dot{P}_2 &= -V_2'(X_2) + Q_2(t) + \sum_{n=1,2,3\ldots}\eta_n(t) + Z_2 - KX_1, \\ \dot{Z}_2 &= -\Gamma_2\frac{P_2}{\tau_{c2}} - \frac{Z_2}{\tau_{c2}}, \quad \dot{\eta}_n = -\frac{\eta_n}{\tau_2} + \frac{1}{\tau_2}\xi_2(t), \end{aligned} \tag{25}$$

where $Z_{1,2} = -\int_0^t dt'\gamma_{1,2}(t-t')P_{1,2}(t')$. The values of $Q_1(t)$ and $Q_2(t)$ can be obtained from Eqs. (16) and (17), and up to second order the correction terms are derived from the following equations:

$$\begin{aligned} \delta\dot{x}_1 &= \delta p_1,\\ \delta\dot{p}_1 &= -V_1''(X_1)\delta x_1 - \frac{1}{2}V_1^{(3)}(X_1)\delta x_1^2 + \delta Z_1 - K\delta x_2, \\ \delta\dot{Z}_1 &= -\frac{\Gamma_1}{\tau_{c1}}\delta p_1 - \frac{1}{\tau_{c1}}\delta Z_1, \end{aligned} \tag{26}$$

$$\begin{aligned} \delta\dot{x}_2 &= \delta p_2,\\ \delta\dot{p}_2 &= -V_2''(X_2)\delta x_2 - \frac{1}{2}V_2^{(3)}(X_2)\delta x_2^2 + \delta Z_2 - K\delta x_1, \\ \delta\dot{Z}_2 &= -\frac{\Gamma_2}{\tau_{c2}}\delta p_2 - \frac{1}{\tau_{c2}}\delta Z_2, \end{aligned} \tag{27}$$

$$\begin{aligned} \langle\delta\dot{x}_1^2\rangle &= \langle\delta x_1\delta p_1 + \delta p_1\delta x_1\rangle,\\ \langle\delta x_1\delta p_1 \dot{+} \delta p_1\delta x_1\rangle &= 2\langle\delta p_1^2\rangle - 2V_1''(X_1)\langle\delta x_1^2\rangle, \\ \langle\delta\dot{p}_1^2\rangle &= -V_1''(X_1)\langle\delta x_1\delta p_1 + \delta p_1\delta x_1\rangle - K\langle\delta p_1\rangle\langle\delta x_2\rangle, \end{aligned} \tag{28}$$

$$\begin{aligned} \langle\delta\dot{x}_2^2\rangle &= \langle\delta x_2\delta p_2 + \delta p_2\delta x_2\rangle,\\ \langle\delta x_2\delta p_2 \dot{+} \delta p_2\delta x_2\rangle &= 2\langle\delta p_2^2\rangle - 2V_2''(X_2)\langle\delta x_2^2\rangle, \\ \langle\delta\dot{p}_2^2\rangle &= -V_2''(X_2)\langle\delta x_2\delta p_2 + \delta p_2\delta x_2\rangle - K\langle\delta p_2\rangle\langle\delta x_1\rangle. \end{aligned} \tag{29}$$

Here, $\delta Z_{1,2} = -\int_0^t \gamma_{1,2}(t-t')\delta p_{1,2}(t')dt'$. Thus, we can investigate the dynamics of the open coupled anharmonic oscillators by solving the 20 differential equations above.

III. APPLICATIONS AND DISCUSSION

In the last section we developed the dynamic equations of the reduced coupled oscillators. The dynamics can be described with the differential equations of quantum-mechanical quantitative averages [Eqs. (24) and (25)] and their quantum corrections [Eqs. (26) and (29)] with the stochastic driving. In this section we investigate a fixed model by using the above theoretical results. Supposing the bath modes in which systems 1 and 2 are embedded are Lorentzian distribution, we then have

$$\kappa_i(\omega)\rho_i(\omega) = \frac{2}{\pi}\left(\frac{\Gamma_i}{1+\omega^2\tau_{ci}^2}\right). \tag{30}$$

Here, $\Gamma_{1,2}$ is the dissipation constant and $\tau_{c1,2}$ refers to the correlation time of the two baths. In the rest of this paper, we set $k_BT = 0.025$, $V_1(X) = V_2(X) = aX^4 - bX^2$, and $\Gamma_1 = \Gamma_2 = 1$, $\tau_{c1} = \tau_{c2} = 1$. It has been shown that in our case the correlation function can be modeled with a single exponential function with $D_1 = 0.025$ and $\tau_1 = \tau_2 = 1$ (see Fig. 1). The time scale is derived from the forms of Eq. (1).

In order to see the dynamics clearly, we first plot the evolutions of $X_1(t)$ and $X_2(t)$ in Fig. 2. Here we set the parameters $a = 0.005$, $b = 0.1$, $K = 0.6$, and the initial conditions are $X_1(0) = -1$, $X_2(0) = 1$, $P_1(0) = 0$, $P_2(0) = 0$, $\delta x_1 = 0$, $\delta x_2 = 0$, $\delta p_1 = 0$, $\delta p_2 = 0$, $\langle\delta x_1^2\rangle = 0.5$, $\langle\delta x_1\delta p_1 + \delta p_1\delta x_1\rangle = 1$, $\langle\delta p_1^2\rangle = 0.5$, $\langle\delta x_2^2\rangle = 0.5$, $\langle\delta x_2\delta p_2 + \delta p_2\delta x_2\rangle = 1$, and $\langle\delta p_2^2\rangle = 0.5$. It is shown in Fig. 2 that $X_1(t)$ and $X_2(t)$ are oscillating with time increase and finally, to their respective equilibrium positions. In the following, by using the dynamic results with the DGCZ criterion, we investigate whether the entanglement existed in the process of evolution for the open coupled systems. Generally, for the type of EPR-like operators [25]

$$u = |\alpha|\,x_1 + \frac{1}{\alpha}x_2, \quad v = |\alpha|\,p_1 - \frac{1}{\alpha}p_2, \tag{31}$$

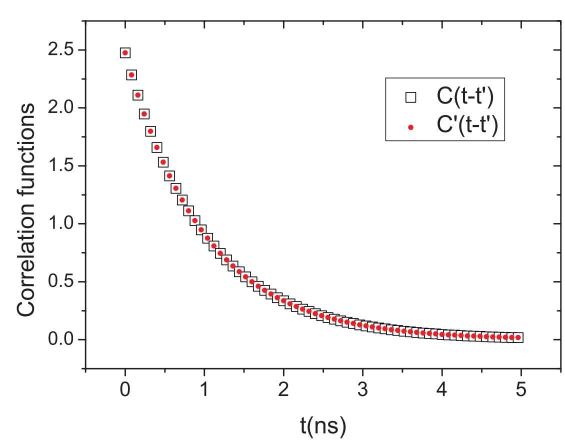

FIG. 1. (Color online) Correlation functions obtained from Eqs. (20) and (21), where $\Gamma_{1,2} = 1$, $\tau_{c1,2} = 1$, $D_{1,2} = 0.025$, $\tau_{1,2} = 1$, and $k_BT = 0.025$.

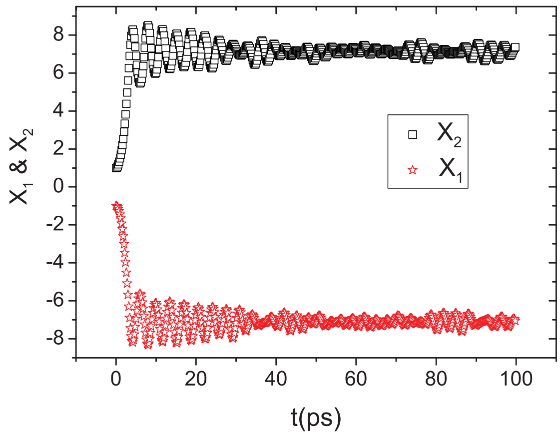

FIG. 2. (Color online) Evolutions of $x_1(t)$ and $x_2(t)$ vs time t in the potential $V_{1,2}(X) = aX_{1,2}^4 + bX_{1,2}^2$, where the coefficient of potential $a = 0.005$, $b = 0.1$, and the initial conditions are given in the text.

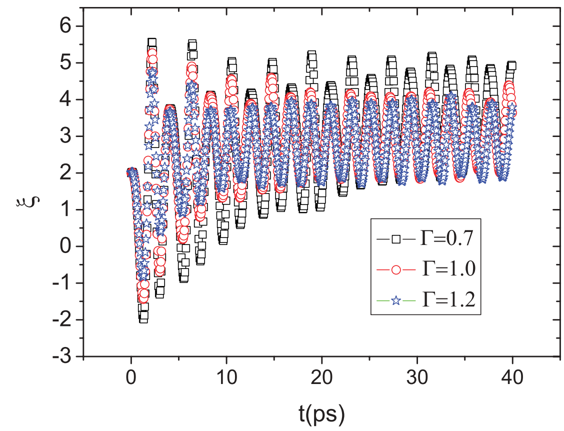

FIG. 3. (Color online) Evolutions of ξ vs time t, where the values of the parameters are similar to Fig. 2, except for Γ.

where α is an arbitrary (nonzero) real number, it has been proven that for any separable quantum state ρ, the total variance of a pair of EPR-like operators defined by Eq. (31) with commutators $[x_k, p_{k'}] = i\delta_{kk'}$ (where i is the imaginary unit, and k, $k' = 1,2$) satisfies the inequality

$$\xi = \langle(\Delta u)^2\rangle_\rho + \langle(\Delta v)^2\rangle_\rho \leqslant 2, \tag{32}$$

as $\alpha = 1$. Namely, if $\xi - 2 = [(\Delta u)^2 + (\Delta v)^2] - 2 < 0$, the quantum state is entangled [26]. Here, $u = x_1 + x_2$ and $v = p_1 - p_2$ are the two Einstin-Podolsky-Rosen–type operators, while Δu and Δv are the corresponding quantum fluctuations. We plot the evolution of ξ with time in Fig. 3, where the values of the parameters are the same as Fig. 2, except for Γ. It is shown that within a short time the two subsystems are entangled. As the time increases to some value, the parameter ξ increases and its minimum value becomes larger than 2, indicating that the two subsystems are entangled in the initial process. But we could not get enough information about the exact degree of entanglement according to the inequality (32). It can also be shown that as the noise $\Gamma_{1,2}$ increases, the entanglement time will decrease and vice versa.

Note that the DGCZ criterion provides a sufficient condition for inseparability of any two-party continuous variable states. From the criterion we obtain only that if the quantity $\xi < 2$ we can judge the systems entangled, but we cannot say anything regarding $\xi > 2$. So for non-Gaussian states, from the quantity ξ we can judge only if the entanglement *exists* in a coupling system. However, as proven in Ref. [22] for all Gaussian states, this criterion turns out to be a necessary and sufficient condition for inseparability. At this time we note that if the quantity $\xi < 2$, the systems are entangled, and if the quantity $\xi > 2$ the systems are separable. It is clear that in our investigated model, the coupled anharmonic oscillators are in their respective baths; thus the state cannot evolve to Gaussian state even from the initial pure Gaussian state. Furthermore, even for the Gaussian states, from the quantity ξ we can only judge if the entanglement *exists or not* in a coupling system. The quantity ξ cannot measure the degrees of entanglement, because as a measure of entanglement, the quantity ξ must satisfy four conditions as given in Ref. [27].

IV. CONCLUSIONS

In this paper we introduce a method to judge whether entanglement exists by using the QLE and the DGCZ criterion. As an example, we investigated the model of the two coupled anharmonic oscillators in their respective baths. This method does not depend on the Markovian approximation and does not need any measure of entanglement. It shows that the quantum Langevin equation with the DGCZ criterion may construct an excellent scheme for estimating whether the system with continuous variables is entangled; however, the exact evolution of the degree of entanglement based on this method needs to be developed.

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