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Quantum electromagnetic waves in nonstationary linear media

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We present a quantum description of electromagnetic waves propagating through time-dependent homogeneous nondispersive conducting and nonconducting linear media without charge sources. Based on the Coulomb gauge and the quantum invariant method, we find the exact wave functions for this problem. In addition, we construct coherent and squeezed states for the quantized electromagnetic waves and evaluate the quantum fluctuations in coordinate and momentum space as well as the uncertainty product for each mode of the electromagnetic field.

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I. INTRODUCTION

The old and fundamental problem of the behavior (classical and quantum) of electromagnetic waves propagating through material media has always attracted the attention of physicists. The story of the solution of this problem is a familiar one. Yet, the solution of this problem has been very important for the development of our understanding of nature.

The quantization of electromagnetic waves is traditionally performed in empty cavities or in free space by the association of a quantum-mechanical harmonic oscillator with each mode of the electromagnetic field [1,2]. The quantum behavior of electromagnetic waves is well understood in the case of electromagnetic fields in empty cavities or in free space only. In the case of electromagnetic waves interacting with external currents and other sources the quantum behavior of these waves is not so clear.

In the past few years, the problem of electromagnetic waves propagating through dispersive and nondispersive material media has become an important subject and much activity has been taking place in this field [3–20]. The great interest in this subject is motivated partly by the advent of modern optical materials such as optical fibers and photonic crystal and partly by the growth of experiments on quantum optics processes taking place inside material [6,8,9]. Here it is worth remarking that in dispersive media the inclusion of losses into the system can be performed either by means of a reservoir having a large number of degrees of freedom which leads to an energy flow from the medium to the reservoir [8,9,15] or through a phenomenological approach in which time-dependent parameters are introduced *ab initio* [16,19,20].

In this paper, we present a quantum description for the problem of electromagnetic waves propagating through homogeneous nondispersive linear media, in the absence of sources, with time-dependent electric permittivity and conductivity. We show that this problem can be analyzed in terms of a damped quantum-mechanical time-dependent harmonic oscillator. Furthermore, on the basis of the Coulomb gauge, quadratic invariants, and the quantum invariant method we solve the time-dependent Schrödinger equation for this problem. In addition, we construct coherent and squeezed states for the quantized electromagnetic waves and evaluate the quantum fluctuations in coordinate and momentum space as well as the uncertainty product for each mode of the electromagnetic field.

We organize this paper as follows. In Sec. II we obtain the classical Hamiltonian for the electromagnetic waves propagating through time-dependent conducting and nonconducting linear media without charge density from Maxwell's equations. In Sec. III, we use quadratic invariants together with the dynamical invariant method to solve the Schrödinger equation for the problem. In Sec. IV, we construct coherent and squeezed states for the quantized electromagnetic waves and evaluate the quantum fluctuation in coordinate and momentum space as well as the uncertainty product for each mode of the electromagnetic field. In Sec. V, we conclude the paper with a short summary.

II. CLASSICAL ELECTROMAGNETIC WAVES IN TIME-VARYING MEDIA

The electromagnetic field dynamics in time-dependent homogeneous conducting linear media, in the absence of charge sources, is governed by the phenomenological Maxwell's equations. Furthermore, the relations between the fields and currents are given as $\vec{D} = \varepsilon(t)\vec{E}$, $\vec{B} = \mu_0\vec{H}$, and $\vec{J} = \sigma(t)\vec{E}$. Here, $\varepsilon(t)$ and $\sigma(t)$ are heuristically introduced as the time-dependent electric permittivity and conductivity, respectively, while μ_0 is the magnetic permeability. In general, the electric permittivity and the magnetic permeability are complex. However, we will restrict our discussion to materials where they are real. This is the case [21,22], for instance, of poor conductors and other materials for frequency below the resonant frequency. Since Maxwell's equations are gauge invariant, we are free to choose the most appropriate gauge for our problem. For convenience we choose to work in the Coulomb gauge [1,2]. In this gauge the vector potential is purely transverse which is convenient for our case since we are dealing with electromagnetic waves. Furthermore, in the Coulomb gauge the divergence of the vector potential $\vec{A}$ is zero and the scalar potential is null in the absence of sources. Consequently, both the electric $\vec{E}$ and magnetic $\vec{B}$ fields are determined from the vector potential *via* the familiar relations

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad \text{and} \quad \vec{E} = -\frac{\partial \vec{A}}{\partial t}. \tag{1}$$

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Therefore, using Maxwell's equations one can verify that the vector potential satisfies the equation [23]

$$\nabla^2\vec{A} - \mu_0(\dot{\varepsilon}+\sigma)\frac{\partial\vec{A}}{\partial t} - \mu_0\varepsilon\frac{\partial^2\vec{A}}{\partial t^2} = 0, \tag{2}$$

where the dot represents a time derivative. Here we notice that the time dependence of the electric permittivity gives rise to the unusual term $\dot{\varepsilon}$ in Eq. (2). This time dependence which, in principle, may be associated with the internal response of the localized charges to an external perturbation, causes an additional attenuation of the electromagnetic field (for $\dot{\varepsilon} > 0$). Furthermore, for nondispersive nonconducting dielectric media, $\sigma(t) = 0$, the media become absorbing because of the time dependence, just as if it were in contact with a reservoir (time-dependent background medium) or if it were a conductor [23].

Let us now consider the solutions of Eq. (2). To treat this equation we consider electromagnetic waves in a certain volume of space. Then, we write the vector potential in terms of the mode $\vec{u}_l(\vec{r})$ and amplitude $q_l(t)$ functions of each cavity mode as [1,2,16,20]

$$\vec{A}(\vec{r},t) = \sum_l \vec{u}_l(\vec{r})q_l(t). \tag{3}$$

The substitution of Eq. (3) into the damped wave Eq. (2) leads to

$$\nabla^2\vec{u}_l(\vec{r}) + \frac{\omega_l^2}{c_0^2}\vec{u}_l(\vec{r}) = 0, \tag{4}$$

$$\frac{\partial^2 q_l}{\partial t^2} + \frac{\dot{\varepsilon}+\sigma}{\varepsilon}\frac{\partial q_l}{\partial t} + \Omega_l^2(t)q_l = 0, \tag{5}$$

where ω_l is the natural frequency of the mode l, $c_0 = 1/\sqrt{\mu_0\varepsilon(0)}$ is the velocity of light inside the medium at $t = 0$, and $\Omega_l(t)$ is a modified frequency defined as $\Omega_l = c(t)\omega_l/c_0$, with $c(t) = 1/\sqrt{\mu_0\varepsilon(t)}$ being the velocity of the electromagnetic wave in the time-dependent medium. Now, it is easy to verify that the equations of motion for the amplitudes $q_l(t)$ given by Eq. (5) can be directly obtained from the classical Hamiltonian

$$H_l(t) = e^{-\wedge(t)}\frac{p_l^2}{2\varepsilon_0} + \frac{1}{2}e^{\wedge(t)}\varepsilon_0\Omega_l^2(t)q_l^2, \tag{6}$$

where q_l and p_l are canonical conjugated variables, with $\wedge(t)$ given by

$$\wedge(t) = \int_0^t \frac{\dot{\varepsilon}(\tau)+\sigma(\tau)}{\varepsilon(\tau)}d\tau. \tag{7}$$

Hence, the total Hamiltonian of the electromagnetic field is a sum of individual Hamiltonians corresponding to each mode, that is, $\sum_l H_l$.

Next we move our attention to the solutions of Eq. (4). Considering the electromagnetic field to be contained in a certain cubic volume V of nonrefracting media, the mode functions are required to satisfy the transversality condition $\vec{\nabla}\cdot\vec{u}_l(\vec{r}) = 0$ and to form a complete orthonormal set [1,2]. Furthermore, assuming periodic boundary conditions on the surface, the mode function may be written in terms of plane waves as [1,2,16,20]

$$\vec{u}_{l\nu}(\vec{r}) = L^{-3/2}e^{\pm i\vec{k}_l\cdot\vec{r}}\hat{e}_{l\nu}, \tag{8}$$

where $L = V^{1/3}$ is the size of the cube, $|\vec{k}_l| = \omega_l/c_0$ is the wave vector, and $\hat{e}_{l\nu}$ are unit vectors in the directions of polarization ($\nu = 1,2$), which must be perpendicular to the wave vector because of the transversality condition. With the spatial mode functions $\vec{u}_l$ completely determined, we only need the canonical variable $q_l(t)$ in order to obtain the vector potential and, consequently, a complete classical description of the electromagnetic field. Then, the electric field confined in the cubic volume of side L can be written as

$$\vec{E}(\vec{r},t) = \frac{e^{-\wedge(t)}}{\varepsilon_0 L^{3/2}}\sum_l\sum_\nu \hat{e}_{l\nu}e^{\pm i\vec{k}_l\cdot\vec{r}}p_l(t). \tag{9}$$

Here we observe that we have quoted some of the results of this section in a recent paper [23].

III. QUANTUM ELECTROMAGNETIC WAVES IN TIME-DEPENDENT CONDUCTING MEDIA

In order to obtain a quantum description of electromagnetic waves propagating in a conducting linear media with time-dependent electric permittivity and conductivity we need to quantize the electromagnetic field. Now as the spatial mode functions $\vec{u}_l(\vec{r})$ are completely determined, the amplitude of each normal mode in Eq. (3) needed to specify a particular field configuration is $q_l(t)$ [1]. Furthermore, for each canonical operator q_l the electromagnetic field is completely specified since $\vec{E}$ and $\vec{B}$ field operators may be derived by inserting the operator $\vec{A}$ given by Eq. (3) into Eq. (1). So, we move our attention to the canonical operator $q_l(t)$ in order to obtain the vector potential. For this purpose, we must solve the Schrödinger equation associated with the Hamiltonian (6)

$$H_l(t)\Psi[q_l,t] = i\hbar\frac{\partial}{\partial t}\Psi[q_l,t], \tag{10}$$

where p_l is now the moment operator $p_l = -i\hbar\partial/\partial q$ with $[q_l,p_l] = i\hbar$. The solutions of Eq. (10) can be obtained with the aid of the dynamical invariant method devised by Lewis and Riesenfeld [24,25]. Following this method, we look for a nontrivial Hermitian operator $I_l(t)$ which satisfies the equation

$$\frac{dI_l}{dt} = \frac{1}{i\hbar}[I_l,H_l] + \frac{\partial I_l}{\partial t} = 0. \tag{11}$$

If the exact invariant $I_l(t)$ (constant of motion) does not contain any time-dependent operator, the Schrödinger equation solutions are straightforwardly written in terms of the orthonormalized eigenfunctions $\phi_n(q_l,t)$ of $I_l(t)$,

$$I_l(t)\phi_n(q_l,t) = \lambda_n\phi_n(q_l,t), \tag{12}$$

and the phase functions $\beta_n(t)$ as

$$\psi_n(q_l,t) = e^{i\beta_n(t)}\phi_n(q_l,t), \tag{13}$$

Here, the λ_n are time-independent eigenvalues and the phase functions $\beta_n(t)$ are to be determined by the equation

$$\hbar\frac{d\beta_n(t)}{dt} = \langle\phi_n|i\hbar\frac{\partial}{\partial t} - H_l|\phi_n\rangle, \tag{14}$$

with the orthonormality condition $\langle\phi_{n'}|\phi_n\rangle = \delta_{n'n}$.

Linear invariant operators satisfying Eq. (11) are innumerable [26,27]. These operators allow one to derive the wave function rather directly since they are readily diagonalized.

However, the wave function obtained is given in terms of plane waves which varies depending on the choice of an arbitrary weight function [26–28]. So, it is hard to investigate classical and quantum correspondence on basis of this wave function [28]. Furthermore, it is not an easy task to construct coherent states based on these linear invariants. For these reasons, in this paper we are interested in dealing with quadratic Hermitian invariants. In this case the derivation of the wave function, compared to that obtained through linear invariants, is lengthier. However, as will be seen later, coherent states based on quadratic invariants are easily constructed. Consequently, one can more easily analyze the classical and quantum correspondence. Now, it is known that a quadratic invariant satisfying (11) is given by [29,30]

$$I_l(t) = \frac{1}{2}\left[\left(\frac{q_l}{\rho_l}\right)^2 + (\rho_l p_l - \varepsilon_0 e^{\wedge(t)}\dot{\rho}_l q_l)^2\right], \tag{15}$$

where $\rho_l(t)$ is a time-dependent real function satisfying the Milne-Pinney equation [29,31,32]

$$\ddot{\rho}_l(t) + \frac{\dot{\varepsilon}+\sigma}{\varepsilon}\dot{\rho}_l(t) + \Omega_l^2(t)\rho_l = \frac{e^{-2\wedge(t)}}{\varepsilon_0^2\rho_l^3}. \tag{16}$$

Next, we look for the eigenstates of $\phi_n(q,t)$ of $I_l(t)$. For this purpose, we consider the unitary transformation [29,30]

$$\phi_n'(q_l,t) = U\phi_n(q_l,t) \tag{17}$$

with

$$U = \exp\left(-\frac{i\varepsilon_0 e^{\wedge(t)}\dot{\rho}_l}{2\hbar\rho_l}q_l^2\right). \tag{18}$$

Making use of this transformation, we can rewrite the eigenvalue equation (12) as

$$I_l'\phi_n'(q_l,t) = \lambda_n\phi_n'(q_l,t), \tag{19}$$

where

$$I_l' = UI_lU^\dagger = -\frac{\hbar^2}{2}\rho^2\frac{\partial}{\partial q_l^2} + \frac{1}{2}\frac{q_l^2}{\rho_l^2}. \tag{20}$$

If we now define a new variable $z_l = q_l/\rho_l$, we can express Eq. (19) as

$$\left(-\frac{\hbar^2}{2}\frac{\partial^2}{\partial z_l^2} + \frac{q_l^2}{2}\right)\varphi_n(z_l) = \lambda_n\varphi_n(z_l), \tag{21}$$

where φ_n is related to ϕ_n' by

$$\varphi_n(z_l) = \varphi_n(q_l/\rho_l) = \rho_l^{1/2}\phi_n'(q_l,t). \tag{22}$$

The factor $\rho_l^{1/2}$ has been introduced to satisfy the normalization condition. Therefore, the solutions $\varphi_n(z_l)$ of Eq. (21) are the eigenfunctions

$$\varphi_n(z_l) = \left(\frac{1}{\pi^{1/2}\hbar^{1/2}n!2^n}\right)^{1/2}\exp\left(-\frac{z_l^2}{2\hbar}\right)H_n\left[\left(\frac{1}{\hbar}\right)^{1/2}z_l\right], \tag{23}$$

with the respective eigenvalues

$$\lambda_n = \hbar\left(n+\frac{1}{2}\right). \tag{24}$$

Here H_n is the Hermite polynomial of order n. So, making use of Eqs. (17), (18), (22), and (23) we find that

$$\phi_n(q_l,t) = \left(\frac{1}{\pi^{1/2}\hbar^{1/2}n!2^n\rho_l}\right)^{1/2} \times \exp\left[\frac{i\varepsilon_0 e^{\wedge(t)}}{2\hbar}\left(\frac{\dot{\rho}_l}{\rho_l} + \frac{ie^{-\wedge(t)}}{\varepsilon_0\rho_l^2}\right)q_l^2\right] \times H_n\left[\left(\frac{1}{\hbar}\right)^{1/2}\frac{q_l}{\rho_l}\right]. \tag{25}$$

The next step is to find the phase function given by Eq. (14). After some basic calculations, we get that

$$\beta_n(t) = -\left(n+\frac{1}{2}\right)\int_0^t \frac{e^{-\wedge(\tau)}}{\varepsilon_0\rho_l^2(\tau)}d\tau. \tag{26}$$

Therefore, we can write the solutions of the Schrödinger equation (10) as

$$\psi_n(q_l,t) = \exp[i\beta_n(t)]\left(\frac{1}{\pi^{1/2}\hbar^{1/2}n!2^n\rho_l}\right)^{1/2} \times \exp\left[\frac{i\varepsilon_0 e^{\wedge(t)}}{2\hbar}\left(\frac{\dot{\rho}_l}{\rho_l} + \frac{ie^{-\wedge(t)}}{\varepsilon_0\rho_l^2}\right)q_l^2\right] \times H_n\left[\left(\frac{1}{\hbar}\right)^{1/2}\frac{q_l}{\rho_l}\right], \tag{27}$$

with the phase function $\beta_n(t)$ given by Eq. (26). Equation (27) represents the exact wave functions for each mode of the electromagnetic field. Furthermore, the evolution of a general Schrödinger state can be written as $\Psi(q_l,t) = \sum_n c_n\psi_n(q_l,t)$, where the c_n are time-independent coefficients. Finally, we observe that for ϵ and σ constants, the results of this section coincide with those of Ref. [30].

IV. COHERENT AND SQUEEZED STATES FOR THE QUANTIZED ELECTROMAGNETIC WAVES

In this section, we construct coherent and squeezed states for the quantized electromagnetic waves propagating in time-dependent conducting media. As will be seen later, these states are indeed equivalent to the squeezed states of the quantized electromagnetic field. In doing so, we introduce the annihilation and creation operators defined as

$$b_l' = \left(\frac{1}{2\hbar}\right)^{1/2}\left[\frac{q_l}{\rho_l} + i\rho_l p_l\right], \tag{28}$$

$$b_l'^\dagger = \left(\frac{1}{2\hbar}\right)^{1/2}\left[\frac{q_l}{\rho_l} - i\rho_l p_l\right], \tag{29}$$

with $[b_l', b_l'^\dagger] = 1$. In terms of these operators, the invariant I_l' given by Eq. (20) can be rewritten as

$$I_l' = \hbar\left(b_l'^\dagger b_l' + \frac{1}{2}\right), \tag{30}$$

whose coherent states have the form [29,33,34]

$$\varphi_\alpha(z_l,t) = \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_n\frac{\alpha^n}{(n!)^{1/2}}\exp\left[i\beta_n(t)\right]\varphi_n(z_l), \tag{31}$$

where α is an arbitrary complex number. Then, using Eqs. (17), (18), (22), and (31) we find that the coherent states for the system described by the Hamiltonian (6) are given by

$$\phi_\alpha(q_l,t) = \frac{1}{\rho_l^{1/2}} \exp\left(\frac{i\varepsilon_0 e^{\wedge(t)}\dot{\rho}_l^2}{2\hbar\rho_l} q_l^2\right) \varphi_\alpha(z_l,t). \tag{32}$$

These states satisfy the eigenvalue equation

$$b_l\phi_\alpha(q_l,t) = \alpha_l(t)\phi_\alpha(q_l,t), \tag{33}$$

with b_l and b_l' related by

$$b_l = U^\dagger b_l' U = \left(\frac{1}{2\hbar}\right)^{1/2}\left[\frac{q_l}{\rho_l} + i(\rho_l p_l - \varepsilon_0 e^{\wedge(t)}\dot{\rho}_l q_l)\right] \tag{34}$$

and

$$\alpha_l(t) = \alpha_l \exp[2i\beta_0(t)], \tag{35}$$

$$\beta_0(t) = -\frac{1}{2}\int_0^t \frac{e^{-\wedge(t)}}{\varepsilon_0\rho_l^2(\tau)} d\tau. \tag{36}$$

Note that, in terms of the operator b_l, the invariant in Eq. (15) can be expressed as $I_l = \hbar(b_l^\dagger b_l + \frac{1}{2})$. Now, after a straightforward calculation we find that the expectation values of q_l and p_l in the coherent state $\phi_\alpha(q_l,t)$ are given by

$$\langle q_l\rangle = (2\hbar|\alpha|^2\rho_l^2)^{1/2}\sin[-2\beta_0(t) + \delta_l], \tag{37}$$

$$\langle p_l\rangle = (2\hbar|\alpha|^2)^{1/2} \times\left\{\varepsilon_0 e^{\wedge(t)}\dot{\rho}_l \sin[-2\beta_0(t) + \delta_l] - \frac{1}{\rho_l}\cos[-2\beta_0(t) + \delta_l]\right\}, \tag{38}$$

where δ_l is the argument of the complex number α. Here we note that expression (37) is a solution of Eq. (5). Hence, the center of the coherent state wave packet follows the motion of a classical particle. Thus, the above result agrees with the original idea of Schrödinger about the coherent states, who was interested in finding quantum mechanical states which followed the motion of a classical particle in a given potential [35]. Also, we can use the result (37) to show that when the electric field (9) is in a state $\phi_\alpha(q_l,t)$, its expectation value looks like a classical field, as it should be. In what follows we evaluate the quantum fluctuations in q_l and p_l in the state $\phi_\alpha(q_l,t)$. After some algebra we find that

$$\langle\Delta q\rangle^2 = \langle q_l^2\rangle - \langle q_l\rangle^2 = \frac{\hbar}{2}\rho_l^2 \tag{39}$$

$$\langle\Delta p\rangle^2 = \langle p_l^2\rangle - \langle p_l\rangle^2 = \frac{\hbar}{2}\left[\frac{1}{\rho_l^2} + (\varepsilon_0 e^{\wedge(t)}\dot{\rho}_l)^2\right]. \tag{40}$$

Thus, the uncertainty product is expressed as

$$(\Delta q_l)(\Delta p_l) = \frac{\hbar}{2}[1 + (\varepsilon e^{\wedge(t)}\rho_l\dot{\rho}_l)^2]^{1/2}. \tag{41}$$

Here, we observe that the uncertainty relation (41), in general, does not attain its minimum value. This occurs, as we have already mentioned, because the states $\phi_\alpha(q_l,t)$ correspond to the squeezed states. This will be seen more clearly below. It is also worth mentioning that if the medium is a dielectric material $[\sigma(t) = 0]$ with constant permittivity $[\varepsilon(t) = \varepsilon_0 = cte]$ and if we take the particular solution $\rho_l = (1/\varepsilon_0\omega_l)^{1/2}$ of the Milne-Pinney equation (16), the uncertainty product attains its minimum value. This occurs because, in this case, both the Hamiltonian (6) and the coherent states $\phi_\alpha(q_l,t)$ reduce to the ordinary harmonic oscillator model. In the following, we show that the state $\phi_\alpha(q_l,t)$ correspond to the squeezed states. For this purpose, let us introduce the annihilation and creation operators a_l and $a_l^\dagger$ of the standard oscillator model given by

$$a_l = \left(\frac{1}{2\hbar\varepsilon_0\omega_l}\right)^{1/2}[\varepsilon_0\omega_l q_l + ip_l], \tag{42}$$

$$a_l^\dagger = \left(\frac{1}{2\hbar\varepsilon_0\omega_l}\right)^{1/2}[\varepsilon_0\omega_l q_l - ip_l]. \tag{43}$$

These operators are related to operators b_l and $b_l^\dagger$, which were defined previously, by the Bogoliubov transformations [36–39]

$$b_l = u(t)a_l + v(t)a_l^\dagger, \tag{44}$$

$$b_l^\dagger = u^*(t)a_l^\dagger + v^*(t)a_l, \tag{45}$$

whose coefficients are expressed as

$$u(t) = \left(\frac{1}{4\varepsilon_0\omega_l}\right)^{1/2}\left(\frac{1}{\rho_l} - i\varepsilon_0 e^{\wedge(t)}\dot{\rho}_l + \varepsilon_0\omega_l\rho_l\right), \tag{46}$$

$$v(t) = \left(\frac{1}{4\varepsilon_0\omega_l}\right)^{1/2}\left(\frac{1}{\rho_l} - i\varepsilon_0 e^{\wedge(t)}\dot{\rho}_l - \varepsilon_0\omega_l\rho_l\right). \tag{47}$$

A straightforward calculation shows that the Bogoliubov coefficients $u(t)$ and $v(t)$ satisfy the relation

$$|u(t)|^2 - |v(t)|^2 = 1. \tag{48}$$

Therefore, from Eqs. (33), (44), and (48), we see that the states $\phi_\alpha(q_l,t)$ are, by definition, equal to well-known squeezed states [30,36,37,39–41]. Furthermore, in terms of the coefficients $u(t)$ and $v(t)$ the quantum fluctuations in $q_l(t)$ and $p_l(t)$ for the squeezed states $\phi_\alpha(q_l,t)$ can be written as

$$(\Delta q_l)^2 = \frac{\hbar}{2\varepsilon_0\omega_l}|u - v|^2, \tag{49}$$

$$(\Delta p_l)^2 = \frac{\hbar\varepsilon_0\omega_l}{2}|u + v|^2, \tag{50}$$

where

$$(\Delta q_l)(\Delta p_l) = \frac{\hbar}{2}|u - v||u + v|. \tag{51}$$

The uncertainty product is minimized if $u = \gamma v$ for γ real [37]. Furthermore, note that the relation (51) is equivalent to Eq. (41), as it should be.

V. SUMMARY

In this work, we have presented a simple and direct quantum description for the problem of electromagnetic waves propagating in time-dependent conducting and nonconducting media without charge sources. We have seen that this problem can be analyzed by associating a damped quantum-mechanical oscillator with each mode of the electromagnetic field. Furthermore, with the aid of the quantum invariant method and a quadratic invariant we have derived exact waves functions for this problem. In addition, we have constructed coherent states for the quantized electromagnetic waves and have calculated the quantum fluctuations in coordinate and momentum space as well as the uncertainty product for each mode of the electromagnetic field. We have also shown that the expectation value of the coordinate q_l follows the motion of a classical particle. What is more, we have seen that this uncertainty product, in general, does not attain its minimum value. Yet, by employing a direct procedure we have shown that this latter result occurs because the coherent states constructed previously correspond to squeezed states. Finally, we expect that the approach developed in this work can be useful to investigate subjects related to the quantization of electromagnetic waves propagating in conducting and nonconducting media with material properties varying in time.

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