Multipartite entanglement of fermionic systems in noninertial frames

Jieci Wang[1,2] and Jiliang Jing[1,*]

[1]*Department of Physics, and Key Laboratory of Low Dimensional Quantum Structures and Quantum Control of Ministry of Education, Hunan Normal University, Changsha, Hunan 410081, China*

[2]*Department of Physics and Center of Physics, University of Minho, Campus of Gualtar, Braga PT-4710-057, Portugal*

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The bipartite and tripartite entanglement of a 3-qubit fermionic system when one or two subsystems accelerate are investigated. It is shown that all the one-tangles decrease as the acceleration increases. However, unlike the scalar case, here one-tangles $\mathcal{N}_{C_I(AB_I)}$ and $\mathcal{N}_{C_I(AB)}$ never reduce to zero for any acceleration. It is found that the system has only tripartite entanglement when either one or two subsystems accelerate, which means that the acceleration does not generate bipartite entanglement and does not affect the entanglement structure of the quantum states in this system. It is of interest to note that the π-tangle of the two-observer-accelerated case decreases much quicker than that of the one-observer-accelerated case and it reduces to a nonzero minimum in the infinite-acceleration limit. Thus we argue that the qutrit systems are better than qubit systems in performing quantum information processing tasks in noninertial systems.

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I. INTRODUCTION

Quantum entanglement is both the central concept and the most desirable resource for a variety of quantum information processing tasks [1–3], such as quantum teleportation, superdense coding, entanglement-based quantum cryptography, error-correcting codes, and quantum computation. In the last decade, although much effort has been made to study the properties of entanglement, our understanding of entanglement is limited to bipartite systems. There is no doubt that multipartite entanglement is a valuable physical resource in large-scale quantum information processing and plays an important role in condensed-matter physics. But, in fact, although the entanglement of multipartite systems can be similarly investigated as a bipartite case, the properties and quantification of entanglement for higher dimensional systems and multipartite quantum systems are issues that must still be resolved.

On the other hand, as a combination of general relativity, quantum field theory, and quantum information theory, relativistic quantum information has been a focus of research in quantum information science in recent years for both conceptual and experimental reasons. Recently, much attention has been given to the study of entanglement shared between inertial and noninertial observers by discussing how the Unruh or Hawking effect will influence the degree of entanglement [4–19]. However, it is worth noting that most investigations of noninertial systems focused on the study of quantum information in bipartite systems when only one of the subsystems accelerated. Fortunately, the tripartite entanglement of the scalar field between noninertial frames was recently studied by Hwang *et al.* [20]. They showed that the tripartite entanglement decreases with increasing acceleration and is different from bipartite entanglement when one observer moves with an infinite acceleration.

In this paper we will discuss both the bipartite and tripartite entanglement of Dirac fields in the noninertial frame when one or two observers are accelerated. We are interested in how the accelerations of these observers will influence the degree of bipartite and tripartite entanglement, and whether the differences between Fermi-Dirac and Bose-Einstein statistics will play a role in the decreasing entanglement. Our setting consists of three observers: Alice, Bob, and Charlie. We first assume Alice is in an inertial frame and Bob and Charlie are observing the system from accelerated frames; we then let Alice and Bob stay stationary while Charlie moves with uniform acceleration. We consider the Dirac fields as shown in Refs. [21–23], which, from an inertial perspective, describe a superposition of the Minkowski monochromatic modes $|0\rangle_M = \bigotimes_i |0_{\omega_i}\rangle_M$ and $|1\rangle_M = \bigotimes_i |1_{\omega_i}\rangle_M \forall i$, where

$$|0_{\omega_i}\rangle_M = \cos r_i \big|0_{\omega_i}\big\rangle_I \big|0_{\omega_i}\big\rangle_{II} + \sin r_i \big|1_{\omega_i}\big\rangle_I \big|1_{\omega_i}\big\rangle_{II}, \tag{1}$$

and

$$\big|1_{\omega_i}\big\rangle_M = \big|1_{\omega_i}\big\rangle_I \big|0_{\omega_i}\big\rangle_{II}. \tag{2}$$

Here $\cos r_i = (e^{-2\pi\omega_i c/a_i} + 1)^{-1/2}$, where a_i is the acceleration of the accelerated observer. The Minkowski $|0\rangle_M$, which is annihilated by operator a_M, is also annihilated by operate a_{M_i} (associated with the vacuum $|0_{\omega_i}\rangle_M$) and also by any combination of Minkowski annihilation operators. It is also worth noting that a Minkowski mode that defines the Minkowski vacuum is related to a highly nonmonochromatic Rindler mode rather than a single mode with the same frequency (see [22,23] for details). Consider that an accelerated observer in the Rindler region I has no access to the field modes in the causally disconnected region II. By tracing over the inaccessible modes we will obtain a tripartite state and we then calculate the tripartite entanglement of the three-qubit state as well as the bipartite entanglement of all possible bipartite divisions of the tripartite system.

The outline of this paper is as follows. In Sec. II we recall some measurements of entanglement in quantum information theory, in particular the negativity and π-tangle. In Sec. III the bipartite and tripartite entanglement of Dirac fields when one or two of the observers are accelerated will be discussed. The conclusions are presented in the last section.

*Corresponding author: jljing@hunnu.edu.cn

II. MEASURES OF TRIPARTITE ENTANGLEMENT

It is well known that there are two remarkable entanglement measures for a bipartite system $\rho_{\alpha\beta}$, the concurrence [24] and the negativity [25]. The former is defined as

$$C_{\alpha\beta} = \max\{0,\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad \lambda_i \geqslant \lambda_{i+1} \geqslant 0, \quad (3)$$

where λ_i are the square roots of the eigenvalues of the matrix $\rho_{\alpha\beta}\tilde{\rho}_{\alpha\beta}$ with $\tilde{\rho}_{\alpha\beta} = (\sigma_y \otimes \sigma_y)\,\rho^*_{\alpha\beta}\,(\sigma_y \otimes \sigma_y)$ describing the "spin-flip" matrix and σ_y the Pauli matrix. The latter is defined as

$$\mathcal{N}_{\alpha\beta} = \left\|\rho_{\alpha\beta}^{T_\alpha}\right\| - 1, \quad (4)$$

where T_α denotes the partial transpose of $\rho_{\alpha\beta}$ and $\|.\|$ is the trace norm of a matrix. Correspondingly, there are two entanglement measures that quantify the genuine tripartite entanglement: three-tangle [24] and π-tangle [26]. The three-tangle (or residual tangle), which has many nice properties but is a highly difficult problem to compute analytically except in a few rare cases, is defined as

$$\tau_{\alpha,\beta,\gamma} = \tau_{\alpha(\beta,\gamma)} - \tau_{\alpha,\beta} - \tau_{\alpha,\gamma}, \quad (5)$$

where $\tau_{\alpha(\beta,\gamma)} = C^2_{\alpha(\beta,\gamma)}$ and $\tau_{\alpha,\beta} = C^2_{\alpha,\beta}$.

To simplify the calculation we merely adopt the π-tangle as the quantification of the tripartite entanglement. For any three-qubit state $|\Phi\rangle_{\alpha\beta\gamma}$, the entanglement quantified by the negativity between α and β, between α and γ, and between α and the overall subsystem $\beta\gamma$ satisfies the following Coffman-Kundu-Wootters (CKW) monogamy inequality [24]

$$\mathcal{N}^2_{\alpha\beta} + \mathcal{N}^2_{\alpha\gamma} \leqslant \mathcal{N}^2_{\alpha(\beta\gamma)}, \quad (6)$$

where $\mathcal{N}_{\alpha\beta}$ is a "two-tangle," which is the negativity of the mixed state $\rho_{\alpha\beta} = \mathrm{Tr}_\gamma(|\Phi\rangle_{\alpha\beta\gamma}\langle\Phi|)$ and $\mathcal{N}^2_{\alpha(\beta\gamma)}$ is a "one-tangle," defined as $\mathcal{N}_{\alpha(\beta\gamma)} = \|\rho^{T_\alpha}_{\alpha\beta\gamma}\| - 1$. The difference between the two sides of Eq. (6) can be interpreted as the residual entanglement

$$\pi_\alpha = \mathcal{N}^2_{\alpha(\beta\gamma)} - \mathcal{N}^2_{\alpha\beta} - \mathcal{N}^2_{\alpha\gamma}. \quad (7)$$

Likewise, we have

$$\pi_\beta = \mathcal{N}^2_{\beta(\alpha\gamma)} - \mathcal{N}^2_{\beta\alpha} - \mathcal{N}^2_{\beta\gamma} \quad (8)$$

and

$$\pi_\gamma = \mathcal{N}^2_{\gamma(\alpha\beta)} - \mathcal{N}^2_{\gamma\alpha} - \mathcal{N}^2_{\gamma\beta}. \quad (9)$$

The π-tangle $\pi_{\alpha\beta\gamma}$ is defined as the average of π_α, π_β, and π_γ, i.e.,

$$\pi_{\alpha\beta\gamma} = \tfrac{1}{3}(\pi_\alpha + \pi_\beta + \pi_\gamma). \quad (10)$$

III. BEHAVIORS OF TRIPARTITE ENTANGLEMENT WHEN ONE OR TWO OBSERVERS ARE ACCELERATED

We consider a tripartite system that consists of three subsystems: Alice is the observer of the first part of the system, and Bob and Charlie are the observers of the second and third parts, respectively. They share a state where all modes are in the vacuum state except that three of them are entangled from the inertial perspective, for example, a Greenberger-Horne-Zeilinger (GHZ) state

$$|\Phi\rangle_{ABC} = \frac{1}{\sqrt{2}}(|0_{\omega_a}\rangle_A|0_{\omega_b}\rangle_B|0_{\omega_c}\rangle_C + |1_{\omega_a}\rangle_A|1_{\omega_b}\rangle_B|1_{\omega_c}\rangle_C), \quad (11)$$

where $|0_{\omega_{a(b,c)}}\rangle_{A(B,C)}$ and $|1_{\omega_{a(b,c)}}\rangle_{A(B,C)}$ are vacuum states and the first excited states from the perspective of an inertial observer. Alice, Bob, and Charlie each carry a monochromatic detector sensitive to frequencies ω_a, ω_b, and ω_c, respectively. Using Eqs. (4) and (10), we can easily get

$$\mathcal{N}_{A(BC)} = \mathcal{N}_{B(AC)} = \mathcal{N}_{C(AB)} = 1,$$
$$\mathcal{N}_{AB} = \mathcal{N}_{BC} = \mathcal{N}_{CA} = 0,$$
$$\pi_{ABC} = 1,$$

where $\mathcal{N}_{A(BC)}$, $\mathcal{N}_{AB}$, and π_{ABC} are the "one-tangle," "two-tangle," and "π-tangle" of state (11) from a inertial viewpoint. Then we let Alice stay stationary while Bob and Charlie move with uniform acceleration. Since Bob and Charlie are accelerated, we should map the second and third partition of this state into the Rindler-Fock space basis. Using Eqs. (1) and (2) we can rewrite Eq. (11) in terms of Minkowski modes for Alice and Rindler modes for Bob and Charlie,

$$\begin{aligned}|\Phi\rangle_{AB_IC_I} = \frac{1}{\sqrt{2}}[&\cos r_b \cos r_c|0\rangle_A|0\rangle_{B_I}|0\rangle_{B_{II}}|0\rangle_{C_I}|0\rangle_{C_{II}}\\ &+ \cos r_b \sin r_c|0\rangle_A|0\rangle_{B_I}|0\rangle_{B_{II}}|1\rangle_{C_I}|1\rangle_{C_{II}}\\ &+ \cos r_b \sin r_c|0\rangle_A|1\rangle_{B_I}|1\rangle_{B_{II}}|0\rangle_{C_I}|0\rangle_{C_{II}}\\ &+ \sin r_b \sin r_c|0\rangle_A|1\rangle_{B_I}|1\rangle_{B_{II}}|1\rangle_{C_I}|1\rangle_{C_{II}}\\ &+ |1\rangle_A|1\rangle_{B_I}|0\rangle_{B_{II}}|1\rangle_{C_I}|0\rangle_{C_{II}}],\end{aligned} \quad (12)$$

where hereafter frequency subscripts are dropped.

Let us first calculate the one-tangle between subsystem A and the overall subsystem B_IC_I by using Eq. (4). Tracing over the inaccessible modes B_{II} and C_{II} we obtain a density matrix

$$\begin{aligned}&\rho_{AB_IC_I}\\ &= \tfrac{1}{2}[\cos^2 r_b \cos^2 r_c|000\rangle\langle000| + \cos^2 r_b \sin^2 r_c|001\rangle\langle001|\\ &\quad+ \sin^2 r_b \cos^2 r_c|010\rangle\langle010| + \sin^2 r_b \sin^2 r_c|011\rangle\langle011|\\ &\quad+ \cos r_b \cos r_c(|111\rangle\langle000| + |000\rangle\langle111| + |111\rangle\langle111|],\end{aligned} \quad (13)$$

where $|lmn\rangle = |l\rangle_A|m\rangle_{B_I}|n\rangle_{C_I}$. Then we can easily get the partial transpose subsystem A of Eq. (13),

$$\begin{aligned}&\rho^{T_A}_{AB_IC_I}\\ &= \tfrac{1}{2}[\cos^2 r_b \cos^2 r_c|000\rangle\langle000| + \cos^2 r_b \sin^2 r_c|001\rangle\langle001|\\ &\quad+ \sin^2 r_b \cos^2 r_c|010\rangle\langle010| + \sin^2 r_b \sin^2 r_c|011\rangle\langle011|\\ &\quad+ \cos r_b \cos r_c(|011\rangle\langle100| + |100\rangle\langle011| + |111\rangle\langle111|],\end{aligned} \quad (14)$$

from which we can get $(\rho^{T_A}_{AB_IC_I})^\dagger$ and the negativity $\mathcal{N}_{A(B_IC_I)}$ is found to be

$$\begin{aligned}\mathcal{N}_{A(B_IC_I)} = \tfrac{1}{2}[&\cos r_b \cos r_c + \cos^2 r_c + \cos^2 r_b \sin^2 r_c\\ &+ \sqrt{\cos^2 r_b \cos^2 r_c + \sin^4 r_b \sin^4 r_c} - 1].\end{aligned} \quad (15)$$

Similarly, we can also get

$$\mathcal{N}_{B_I(AC_I)} = \tfrac{1}{2}[\cos r_b \cos r_c + \cos^2 r_b + \sin^2 r_b \sin^2 r_c + \cos r_c\sqrt{\cos^2 r_b + \sin^4 r_b \cos^2 r_c} - 1] \quad (16)$$

and

$$\mathcal{N}_{C_I(AB_I)} = \tfrac{1}{2}[\cos r_b \cos r_c + \sin^2 r_b + \cos^2 r_b \cos^2 r_c + \cos r_b\sqrt{\cos^2 r_c + \sin^4 r_c \cos^2 r_b} - 1]. \quad (17)$$

The properties of all the one-tangles of $\rho_{AB_IC_I}$ are shown in Fig. 1 with $r_b = r_c = r$. It is shown that all the one-tangles are equal to one when $r = 0$, which is exactly the value of the one-tangles in Eq. (11) obtained in the inertial frame. All of the one-tangles decrease as the accelerations of Bob and Charlie increase, which is similar to the behavior of the bipartite entanglement of the Dirac field [6] and the tripartite one-tangle of the scalar field when one of the observers is accelerated [20]. Note that $\mathcal{N}_{B_I(AC_I)} = \mathcal{N}_{C_I(AB_I)}$ for all accelerations, which indicates that Bob and Charlie's subsystems are symmetrical in this case. It is worthwhile to note that unlike the scalar case the one-tangle $N_{C_I(AB)}$ goes to zero when Charlie moves with infinite acceleration; here $N_{C_I(AB_I)}$ and $N_{C_I(AB)}$ never go to zero for any acceleration. We argue that this difference is due to the differences between Fermi-Dirac and Bose-Einstein statistics [10] rather than because the observers cannot access the entanglement of the subsystems that moves with infinite acceleration with respect to them, as the authors stated in Ref. [20]. What is surprising is that in this case both Bob and Charlie move with infinite acceleration ($r = \pi/4$), $\mathcal{N}_{A(B_IC_I)} = \mathcal{N}_{B_I(AC_I)} = \mathcal{N}_{C_I(AB_I)} = \frac{1-\sqrt{5}}{8}$, which means that there is no difference between the subsystems A, B_I, and C_I in this limit.

Now, let us compute the two-tangle between subsystems A and B_I. Tracing the qubit of subsystem C_I we obtain

$$\rho_{AB_I} = \frac{1}{2}\begin{pmatrix} \cos r_b^2 \cos^2 r_c + \cos r_b^2 \sin^2 r_c & 0 & 0 & 0 \\ 0 & \sin^2 r_b \cos^2 r_c + \sin r_b^2 \sin^2 r_c & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Using this matrix and Eq. (4) we can obtain the negativity $\mathcal{N}_{AB_I} = 0$, which means there is no bipartite entanglement between mode A and B_I in spite of the acceleration of Bob and Charlie. Similarly, it is found that $\mathcal{N}_{AC_I} = \mathcal{N}_{B_IC_I} = 0$. Note that the CKW inequality [24], $\mathcal{N}^2_{\alpha\beta} + \mathcal{N}^2_{\alpha\gamma} \leqslant \mathcal{N}^2_{\alpha(\beta\gamma)}$, is saturated for any acceleration parameter r.

Then by use of Eqs. (7)–(10), the π-tangle of our system is found to be

$$\pi_{AB_IC_I} = \tfrac{1}{3}(\pi_A + \pi_{B_I} + \pi_{C_I}) = \tfrac{1}{3}\left[\mathcal{N}^2_{A(B_IC_I)} + \mathcal{N}^2_{B_I(AC_I)} + \mathcal{N}^2_{C_I(AB_I)}\right], \quad (18)$$

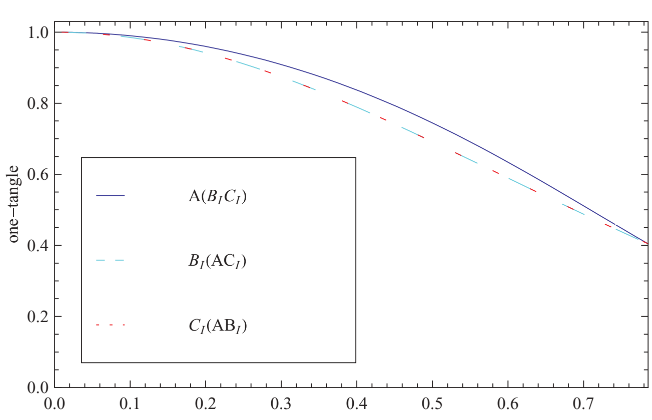

FIG. 1. (Color online) The negativity $\mathcal{N}_{A(B_IC_I)}$(solid line), $\mathcal{N}_{B_I(AC_I)}$ (dashed line), and $\mathcal{N}_{C_I(AB_I)}$ (dotted line) of a two-observer-accelerated case as a function of the acceleration parameter $r = r_b = r_c$.

where $\mathcal{N}_{A(B_IC_I)}$, $\mathcal{N}_{B_I(AC_I)}$, and $\mathcal{N}_{C_I(AB_I)}$ are given by Eqs. (15)–(17), respectively.

In order to better understand the multipartite entanglement in the noninertial frames, we also compute the entanglement of a tripartite system that includes two inertial subsystems and one noninertial subsystem; i.e., let Alice and Bob stay stationary and Charlie moves with uniform acceleration. They share the same GHZ state Eq. (11) at the same point in Minkowski spacetime. According to the preceding calculations, we can obtain

$$\mathcal{N}_{A(BC_I)} = \mathcal{N}_{B(AC_I)} = \cos r_c,$$
$$\mathcal{N}_{C_I(AB)} = \tfrac{1}{2}(\cos r_c + \cos^2 r_c + \sqrt{\cos^2 r_c + \sin^4 r_c} - 1), \quad (19)$$
$$\mathcal{N}_{AB} = \mathcal{N}_{BC_I} = \mathcal{N}_{C_IA} = 0,$$

where $\mathcal{N}_{A(BC_I)}$ and $\mathcal{N}_{AB}$ are the one-tangle and two-tangle of the one-observer-accelerated case. It is worth noticing that the CKW inequality is also saturated for any acceleration parameter r in this case. From these facts we arrive at the conclusion that this inequality is valid in both inertial and noninertial frames.

We plot the one-tangles of this case in Fig. 2 and find that (i) all of them decrease as the acceleration of Charlie increases; (ii) $\mathcal{N}_{A(BC_I)} = \mathcal{N}_{B(AC_I)}$ for all accelerations; and (iii) the one-tangle $N_{C_I(AB)}$ never goes to zero for any acceleration. However, it is interesting to note that in this case $\mathcal{N}_{A(BC_I)} = \mathcal{N}_{B(AC_I)} \neq \mathcal{N}_{C_I(AB)}$ when Charlie moves with infinite acceleration, which is very different from the two-observer-accelerated case. We are not sure whether this is an individual case that only appears in the fermionic systems because it is probably related to the incomplete definition of

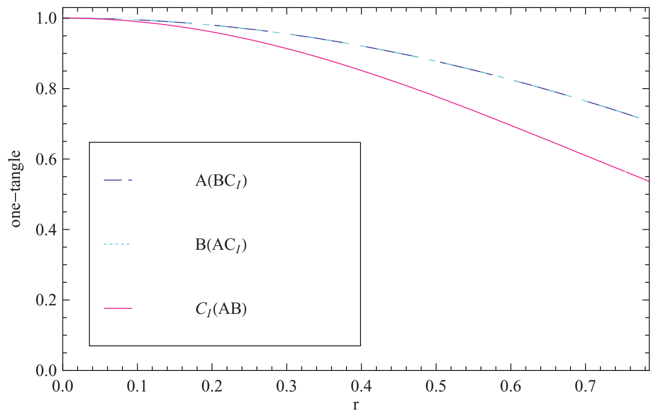

FIG. 2. (Color online) The one-tangles $\mathcal{N}_{A(BC_I)}$(dashed line), $\mathcal{N}_{B(AC_I)}$ (dotted line), and $\mathcal{N}_{C_I(AB)}$ (solid line) of one-observer-accelerated case as a function of the acceleration parameter $r = r_c$.

the one-tangle in the noninertial frames. Thus, it seems to be interesting to repeat the calculation of this paper for other systems and make use of other entanglement measurements. It is shown again that all the two-tangles equal zero in this case, which is exactly the same as the two-tangles obtained in the inertial frame. That is to say, either one or two subsystems of the tripartite state are accelerated, and there is no bipartite entanglement in this system. The acceleration does not generate a bipartite entanglement and the entanglement structure of the quantum state does not change. It is interesting to note that, in Ref. [6], there was no tripartite entanglement between observers Alice, Rob, and Anti-Rob; i.e, the entire entanglement is bipartite when one of the observers is static and the other accelerated. However, here we find that there is no bipartite entanglement; all the entanglements of this system are in the form of tripartite entanglements.

By use of Eq. (12) we get the π-tangle of the one-observer-accelerated system $\pi_{ABC_I} = [\mathcal{N}^2_{A(BC_I)} + \mathcal{N}^2_{B(AC_I)} + \mathcal{N}^2_{C_I(AB)}]/3$. For comparison, we plot π_{ABC_I} for this case and $\pi_{AB_IC_I}$ of the two-observer-accelerated case in Fig. 3.

In Fig. 3 we plot the π-tangle π_{ABC_I} of the one-observer-accelerated case and $\pi_{AB_IC_I}$ of the two-observer-accelerated

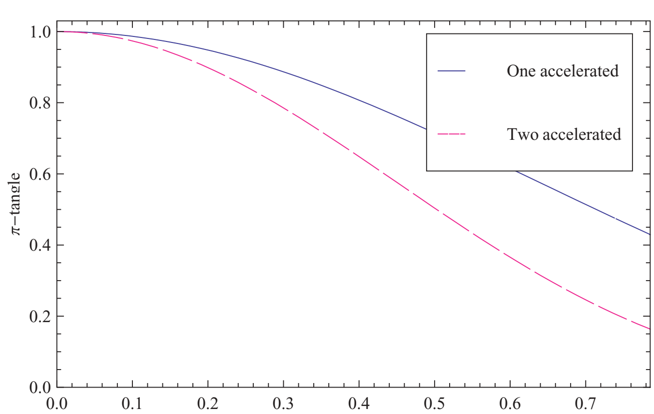

FIG. 3. (Color online) The π-tangle π_{ABC_I} of the one-observer-accelerated case (solid line) vs $\pi_{AB_IC_I}$ of the two-observer-accelerated case (dashed line), as a function of the acceleration parameter $r = r_b = r_c$.

case as a function of the acceleration parameter $r = r_b = r_c$, which shows how the acceleration changes tripartite entanglement. In the case of zero acceleration, $\pi_{ABC_I} = \pi_{AB_IC_I} = 1$. With increasing acceleration, the π-tangles decrease monotonically for both of these two cases. It is shown that the decrease in speed of the tripartite entanglement of the two-observer-accelerated case is much quicker than that of the one-observer-accelerated case as expected. Recall that the loss of entanglement was explained as the information formed in the inertial system was leaked into the causally disconnected region [14,19,22] due to the Unruh effect. The quicker decrease of entanglement in the two-observer-accelerated case is attributed to the information that both Bob and Charlie's subsystems are redistributed into the unaccessible regions as the acceleration grows. In the limit of infinite acceleration, the tripartite entanglement of the two-observer-accelerated case reduces to a lower minimum but never vanishes. It is interesting to note that either the scalar system or the Dirac system and either one or two observers are accelerated, and the tripartite entanglement never vanishes for any acceleration. Thus we can arrive at the striking conclusion that the quantum entanglement in a tripartite system is a better resource for performing quantum information processing such as teleportation. We can also perform such quantum information tasks by use of the tripartite entanglement when some observers are falling into a black hole while others are hovering outside the event horizon.

IV. SUMMARY

The effect of acceleration on bipartite and tripartite entanglements of a three-qubit Dirac system when one or two subsystems are accelerated is investigated. It is shown that all the one-tangles decrease as the accelerations of Bob and Charlie increase. However, unlike the scalar case in which the one-tangle $\mathcal{N}_{C_I(AB)}$ goes to zero when Charlie moves with infinite acceleration, here $\mathcal{N}_{C_I(AB_I)}$ and $\mathcal{N}_{C_I(AB)}$ never reduce to zero for any acceleration. It is also shown that the CKW inequality is valid in noninertial systems. It is interesting to note that $\mathcal{N}_{A(B_IC_I)} = \mathcal{N}_{B_I(AC_I)} = \mathcal{N}_{C_I(AB_I)}$ in the infinite-acceleration limit, which means that there is no difference between the subsystems A, B_I, and C_I in this limit. It is found that either one or two subsystems of the tripartite state accelerated and there is no bipartite entanglement in this system; i.e., all the entanglement of this system is in the form of tripartite entanglement. The acceleration does not generate a bipartite entanglement in this system and does not change the entanglement structure of the quantum state. It is also found that the π-tangle of the two-observer-accelerated case decreases much quicker than that of the one-observer-accelerated case and it reduces to a nonzero minimum in the infinite-acceleration limit. It is worth mentioning that for both scalar [20] and Dirac fields, and when either one or two observers are accelerated, the tripartite entanglement does not vanish for any acceleration. That is to say, quantum entanglement in a tripartite system is a better resource than bipartite entanglement for performing quantum information processing such as teleportation. We can also perform such quantum information tasks by using a tripartite entanglement when one or two observers are falling into a black hole while others hover outside the event horizon. The discussions of this paper can also be applied to the investigations of multipartite entanglement and quantum correlations in curved spacetime [17,18,22] as well as to the properties of multipartite Gaussian entanglement in noninertial frames [14]. Therefore, further investigation by using the results in this paper will not only help us to understand genuine multipartite entanglements but also help to give us a better insight into the entanglement entropy and information paradox of black holes.

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[1] A. Peres and D. R. Terno, Rev. Mod. Phys. **76**, 93 (2004).
[2] D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).
[3] D. Bouwmeester, A. Ekert, and A. Zeilinger, *The Physics of Quantum Information* (Springer-Verlag, Berlin, 2000).
[4] P. M. Alsing and G. J. Milburn, Phys. Rev. Lett. **91**, 180404 (2003).
[5] I. Fuentes-Schuller and R. B. Mann, Phys. Rev. Lett. **95**, 120404 (2005).
[6] P. M. Alsing, I. Fuentes-Schuller, R. B. Mann, and T. E. Tessier, Phys. Rev. A **74**, 032326 (2006).
[7] T. C. Ralph, G. J. Milburn, and T. Downes, Phys. Rev. A **79**, 022121 (2009).
[8] J. Doukas and L. C. L. Hollenberg, Phys. Rev. A **79**, 052109 (2009).
[9] S. Moradi, Phys. Rev. A **79**, 064301 (2009).
[10] E. Martín-Martínez and J. León, Phys. Rev. A **80**, 042318 (2009); **81**, 032320 (2010); **81**, 052305 (2010).
[11] J. Wang, J. Deng, and J. Jing, Phys. Rev. A **81**, 052120 (2010); J. Wang and J. Jing, *ibid.* **82**, 032324 (2010).
[12] D. C. M. Ostapchuk and R. B. Mann, Phys. Rev. A **79**, 042333 (2009).
[13] A. G. S. Landulfo and G. E. A. Matsas, Phys. Rev. A **80**, 032315 (2009).
[14] G. Adesso, I. Fuentes-Schuller, and M. Ericsson, Phys. Rev. A **76**, 062112 (2007).
[15] J. León and E. Martín-Martínez, Phys. Rev. A **80**, 012314 (2009).
[16] R. B. Mann and V. M. Villalba, Phys. Rev. A **80**, 022305 (2009).
[17] Q. Pan and J. Jing, Phys. Rev. A **77**, 024302 (2008); Phys. Rev. D **78**, 065015 (2008).
[18] J. Wang, Q. Pan, and J. Jing, Ann. Phys. (NY) **325**, 1190 (2010); Phys. Lett. B **692**, 202 (2010).
[19] J. Wang, Q. Pan, S. Chen, and J. Jing, Phys. Lett. B **677**, 186 (2009); Quantum Inf. Comput. **10**, 0947 (2010).
[20] M.-R. Hwang, D. Park, and E. Jung, Phys. Rev. A **83**, 012111 (2011).
[21] M. Aspachs, G. Adesso, and I. Fuentes, Phys. Rev. Lett **105**, 151301 (2010).
[22] E. Martín-Martínez, L. J. Garay, and J. León, Phys. Rev. D **82**, 064006 (2010); **82**, 064028 (2010).
[23] D. E. Bruschi, J. Louko, E. Martín-Martínez, A. Dragan, and I. Fuentes, Phys. Rev. A **82**, 042332 (2010).
[24] V. Coffman, J. Kundu, and W. K. Wootters, Phys. Rev. A **61**, 052306 (2000).
[25] G. Vidal and R. F. Werner, Phys. Rev. A **65**, 032314 (2002); M. B. Plenio, Phys. Rev. Lett. **95**, 090503 (2005).
[26] Y. C. Ou and H. Fan, Phys. Rev. A **75**, 062308 (2007).