Photon-echo quantum memory with complete use of natural inhomogeneous broadening

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The photon-echo quantum memory is based on a controlled rephasing of the atomic coherence excited by a signal light field in the inhomogeneously broadened resonant line. Here, we demonstrate an active mechanism of the atomic rephasing that provides a perfect retrieval of the stored light field in the photon-echo quantum memory based on the use of arbitrary initial inhomogeneous broadening of the resonant line. We show that the rephasing mechanism can exploit all resonant atoms, thereby maximally increasing an optical depth of the resonant transition, which is one of the critical parameters for the realization of highly efficient quantum memory. We also demonstrate that the rephasing mechanism can be used for various realizations of the photon-echo quantum memory, thereby creating many possibilities for its practical realization.

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I. INTRODUCTION

Quantum memory (QM) is one of the key quantum devices for the practical realization of various basic protocols in quantum communication [1,2] and quantum computation [3,4]. In the past decade, considerable progress has been achieved in the optical QMs based on the atoms in cavities [5], nonresonant Raman transitions [6–9], electromagnetically induced transparency [10–16], and photon-echo QM techniques [17–22].

The photon-echo approach offers promising possibilities for the storage of arbitrary multimode light fields [23–26], as demonstrated recently in a storage of 64 [27] and 1090 [28] temporal modes. Record quantum efficiencies of 69% [29] and 87% [30] have also been demonstrated for the QM of the traveling light fields in solid state and gaseous media. Moreover, even higher quantum efficiency (>90%) is predicted for the storage of 100 temporal modes in the optimal QED cavity for moderate atomic parameters [31]. However, there are serious experimental problems in the realization of the photon-echo QMs with practically vital properties that are discussed in Refs. [32–35]. It is especially worth noting that recently developed variants of the photon-echo QM use quite complicated experimental methods (see the forthcoming discussion) for the realization of very delicate spectral manipulations of the inhomogeneously broadened (IB) lines, which restricts the quantum efficiency of the QMs, storage time, or spectral width of the signal light field. In this paper we propose an *active mechanism of rephasing* (AMR protocol) of the atomic coherence excited on the natural IB atomic transition that offers experimental possibilities for the practical realization of the photon-echo QM.

First, we briefly outline the experimental methods, providing temporal and spectral manipulations of the atomic coherence excited in the photon-echo QM media. Then, we propose a basic scheme of the AMR protocol by using Raman-type photon-echo QM (Raman-echo QM) proposed recently in Refs. [36,37], further developed in Refs. [26,38], and experimentally demonstrated in Refs. [30,39]. Finally, we describe how the AMR procedure can be used for the original photon-echo QM and discuss two perfect realizations of the photon-echo QMs in which the AMR protocol is protected from the negative influence of extra quantum noise. In conclusion, we summarize the advantages of the AMR protocol and outline some of its interesting applications.

II. ATOMIC REPHASING IN THE PHOTON-ECHO QUANTUM MEMORIES

In accordance with the basic idea [17], the photon-echo QM exploits complete absorption of the signal light pulse on the resonant IB transition, thereby providing a direct pure mapping of the quantum information carried by the signal field on the excited coherence of the multiatomic ensemble. In a free space scheme, the complete absorption of the input light pulse occurs at a large optical depth for each isochromatic atomic group of the IB line, which is one of the critical requirements for the realization of the effective photon-echo QM. A subsequent efficient retrieval of the stored light field is realized in the echo signal irradiated in the backward direction in comparison with the direction of the input signal field propagation. Such a scenario of the echo field generation is realized in accordance with most desirable reversible dynamics of the light field retrieval. The retrieval is launched by inversion of the frequency detunings for each jth atom $\Delta_j(t > t') = -\Delta_j(t < t')$ of the IB resonant atomic transition at some moment of time t' [the procedure is called the controlled reversibility of IB (CRIB)] and is provided by the phase-matching condition for the echo field emission.

Concrete realizations of the CRIB procedure can be fulfilled in various ways; for example, it occurs automatically in the atomic gases due to opposite Doppler frequency shifts of the echo field irradiated in the backward direction to the input signal field propagation [17]. However, this scheme does not provide a long-lived QM, so it is more interesting for some quantum manipulations of the stored light field. The CRIB procedure can be realized in some crystals by active inversion of local magnetic fields [18] caused by the dipole interaction with nearest nuclei or electron spins. Very promising variant of the CRIB procedure uses the external electric or magnetic

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fields for control of the solid-state photon-echo QM media, provided by preliminary spectral tailoring of the original IB resonant line into a narrowed single pike. Here, the CRIB procedure is fulfilled by changing a polarity of the external electric (magnetic) field gradient, effecting the inversion of linear Stark (Zeeman) shifts of the atomic transition [20,21,33]. However, preliminary tailoring of the original IB line is accompanied by a large reduction of the active atoms, which considerably reduces an effective optical depth on the atomic transition.

Reduction of the optical depth can be minimized by using the so-called atomic frequency comb (AFC) structure of the IB transition [22], which offers promising possibilities for broadband photon-echo QM [27,28], as also demonstrated recently for the entangled states of light [40,41]. However, even for the ideal AFC, the optimal effective optical depth will be >10 times smaller in comparison with the original optical depth of the IB resonant line. In addition, some specific experimental problems must be resolved in the AFC protocol. In particular, the retrieval time cannot be shorter than some given value, determined by the AFC structure, that excludes a temporal flexibility in the readout of the stored information. Perfect tailoring of the AFC structure within the IB line by using the laser hole burning technique is also a serious experimental problem in the presence of additional atomic sublevels situated closely to the active levels used in the QM.

III. BASIC EQUATIONS

The basic scheme of the light-atoms interaction for the AMR protocol is presented in Figs. 1, 2, and 3. At time $t = 0$, the input signal light field $\hat{A}_1(t,z)$ with total temporal duration $\delta t \ll T_2$ (T_2 is a decoherent time of the Raman transition), carrier frequency ω_1, and spectral width $\delta\omega$ enters along the $+z$ direction in the medium with three-level atoms prepared in the long-lived level $|1\rangle = \prod_{j=1}^{N} |1\rangle_j$. The control (writing) field with Rabi frequency Ω_1 is switched on before the entrance of the input pulse and propagates along the wave vector $\vec{K}_1$ at a small angle to z axis with carrier frequency ω_1^c. The signal and

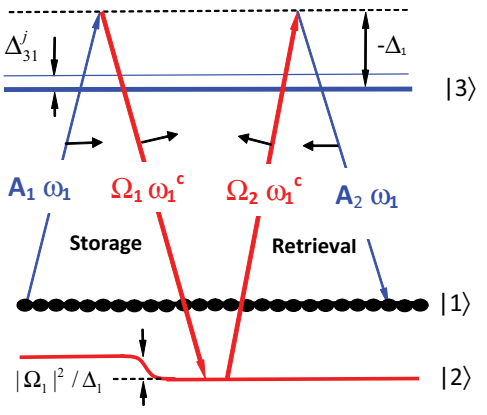

FIG. 1. (Color online) Energies of the atomic levels and Raman transition $1 \leftrightarrow 2$ due to interaction with probe field A_1 and writing field Ω_1 (two left arrows) and with echo field A_2 and reading field Ω_2 (two right arrows). Black small arrows show the wave vectors of the fields. $|\Omega_1|^2/\Delta_1$ is the Stark shift of the Raman transition.

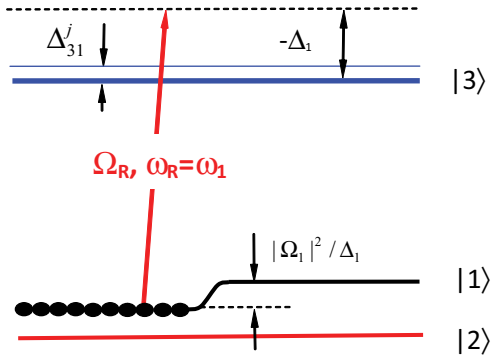

FIG. 2. (Color online) Rephasing pulse Ω_R switched on adiabatically on the transition $1 \leftrightarrow 3$ after storage of the signal field. The pulse Ω_R causes a Stark shift $|\Omega_R/(\Delta_1 + \Delta_{13}^j)|^2$ with an opposite sign in comparison with the Stark shifts induced by the control fields Ω_1 and Ω_2, leading to rephasing of the excited Raman coherence $\hat{R}_{12}^j$.

writing fields are in Raman resonance $\omega_1 - \omega_1^c \approx \omega_{21}$ with sufficiently large spectral detuning $\Delta_1 = \omega_{31} - \omega_1$ from the optical transition $|1\rangle \leftrightarrow |3\rangle$, so that $\Delta_1 \gg \delta\omega, \Delta_{\text{in}}^{(31)}$ (where $\Delta_{\text{in}}^{(31)}$ is an IB for the transition $|1\rangle \leftrightarrow |3\rangle$ and $\delta\omega$ is a spectral width of the signal light field).

We assume a very weak intensity of the signal field (in particular, it can be a single photon field), so the excited population of atomic levels 2 and 3 can be ignored. To be concrete, we will analyze a rare-earth type of three-level scheme in the inorganic crystals, in which large IB ($\sim 10^8$-10^{10}s^{-1}) can be easily realized for optical transition $|1\rangle \leftrightarrow |3\rangle$ while a spectral width of the transition $|1\rangle \leftrightarrow |2\rangle$ reaches few kHz [33,35]. Thus, the spectral broadening of the transition $|1\rangle \leftrightarrow |2\rangle$ is negligible in a microsecond time scale. In this case, we get the following linearized system of Heisenberg equations for the weak signal (echo) fields $\hat{A}_{12}(\tau,z)$ and for long-lived atomic coherence $\hat{R}_{12}^j$ between the states $|1\rangle$ and $|2\rangle$:

$$-(-1)^{\nu}\frac{\partial}{\partial z}\hat{A}_{\nu}(\tau_{\nu},z) = i\frac{\beta_{\nu}}{2}\left(\chi\hat{A}_{\nu}(\tau_{\nu},z) + \frac{\Omega_{\nu}(t)}{g_{\nu}}\left\langle\frac{\hat{R}_{12}^j}{\Delta_{\nu}+\Delta_{31}^j}\right\rangle\right), \quad (1)$$

$$\frac{\partial}{\partial\tau_{\nu}}\hat{R}_{12}^j = i\frac{\Omega_{\nu}^*(\tau_{\nu})g_{\nu}\hat{A}_{\nu}(\tau_{\nu},z_j)}{\Delta_{\nu}+\Delta_{31}^j} - i\delta\omega_{\nu}\left(\Delta_{31}^j\right)\hat{R}_{12}^j, \quad (2)$$

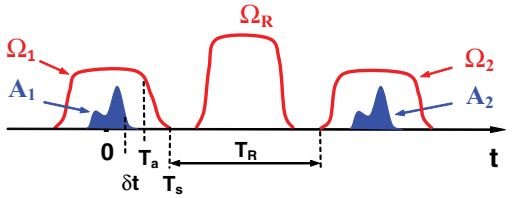

FIG. 3. (Color online) Basic AMR protocol. Temporal sequence of the interaction with weak signal (A_1) and echo (A_2) fields (filled blue shapes); writing (Ω_1) and reading (Ω_2) control fields are applied together with the weak light fields; Ω_R is a rephasing control laser pulse.

where we have used the moving system of coordinates $t < t'$ $(\nu = 1)$, $\tau_1 = t - z/v_1$, and $z = z$, and for retrieval $t > t'$ $(\nu = 2)$, $\tau_2 = t + Z/v_2$, $z = z$, $\delta\omega_\nu(\Delta_{31}^j) = \delta_\nu + |\Omega_\nu(t)|^2 f_\nu(\Delta_{31}^j)$, $\delta_\nu = \Delta_{21} - \frac{|\Omega_\nu(\tau_\nu)|^2}{\Delta_\nu}$, and $f_\nu(\Delta_{31}^j) = \frac{1}{\Delta_\nu} - \frac{1}{\Delta_\nu + \Delta_{31}^j}$, where $\nu = 1(2)$ corresponds to the signal (echo) field. For the signal and echo fields we have $\hat{E}_\nu(\tau_\nu, z) = \hat{A}_\nu(\tau_\nu, z)\exp[-i\omega_\nu(t + (-1)^\nu n_\nu z/c)]$, where n_ν is the refractive index for the signal and echo fields; for the control fields with Rabi frequency $\tilde{\Omega}_\nu(t, \vec{r}) = \Omega_\nu(\tau_\nu)\exp(-i\omega_\nu^c t + i\vec{K}_\nu\vec{r})$ and $v_\nu = \partial\omega/\partial k|_{\omega=\omega_\nu}$ are group velocities for the signal (echo) in the absence of interaction with atoms, and $\beta_\nu = 2\pi(n_o S)|g_\nu|^2/v_\nu$ with atomic density n_o, photon-atom coupling constants g_ν, and cross section of the signal (echo) fields S; and for the atomic coherences $\hat{P}_{12}^j(t) = \hat{R}_{12,\nu}^j(t)\exp\{i\varphi_\nu(\vec{r}, z) - i(\omega_\nu - \omega_\nu^c)[t + (-1)^\nu n_\nu z/c]\}$, $\Delta_{21} = \omega_{21} - \omega_1 + \omega_1^s$, $\varphi_\nu(\vec{r}, z) = -[(-1)^\nu n_\nu \omega_\nu^c z/c + \vec{K}_\nu\vec{r}]$, where $\vec{K}_\nu$ is the wave vector of the control fields, and $\langle\cdots\rangle$ means an ensemble averaging over spectral detunings Δ_{31}^j of IB on the transition $1 \leftrightarrow 3$ for atoms with spatial coordinates $z_j \approx z$: $\langle\cdots\rangle = \int d\Delta_{31,\nu}^j\, G(\Delta_{31,\nu}^j)\ldots, \chi = \langle\frac{1}{\Delta_\nu + \Delta_{31}^j}\rangle$.

In Eqs. (1) and (2), we have used a slowly varied optical coherence $\hat{R}_{13}(t)$ that follows adiabatically to temporal evolution of the signal (echo) field and atomic coherence $\hat{R}_{12}^j(t)$ as $\hat{R}_{13}^j(t) \cong \frac{g_\nu\hat{A}_\nu(t, z_j) + \Omega_\nu(t)\hat{R}_{12}^j(t)}{\Delta_1 + \Delta_{31}^j}$ and $\hat{R}_{11}^j(t) \approx 1, \hat{R}_{22}^j(t) = \hat{R}_{33}^j(t) = 0$.

IV. STORAGE

Similarly to the main idea of photon-echo QM [17], we assume that IB broadening on the Raman transition outreaches the light field width and that the resonant transition has sufficiently large optical depth. By taking these spectral conditions into account, we launch the signal field into the medium at $\tau = 0$. The probe pulse will be completely absorbed almost during the time duration of the light pulse $\tau \cong \delta t$. After the absorption we slowly switch off the control field $\Omega_\nu(t > \delta t) \to 0$, so that only the atomic coherence on the transition $1 \leftrightarrow 2$ will be created in the atomic system. By taking the linear equations (1) and (2) into account, we will analyze only the behavior of the observable values of the light field $A_\nu(\tau_\nu, z)$ and the atomic coherence $R_{12}^j(\tau_\nu)$ that is sufficient for understanding the main properties of the analyzed QM. By assuming that the control field amplitude $\Omega_{\nu=1}(\tau_\nu)$ is constant during the interaction with probe field $A_1(\tau_\nu, z)$, we find $A_1(\tau_1 > \delta t, z) \cong 0$ for the optically dense media and the excited atomic coherence $R_{12}^j(T_a > \delta t) = i\frac{|\Omega_1|^2}{\Delta_1 + \Delta_{31}^j}\exp[-i\delta\omega_1(\Delta_{32}^j)(T_a - z/v_1)]\tilde{A}_1[\delta\omega_1(\Delta_{31}^j), z]$, where

$$\tilde{A}_1(\omega, z) = \exp\left(\frac{\beta_1}{2}[i\chi - B_1(\omega)]z\right)\tilde{A}_1(\omega, 0), \qquad (3)$$

$$B_1(\omega) = -\frac{1}{g_1}\int du\,\tilde{G}_1(u)/[\gamma + i(\Delta_{21} - u - \omega)], \qquad (4)$$

where $\tilde{A}_1(\omega, z) = \int_{-\infty}^{\infty}\exp(-i\omega\tau)A_1(\tau, z)d\tau$, $\tilde{G}_1(u) = G(\frac{|\Omega_1|^2}{u} - \Delta_1)$, and γ is a negligibly small decay constant of the atomic coherence.

By using Eqs. (3) and (4), we find the absorption coefficient on the frequency detuning ω

$$\beta_1\mathrm{Re}[B_1(\omega)] = \frac{\pi\beta_1}{g_1}G\left(\frac{|\Omega_1|^2}{\Delta_{21} - \omega} - \Delta_1\right). \qquad (5)$$

As seen in Eq. (5), a spectral profile of the absorption coefficient does not reproduce a line shape of IB on the optical transition $|1\rangle \leftrightarrow |3\rangle$ in a most general choice of the possible parameters Ω_1 and Δ_1. The maximum absorption coefficient takes place on the frequency detuning $\omega_o = \Delta_{21} - \frac{|\Omega_1|^2}{\Delta_1}$, where the function of IB on the transition $|1\rangle \leftrightarrow |3\rangle$ gets a maximum $G(0)$, and $|\Omega_1|^2/\Delta_1$ is a Stark shift of the IB Raman transition $|1\rangle \leftrightarrow |2\rangle$ induced by the control field Ω_1. It is also clear that $\mathrm{Im}[B_1(\omega)] \approx 0$ for a small value $|\omega - \omega_o| \ll \Delta_{\mathrm{in}}^{31}$, respectively.

By using Eq. (5), we find that large absorption will occur within a spectral range $\Delta_{\mathrm{in}}^R = \Delta_{\mathrm{in}}^{(31)}|\Omega_1|^2/[\Delta_1^2 - (\Delta_{\mathrm{in}}^{(31)})^2/2]$ around the frequency ω_o and the absorption decreases continuously to zero with an increase of the frequency detuning $|\omega - \omega_o| > \Delta_{\mathrm{in}}^R$ (where $|\Delta_1| > \Delta_{\mathrm{in}}^{(31)}$). Δ_{in}^R is a linewidth of the inhomogeneously broadened Raman transition between the states $|1\rangle$ and $|2\rangle$ that is caused by the Stark effect in the atomic system with large IB on the optical transition $|1\rangle \leftrightarrow |3\rangle$. By assuming sufficiently large Raman linewidth in comparison with the spectral width of the signal field $\Delta_{\mathrm{in}}^R > \delta\omega$, we get a complete absorption of the signal light field for sufficiently large optical depth field $\beta_1\mathrm{Re}[\mathrm{B}_1(\omega)]\mathrm{L} \gg 1$ within all spectral ranges of the signal $\delta\omega < \Delta_{\mathrm{in}}^R$. We note that the signal field can contain many light pulses (for example, two light pulses are depicted in Figs. 3 and 4) with total temporal duration $\delta t \gg \delta\omega^{-1}$ and $\delta t\Delta_{\mathrm{in}}^R \gg 1$, respectively, which should be shorter only in comparison with the decoherence time $\delta t \ll T_2$ of the Raman transition $|1\rangle \to |2\rangle$.

After absorption of the signal field (at time $\tau_1 = T_a$), we switch off the control field $(\Omega_1 = 0)$ during $T_a < t < T_s$ for long-lived storage of the transferred signal field state so that $R_{12}^j(T_s) = \exp[-i\int_{T_a}^{T_s}\delta\omega_1(\Delta_{31}^j, \tau_1)d\tau_1]R_{12}^j(T_a)$. We note that switching off the control field Ω_1 freezes further dephasing of the atomic coherence. In the following section, we propose an AMR procedure for rephasing of the excited coherence R_{12} by launching one additional nonresonant control laser pulse.

V. AMR PROTOCOL FOR CONTROL OF ATOMIC COHERENCE

The principal spectral scheme of the rephasing process is depicted in Fig. 2. We launch a nonresonant control light pulse $\Omega_R(\tau)$ coupling only the atomic levels 1 and 3. Carrier frequency of the rephasing pulse coincides with carrier frequency of the signal field. It is well known that the selective interaction of the control field with the transition $|1\rangle \to |3\rangle$ can be experimentally realized by exploiting the properties of allowed and forbidden atomic transitions or frequency vicinity between the carrier frequency and the atomic transition (see also the discussion that follows). Here, the evolution of atomic coherence R_{12}^j is determined by the following equation:

$$\frac{\partial}{\partial\tau_\nu}R_{12}^j = -i\delta\omega_R\left(\Delta_{31}^j, \tau\right)R_{12}^j, \qquad (6)$$

where the frequency detuning

$$\delta\omega_R\left(\Delta_{31}^j,\tau\right)=\delta_R(\tau)-|\Omega_R(\tau)|^2 f_1\left(\Delta_{31}^j\right), \tag{7}$$

where $\delta_R(\tau)=\Delta_{21}+\frac{|\Omega_R(\tau)|^2}{\Delta_1}$. We note that the frequency shift $\delta\omega_R(\Delta_{31}^j,\tau)$ gets an opposite frequency dependence on the atomic detuning Δ_{31}^j, in comparison with the frequency shift in Eq. (2) that occurred during the signal absorption. An opposite sign of the frequency shift in Eq. (7) is a result of the fact that the rephasing nonresonant field couples the atomic states $|1\rangle\leftrightarrow|3\rangle$ but not the states $|2\rangle\leftrightarrow|3\rangle$ as is the case for the absorption and echo emission stages. The perfect ratio between the two frequency shifts holds for all possible parameters of the Raman transition Δ_1, $\Delta_{\rm in}^{(\rm in)}$, and Ω_1, even where the resulting line shape of the IB on the Raman transition will not reproduce the original spectral shape of the IB on the optical transition $|1\rangle\leftrightarrow|3\rangle$. Here, we have also assumed a slowly (adiabatically) varying amplitude of the control field $\Omega_R(\tau)$ that excludes any real atomic transition $|1\rangle\leftrightarrow|3\rangle$. However, we will also discuss an additional method that must be applied for the complete elimination of negative influence caused by the spontaneous induced transition $|1\rangle\rightarrow|2\rangle$ during the rephasing procedure.

We apply the rephasing field $\Omega_R(\tau)$ only for finite temporal duration T_R. The switching of the rephasing pulse results in the following atomic coherence:

$$\begin{aligned}R_{12}^j(T_s+T_R)&=\exp\left(-i\int_{T_s}^{T_s+T_R}\delta\omega_R\left(\Delta_{31}^j,\tau\right)d\tau\right)R_{12}^j(T_s)\\&=i\frac{|\Omega_1|^2}{\Delta_1+\Delta_{31}^j}\exp\left[-i\theta+if_1\left(\Delta_{31}^j\right)P(S,R)\right]\\&\quad\times\tilde{A}_1\left[\delta\omega_1\left(\Delta_{31}^j\right),z\right],\end{aligned} \tag{8}$$

where $\theta=\int_{-\infty}^{T_s}\delta_1 d\tau+\int_{T_s}^{T_s+T_R}\delta_R d\tau$ is a constant phase shift, and the factor $P(S,R)=\int_{T_s}^{T_s+T_R}|\Omega_R(\tau)|^2 d\tau-\int_{-\infty}^{T_s}|\Omega_1(\tau_1)|^2 d\tau_1$ determines conditions of the atomic rephasing. For some fixed temporal duration T_R', the factor $P(S,R)=0$ means a complete recovering of the atomic coherence R_{12}. In the following we will use larger temporal duration $T_R>T_R'$, in which the rephased coherence is realized again but with opposite atomic phase shifts for each isochromatic group. For simplicity we use equaled magnitudes of the control fields $\Omega_1=\Omega_R$ with adiabatic switching of the rephasing pulse at time $t=T_s+T_R$ [$\Omega_R(\tau>T_s+T_R)=0$]. By assuming a sufficiently large temporal duration T_R (for example, $T_R=2T_s$ or larger), we have prepared the atomic system (R_{12}) for readout of the stored signal light field.

VI. ECHO SIGNAL IRRADIATION

Here we launch the readout control pulse $\Omega_2(\tau_2)$ at $\tau_2>T_s+T_R$ in almost the opposite direction in comparison with the first writing control pulse, in order to provide the phase-matching condition and propagation of the echo field in the backward direction to the signal light pulse (see also the details in Ref. [36]). In this case we prepare the initial atomic state on the second ground level and exploit larger wave vectors of the writing and reading control laser fields, which leads us to the following initial atomic coherence: $R_{12,\rm in}^j(T_s+T_R)=R_{12}^j(T_s+T_R)\exp[i\delta k(\Delta_{31}^j)z]$, where the appropriate value of δk provides the phase-mismatch condition (see the following) due to the use of a difference of energies between levels 1 and 2. In order to satisfy a temporally reversible behavior, we exploit the same amplitude of the reading control field $\Omega_2=\Omega_1$ during the echo signal emission and the same frequency detuning $\Delta_2=\Delta_1$.

Evolution of the light field dynamics is determined by Eqs. (1) and (2) with index $\nu=2$. Initially, the launched reading pulse will only recover the macroscopic atomic coherence R_{12} during temporal interval T_s–T_a so the complete rephasing of the atomic coherence will occur later at $t\cong T_R+T_s=3T_s$. By taking into account the initial state in Eq. (8), we find the following equation for the Fourier component of the echo field $\tilde{A}_2(\omega,z)$:

$$\begin{aligned}&-\frac{\partial}{\partial z}\tilde{A}_2(\omega,z)\\&=\frac{\beta_\nu}{2}[i\chi-B_1(\omega)]\tilde{A}_2(\omega,z)-\exp[i(\omega T_R-\theta)]\frac{\pi\beta}{g_1}\\&\quad\times\int\frac{d\Delta e^{i\delta k(\Delta)z}G(\Delta)|\Omega_1|^2}{(\Delta_1+\Delta)^2}\exp\Bigg(+iP(S,R)f_1(\Delta)+\frac{\beta_\nu}{2}\\&\quad\times\{i\chi-B_1[\delta\omega_1(\Delta)]\}z\Bigg)\delta(\omega-\delta\omega_1(\Delta))\tilde{A}_1(\delta\omega_1(\Delta),0),\end{aligned} \tag{9}$$

where θ is some constant phase shift.

By integrating (9) over the delta function $\delta(\omega-\delta\omega_1(\Delta))$ with substitution $u=\frac{|\Omega_1|^2}{\Delta+\Delta_1}$, we find the following solution:

$$\begin{aligned}\tilde{A}_2(\omega,z=0)=&-\exp[-i\theta+i\omega(T_R+T_s)]\\&\times\frac{\pi}{g_1}\frac{G\left(\frac{|\Omega_1|^2}{\Delta_{21}-\omega}-\Delta_1\right)}{\{B_1(\omega)-i[\chi+\delta k(\omega)/\beta_1]\}}\tilde{A}_1(\omega,0),\end{aligned} \tag{10}$$

where we have taken into account $T_s=P(S,R)/|\Omega_1|^2$ and large optical depth of the Raman transition $\mathrm{Re}[B_1(\omega)]\beta L\gg1$.

The function $G(\frac{|\Omega_1|^2}{\Delta_{21}-\omega}-\Delta_1)$ reaches a maximum while $\mathrm{Im}[B_1(\omega)]\cong(\omega-\omega')B_1(\omega')'_{\omega'}$ close to the frequency detuning $\omega'=\Delta_{21}-|\Omega_1|^2/\Delta_1$ (center of the input pulse spectrum). Therefore, as seen in Eq. (9), we can satisfy the phase-matching condition by using the relation $\delta k(\omega)\cong-\beta_1\chi+\delta\kappa'_{\omega'}(\omega-\omega')$. Thus, the denominator in Eq. (9) can be simplified for narrow spectral width of the input light field as follows:

$$\begin{aligned}&\{B_1(\omega)-i[\chi+\delta k(\omega)/\beta_1]\}\\&\cong\frac{\pi}{g_1}G\left(\frac{|\Omega_1|^2}{\Delta_{21}-\omega}-\Delta_1\right)+i(\omega-\omega')[B_1(\omega')'_{\omega'}-\delta\kappa'_{\omega'}]\\&\cong\frac{\pi}{g_1}G\left(\frac{|\Omega_1|^2}{\Delta_{21}-\omega}-\Delta_1\right)\exp[i(\omega-\omega')\delta\tau],\end{aligned} \tag{11}$$

where $\delta\tau\cong\frac{g_1}{\pi}[B_1(\omega')'_{\omega'}-\delta\kappa'_{\omega'}/\beta_1]/G(0)$. Finally, after the Fourier transformation, we find the echo field

$$\begin{aligned}&A_2(\tau,z=0)\\&\quad=-\exp[i(\omega'\delta\tau-\theta)]A_1(\tau-T_R-T_s+\delta\tau,z=0).\end{aligned} \tag{12}$$

As seen in Eq. (12), the echo field completely reproduces the input signal field similarly to the AFC protocol, while we recall that the usual scenario of the photon-echo QM [17] leads to the temporally reversed shape of the signal field. The original temporal shape of the echo field in the AMR protocol is caused by the same temporal behavior of the atomic coherence on each spectral component of the IB line. Here, we must note that the absence of the temporal reversibility in light-atoms dynamics can lead to irreversible behavior due to spectral dispersion in echo field emission [see denominator in Eq. (10)]. However, the weak dispersion leads only to an additional time delay $-\delta\tau$ and phase shift $\omega'\delta\tau$, which is possible for a sufficiently narrow spectral width of the signal field.

The described scheme of QM needs additional analysis and some improvement, since the rephasing laser pulse induces spontaneous Raman transitions $|1\rangle \rightarrow |2\rangle$ so that a direct use of the schemes depicted in Figs. 1–3 leads to extra quantum noises in the irradiated echo field. In the following, we will describe the procedure providing complete elimination of any drawbacks caused by the spontaneous transitions.

Let us consider four-level realization of the described QM protocol depicted in Fig. 4, where an additional (buffer) level 4 could be some hyperfine sublevel similar to other ground levels 1 and 2. Here, we assume that the transition $|1\rangle \leftrightarrow |4\rangle$ is forbidden for the Raman transitions realized during the storage and echo field retrieval. Before rephasing of the excited atomic coherence R_{12} by the laser pulse Ω_R, we transfer the coherence R_{12} to the long-lived coherence

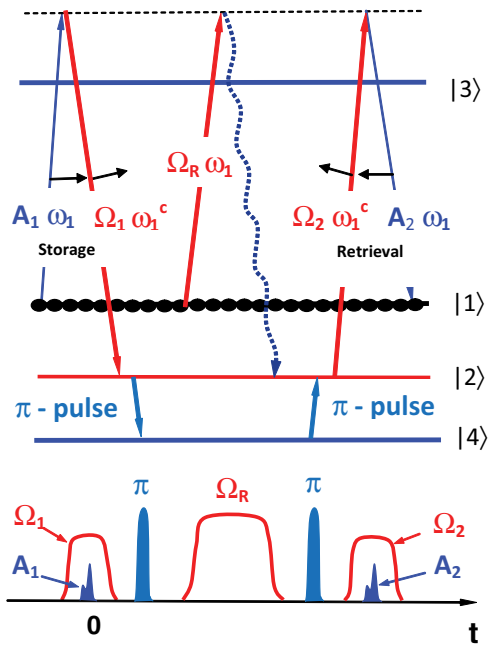

FIG. 4. (Color online) Atomic transitions in four level systems where the first two arrows indicate the input A_1 and writing Ω_2 fields, then π pulse transfers the atoms from level 2 to level 4; laser pulse Ω_R rephases the atomic coherence R_{14} and leads to spontaneous transitions of atoms on level 2 (dotted waved arrow directed to level 2); the additional atomic population $\delta\rho$ of level 2 is transferred to level 4 by the second π pulse; the last two arrows indicate the reading Ω_2 and echo A_2 fields.

R_{14} by a resonant π pulse on the transition $|2\rangle \leftrightarrow |4\rangle$ so that $R_{14}^j = \exp(-i\phi_1)R_{12}^j$ (where ϕ_1 is a constant phase of the first π pulse). Then we apply the laser pulse Ω_R, which rephases the coherence R_{14} that follows the equations coinciding with Eqs. (6) and (7). Rephasing of coherence R_{14} is accompanied by some additional population $\delta\rho_2$ of level 2 due to the spontaneous Raman transitions caused by the rephasing pulse Ω_R (see Fig. 4). Since the total temporal duration δt of the rephasing time is much shorter in comparison with the lifetime $T_1^{(3)}$ of level 3, the induced population of level 2 also will be weaker in comparison with the original population of level 1 $\delta\rho_2 \approx (\delta t/T_1^{(3)})|\Omega_1|^2/[\Delta_1^2 - (\Delta_{\rm in}^{(31)})^2/2] \ll 1$. Then we can apply a second π pulse on the transition $|2\rangle \leftrightarrow |4\rangle$ for transfer of the rephased coherence R_{14} to the coherence R_{12} (so that $R_{12}^j = \exp[-i(\phi_1 - \phi_2)]R_{12}^j$) and remove the atomic population $\delta\rho_2$ to level 4 (ϕ_2 is a constant phase of the second π pulse). The coherence R_{12} is thereby prepared for retrieval of the stored information in the echo field A_2 without any quantum noise, since all of the atoms excited by the spontaneous Raman transitions will stay on level 4. We see that the use of two π pulses will lead only to an additional constant phase shift ϕ_1–ϕ_2 in the irradiated echo field, and extra noise will not change the light-atoms equation due to negligible reduction of the atomic population on level 1 at the echo field irradiation. In general, we must recover the initial population of level 1 for the next cycle of the QM process. Therefore, we use a π pulse on the transition $|4\rangle \rightarrow |2\rangle$ and transfer the atomic population $\delta\rho_2$ from level 2 to level 1 by applying an additional laser field Ω_2 on the optical transition $|2\rangle \rightarrow |3\rangle$.

Finally, we note that the described procedure of AMR can be applied for the original scheme of the photon-echo QM [17,18] in which the input signal pulse is absorbed on the optical transition $|1\rangle \leftrightarrow |3\rangle$. In this case, we have to use sufficiently large spectral detuning $\Delta_1 \gg |\Delta_{13}^j|$ for the rephasing pulse so that the frequency detunings during the rephasing stage will be given by $\Omega_R^2/(\Delta_1 + \Delta_{13}^j) \cong \Omega_R^2/\Delta_1 - \Delta_{13}^j|\Omega_R/\Delta_1|^2$. Here, we get the same spectral shape of the IB as takes place for the absorption of the signal light field. Moreover, by taking into account that factor $f_R = |\Omega_R/\Delta_1|^2$ can be close to unity, we can rephase the excited atomic coherence R_{13} (after transfer to the long-lived coherence R_{12}) within the same temporal scale, thereby eliminating the negative drawbacks caused by the spontaneous transitions on level 2, as described in the preceding section.

VII. CONCLUSION

We have described a scheme of the photon-echo QM in which the rephasing of the atomic coherence (AMR procedure) is realized by the use of nonresonant interaction with additional control laser field, without the use of the CRIB procedure or AFC structure of the inhomogeneously broadened resonant transition. We have also demonstrated that the proposed atomic rephasing can be realized without negative influence of the quantum noises by the use of an additional buffer level 4 for the atoms excited by the spontaneous Raman transitions during the rephasing stage. We have shown that the AMR procedure can be used for Raman-echo QM and for the usual

photon-echo QM. In the latter case we use larger spectral detuning $|\Delta_{31}^{j}| \ll \Delta_1$, in which the Raman transition will get a spectral IB shape that differs only by the factor $f_R = |\Omega_R|^2/\Delta_1|^2$ from the original shape of the IB line. Therefore, the time of echo field irradiation will be scaled only by the factor f_R. Here we note that the use of the analyzed Raman scheme provides a larger spectral range of the frequency detuning Δ_1 due to the same spectral shapes of the IBs on the absorption (retrieval) and rephasing stages.

The proposed AMR procedure provides a possibility of photon-echo QMs for atomic systems by the use of the initial natural inhomogeneous broadenings that now offers practical perspectives for realization of the efficient optical quantum memories and repeaters. We believe that the proposed scheme of photon-echo QMs will be interesting for quantum manipulations of the light fields, in particular for the purposes of quantum compression [39,42] and frequency conversion [36]. We also anticipate considerable advantages of the AMR procedure for the Raman-echo QM on surface plasmon polariton fields, which is very promising for nanoscale storage of the light fields [43].

Finally, we can compare some basic properties of the proposed AMR protocol with the properties of other variants of the photon-echo quantum memory. As discussed earlier, the AMR protocol can be characterized by higher optical depth and, therefore, higher quantum efficiency, in comparison with the CRIB or AFC protocols for the same parameters of the atomic system. We also note that the time-bandwidth factor of the AMR protocol can be larger in comparison with the factor of CRIB protocol due to the use of a larger linewidth of the Raman transition, which can be comparable with the original inhomogeneous broadening ($\Delta_{\text{in}}^{R} \sim \Delta_{\text{in}}^{31}$). Similarly, we get an advantage of AMR protocol use for the storage capacity of many temporal light modes. It is worth noting that special studies should be done for the necessity of the modified AMR protocol, in comparison with its simplified version in the quantum storage of some special classes of the signal light fields. The noise fields are irradiated spontaneously in all spatial directions and are spread over the larger temporal duration in comparison with the echo field pulses. Thus,the direct noise influence on the echo field pulses can be partially suppressed even by the use of the temporal and spatial selection of the echo signal detection. Moreover, some special experimental methods can be applied for additional suppression of the spontaneous Raman transitions during the AMR of the excited atomic coherence. For example, it is interesting to exploit the dynamically tuned QED cavities [44] and external quasistatic magnetic (and electrical) fields for controlling the strengths of the optical (Raman) transitions. In short, the use of the modified AMR protocol would be important for highly protected storage of the multimode quantum light fields. A complete analysis of the extra noise problem should be the subject of a more detailed, intensive investigation for two proposed schemes of the AMR protocol in which a number of possible sources of the extra quantum noise in the real experimental conditions should also be taken into account, similarly to a recent analysis [45] of the well-known variants of the photon-echo quantum memory.

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