Addendum to "Single-photon logic gates using minimum resources"

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The authors call attention to a previous work [Lin and He, Phys. Rev. A **80**, 042310 (2009)] on the realization of multiqubit logic gates with controlled-path and merging gates. We supplement the work by showing how to efficiently implement quantum algorithms in this approach and by providing guide rules for the task.

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A core element for a quantum computer is a quantum logic gate. The realization of photonic logic gates is one of the main directions in the research of optical quantum computing. Compared with the linear optical approach [1,2], deterministic photonic gates enjoy the advantages of efficiency and simplicity. The realizations of such gates are proposed mostly with nonlinear optical processes, such as cross-phase modulation (XPM) in Kerr media [3]. In this approach, qubits in the basis $\{|0\rangle,|1\rangle\}$ can simply be encoded with photon number, polarization, or spatial modes of single photons, and a gate operation dispenses with complicated ancilla photonic states. For example, an XPM between two photons directly implements a controlled phase gate with the transformation $|1\rangle_1 \otimes |1\rangle_2 \rightarrow e^{i\theta}|1\rangle_1 \otimes |1\rangle_2$ (with $|0\rangle_1 \otimes |0\rangle_2$, $|0\rangle_1 \otimes |1\rangle_2$ and $|1\rangle_1 \otimes |0\rangle_2$ being unchanged). However, due to technical challenges, a value of θ on the order of a π radian could be difficult to come by. One feasible substitute is the weak nonlinearity approach [4,5]. It is to couple coherent states $|\alpha\rangle$ of a large amplitude $|\alpha|$ to single photons for the transformation $|\alpha\rangle_1|1\rangle_2 \rightarrow |\alpha e^{i\theta}\rangle_1|1\rangle_2$, where θ could be rather small. Then, the processed coherent states are measured for projecting out the proper output states of the single photons, which should be obtained by gate operation. Recently, we developed the approach further by proposing an architecture using two element gates—controlled-path (C-path) and merging gates—for single-photon logic gates [6]. Multiple qubit control gates, such as the Fredkin and the Toffoli gates, which recently were under extensive studies [7–14], can efficiently be realized by the combinations of the two element gates. This Brief Report supplements the previous study from the view of constructing the realistic circuits for implementing quantum algorithms.

A quantum circuit consists of various ingredients (e.g., single-qubit gates, two-qubit gates, multicontrol gates, etc.) Individually, all control gates involving more than two qubits can be realized with pair(s) of C-path and merging gates, together with the necessary single- qubit gates [6]. A prominent feature in a realistic quantum circuit is that more than one control operation could be acted on a particular target qubit, so the photon to encode the target qubit should be separated by C-path gates and merged by merging gates again and again if we straightforwardly apply the elementary gates to implement a circuit operation. Actually, such repetition can be saved if one modifies C-path and merging gates a little bit. Then, a target photon could be merged only after all control operations have been performed on it, thus, greatly simplifying circuit structure by reducing the number of the merging gates. Such simplification is particularly relevant to circuit operations involving a large number of qubits (e.g., the implementation of quantum algorithms). To fulfill the simplification, a new element—the eraser for eliminating the unwanted photon correlations between the successive C-path gate operations, should be introduced, as we will explain in the following.

The control photon in an original C-path or merging gate carries only one spatial mode [6,15]. Here, we make a modification in the design so that a photon with more than one spatial mode could control the path of another photon (a similar modification in a special case is given in Ref. [16]). In Fig. 1(a), we suppose that the input state (resulting from the action of the previous logic gates) for the gate is

$$|\psi\rangle_{CT}^2 = |H\rangle_{C_1}|\phi_1\rangle_T + |H\rangle_{C_2}|\phi_2\rangle_T + |V\rangle_{C_1}|\phi_3\rangle_T + |V\rangle_{C_2}|\phi_4\rangle_T, \quad (1)$$

where C_1, C_2 denote the different spatial modes of the control photon, H, V denote the polarization modes, and the components of the target photon are $|\phi_i\rangle = \alpha_i|H\rangle + \beta_i|V\rangle$, where $\sum_{i=1}^4(|\alpha_i|^2 + |\beta_i|^2) = 1$. The special forms of such inputs with $\alpha_{1,3} = \beta_{1,3} = 0$ or $\alpha_{2,4} = \beta_{2,4} = 0$ can be processed by the original C-path gate in Refs. [6,15,16]. We first use a 50:50 beam splitter (BS) to divide the target photon into two different spatial modes:

$$\frac{1}{\sqrt{2}}[|H\rangle_{C_1}(|\phi_1\rangle_1 + |\phi_1\rangle_2) + |H\rangle_{C_2}(|\phi_2\rangle_1 + |\phi_2\rangle_2) + |V\rangle_{C_1}(|\phi_3\rangle_1 + |\phi_3\rangle_2) + |V\rangle_{C_2}(|\phi_4\rangle_1 + |\phi_4\rangle_2)], \quad (2)$$

where the indices 1 and 2 denote two different paths. And then, following the coupling patterns of XPM in Fig. 1(a) [i.e., the first (second) coherent state $|\alpha\rangle$ is coupled to mode 1 (2) of the target and V (H) mode on both C_1 and C_2 for the control photon], we will obtain the following total state (the global coefficient is neglected):

$$\begin{aligned}&|H\rangle_{C_1}(|\phi_1\rangle_1|\alpha e^{i\theta}\rangle|\alpha e^{i\theta}\rangle + |\phi_1\rangle_2|\alpha\rangle|\alpha e^{2i\theta}\rangle)\\&+|H\rangle_{C_2}(|\phi_2\rangle_1|\alpha e^{i\theta}\rangle|\alpha e^{i\theta}\rangle + |\phi_2\rangle_2|\alpha\rangle|\alpha e^{2i\theta}\rangle)\\&+|V\rangle_{C_1}(|\phi_3\rangle_1|\alpha e^{2i\theta}\rangle|\alpha\rangle + |\phi_3\rangle_2|\alpha e^{i\theta}\rangle|\alpha e^{i\theta}\rangle)\\&+|V\rangle_{C_2}(|\phi_4\rangle_1|\alpha e^{2i\theta}\rangle|\alpha\rangle + |\phi_4\rangle_2|\alpha e^{i\theta}\rangle|\alpha e^{i\theta}\rangle).\end{aligned} \quad (3)$$

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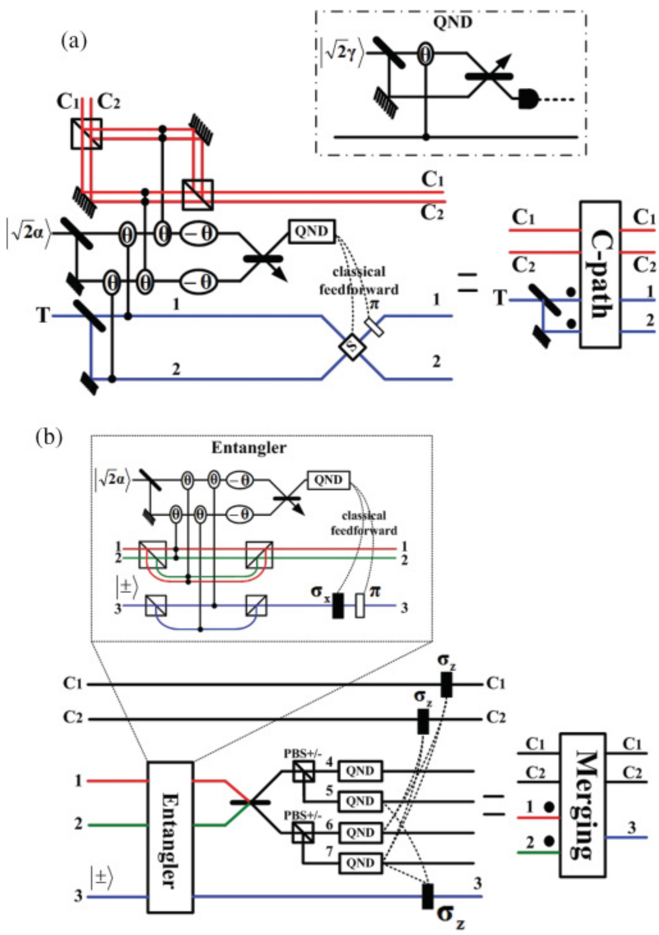

FIG. 1. (Color online) (a) Layout for a modified C-path gate. The control photon has two different spatial modes (denoted by a black dot), which will interact with two other spatial modes of the target photon through Kerr media. The XPM phases are θ, and a phase shift $-\theta$ is applied to the two coherent states, respectively. The part in the dashed-dotted line shows the structure of the quantum-nondemolition measurement (QND) module. S denotes a switch operation, and π denotes a phase shift of such a value. $|\sqrt{2}\alpha\rangle$ and $|\sqrt{2}\gamma\rangle$ are coherent states. (b) Layout of the corresponding merging gate, which implements the inverse operation for the C-path gate.

After a phase shifter $-\theta$ and a 50:50 BS implementing the transformation $|\alpha_1\rangle|\alpha_2\rangle \rightarrow |\frac{\alpha_1-\alpha_2}{\sqrt{2}}\rangle|\frac{\alpha_1+\alpha_2}{\sqrt{2}}\rangle$ are performed on the two ancilla beams, the eight terms in the preceding equation can be projected into two groups of output states by a photon number projection $|n\rangle\langle n|$ on one of the output beams, which could be in the state $|\beta\rangle = |\pm\sqrt{2}\alpha\sin\theta\rangle$ or $|0\rangle$. By the path switch S and a phase shift π on the target photon, which is conditioned on the measurement results $n \neq 0$, the two-photon state from both of the groups can be transformed to

$$|\phi\rangle = |H\rangle_{C_1}|\phi_1\rangle_1 + |H\rangle_{C_2}|\phi_2\rangle_1 + |V\rangle_{C_1}|\phi_3\rangle_2 + |V\rangle_{C_2}|\phi_4\rangle_2, \tag{4}$$

thus, realizing a deterministic control of the target photon's paths by the polarizations (H and V) of the control photon carrying two spatial modes. The projection $|n\rangle\langle n|$ is implemented by a QND module, in which a beam in the state $|\gamma\rangle$ (where $|\gamma|$ is large) is coupled to the above-mentioned output ancilla beam through an XPM implementing the transformation $|\beta\rangle|\gamma\rangle \rightarrow e^{-(1/2)|\beta|^2}(|0\rangle|\gamma\rangle + \beta|1\rangle|\gamma e^{i\theta}\rangle + \cdots)$. Even if $\theta \ll 1$, the output coherent states $|\frac{\gamma e^{in\theta}\pm\gamma}{\sqrt{2}}\rangle$ can still be well separated with respect to their Poisson distributions of photon numbers, given a sufficiently large $|\gamma|$. Thus, a number of nonresolving detectors even without high detecting efficiency or a quadrature measurement can indirectly realize the deterministic photon number resolving detection corresponding to $|n\rangle\langle n|$ (see Ref. [16] for the details). The progress on the physical realization of such XPM-based QNDs refers to, for example, Refs. [17–20]. The number of the controlling spatial modes for the C-path gate can be straightforwardly generalized to larger than 2.

Similarly, we can modify a merging gate, which performs the inverse operation of the previous C-path gate, see Fig. 1(b). By such a merging gate with multispatial control modes, the output state in Eq. (4) can be transformed to

$$|H\rangle_{C_1}|\phi_1\rangle_3 + |H\rangle_{C_2}|\phi_2\rangle_3 + |V\rangle_{C_1}|\phi_3\rangle_3 + |V\rangle_{C_2}|\phi_4\rangle_3 \tag{5}$$

(i.e., the merging of the target photon modes on paths 1 and 2 to path 3).

Now, we will put the element gates together to build a quantum circuit. Without loss of generality, we illustrate the architecture by a three-qubit circuit shown in the dashed-dotted line of Fig. 2. The input state for the circuit is (the global coefficient is neglected)

$$|\psi_{in}\rangle = d_1|HHH\rangle + d_2|HHV\rangle + \cdots + d_8|VVV\rangle, \tag{6}$$

which could either be entangled or not be entangled. Three controlled unitary (CU) operations and one Toffoli operation will be performed on the state. In the space where the qubits are encoded with the polarization modes of single photons, the CU operations are represented by the operators $|H\rangle\langle H| \otimes \mathbb{I} + |V\rangle\langle V| \otimes U_i$, where $\mathbb{I} = |H\rangle\langle H| + |V\rangle\langle V|$ and $i = 1,2,4$. The Toffoli gate performs the operation $(\mathbb{I}\otimes\mathbb{I} - |VV\rangle\langle VV|)\otimes\mathbb{I} + |VV\rangle\langle VV| \otimes U_3$.

The first CU gate can be straightforwardly realized with a C-path gate plus a single-qubit operation U_1. Before the implementation of the second CU operation involving photons b and c, we do not merge the spatial modes of photon b, and, instead, we directly use them to control the operation on the third photon in the next gate operation. The second CU operation can be implemented by a generalized C-path gate in Fig. 1(a), associated with the single-photon operation U_2 performed on the spatial mode c_2.

The triple-photon state, after being processed by the first two CU gates, is in the form

$$|H\rangle_a|H\rangle_{b_1}|\phi_1\rangle_{c_1} + |V\rangle_a|H\rangle_{b_2}|\phi_2\rangle_{c_1} + |H\rangle_a|V\rangle_{b_1}|\phi_3\rangle_{c_2} + |V\rangle_a|V\rangle_{b_2}|\phi_4\rangle_{c_2}, \tag{7}$$

where the specific forms of $|\phi_i\rangle_{c_j}$ are determined by the operations U_i, V_i and the coefficients d_i of the input state. The polarization modes of photon b are entangled with the spatial modes of photon c in the foregoing expression ($|H\rangle$ of photon b is always in the same terms with the spatial mode 1 of photon c, etc.). However, a proper state to be processed by the next C-path gate should be in the form of Eq. (2), where each polarization mode (irrespective of its spatial mode) for the control photon is in a tensor product with the superposition

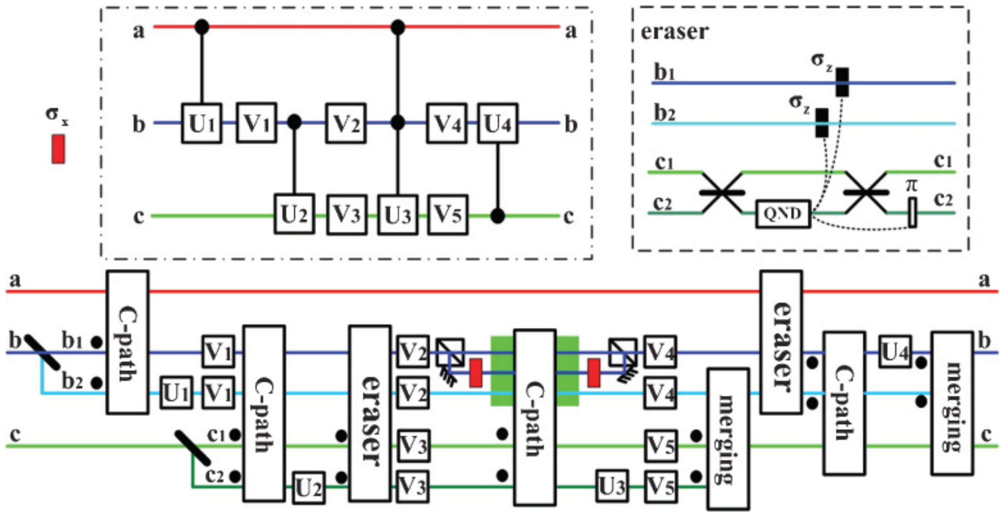

FIG. 2. (Color online) Example of a triple-qubit gate. The circuit in the dashed-dotted line is implemented by the modified C-path and merging gates. A necessary ingredient for the implementation is the eraser shown in the dashed lines.

of both spatial mode terms of the target photon. Therefore, it is necessary to eliminate such unwanted correlation between the polarizations of photon b and the spatial modes of photon c.

We introduce a circuit ingredient illustrated in the dashed lines of Fig. 2 for this purpose. Here, the QND module is the same as previously described. After the interference of the spatial modes c_1 and c_2 by a 50:50 BS, a QND module projects the state to

$$|HH\rangle_{a,b_1}|\phi_1\rangle_{c_1} + |VH\rangle_{a,b_2}|\phi_2\rangle_{c_1} + |HV\rangle_{a,b_1}|\phi_3\rangle_{c_1} + |VV\rangle_{a,b_2}|\phi_4\rangle_{c_1}$$

or

$$|HH\rangle_{a,b_1}|\phi_1\rangle_{c_2} + |VH\rangle_{a,b_2}|\phi_2\rangle_{c_2} - |HV\rangle_{a,b_1}|\phi_3\rangle_{c_2} - |VV\rangle_{a,b_2}|\phi_4\rangle_{c_2}. \quad (8)$$

Then, one more BS, as well as the conditional operation σ_z on both spatial modes of photon b and a conditional phase shift π on mode c_2, depending on the classically feedforwarded detection results of the QND, will be applied. The whole operation will result in the state,

$$\frac{1}{\sqrt{2}}[|HH\rangle_{a,b_1}(|\phi_1\rangle_{c_1} + |\phi_1\rangle_{c_2}) + |VH\rangle_{a,b_2}(|\phi_2\rangle_{c_1} + |\phi_2\rangle_{c_2}) + |HV\rangle_{a,b_1}(|\phi_3\rangle_{c_1} + |\phi_3\rangle_{c_2}) + |VV\rangle_{a,b_2}(|\phi_4\rangle_{c_1} + |\phi_4\rangle_{c_2})], \quad (9)$$

which is similar to the form of the input in Eq. (2). We call this circuit ingredient an eraser.

Now, the correlation between the polarization and the spatial modes of photons a and b still exists [i.e., mode 1 of photon b is always in the same terms as the H mode of photon a, etc., see Eq. (9)]. This happens to be an advantage for implementing the following Toffoli gate. Then, a C-path gate only by photon b's polarizations on its all-spatial paths will control the path of photon c, thus, realizing a Toffoli gate with a simplified structure from that in Ref. [6].

It is not necessary to erase the correlation between the photons immediately after implementing the Toffoli gate, but the spatial modes of the third photon should be merged by a merging gate. There will be no control operation to be performed on this photon (photon c). The merging of its spatial modes will simplify the further control operation by itself. Before the final CU operation, one more eraser should be applied to eliminate the unwanted correlation between photons a and b.

The imperfections of the circuit operation could arise from losses of the photonic states. The decoherence effects on the photonic states due to the losses in XPM and transmission are studied with the corresponding master equations in Refs. [21–23]. In the implementation of the earlier circuit, the losses will mainly come from the XPM between single-photon and coherent states if there is a close-to-ideal performance of the linear optical components. In Ref. [21], it is shown that an XPM in lengthy optical fiber is impossible to avoid considerable decoherence, but the acceptable fidelities (with the ideal pure output states) could be achievable with the reasonable system parameters for the XPM in media under electromagnetically induced transparency conditions. A feasible realization of the XPM between coherent and single-photon states still awaits to be clarified further for building such photonic circuits.

The generalization of the circuits involving more qubits is straightforward. Here, we give two examples for the implementation of quantum algorithms. The first is Grover's searching algorithm [24], which could be implemented with $O(n^2)$ two-qubit gates [25]. The essential part of the algorithm is the following operation:

$$U_s = 2|s\rangle\langle s| - \mathbb{I} = H^{\otimes n}(2|0\rangle\langle 0| - \mathbb{I})H^{\otimes n}, \quad (10)$$

where $\mathbb{I}$ is the identity operator for n qubits, and $|s\rangle = H^{\otimes n}|0\rangle(H^{\otimes n}$ denotes n Hadamard operations on n qubits, respectively). The operator $2|0\rangle\langle 0| - \mathbb{I}$ in the foregoing equation is an $n-1$-control Toffoli-σ_z gate (i.e., logic 1 of the first $n-1$ qubits conditions the operation σ_z on the nth qubit). Therefore, as a simple generalization from the two-qubit Toffoli gate in Fig. 2, it can be implemented with $n-1$ pairs of C-path and merging gates. Since the paths of the kth photon to encode the qubit are only controlled by the polarization modes of the $k-1$th photon, there are, at most, $5 + 7(n-2)$

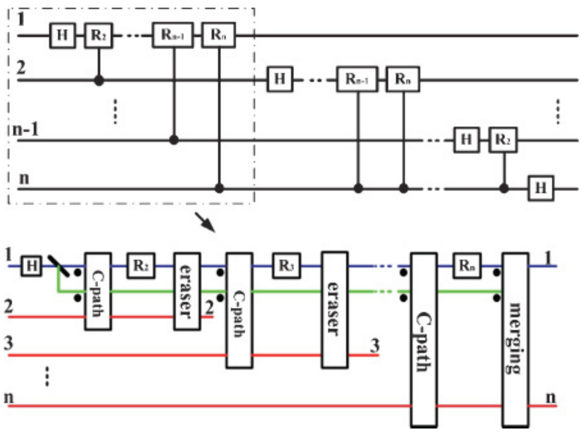

FIG. 3. (Color online) (Upper) QFT. The QFT circuit consists of a series of controlled rotations [27]. (Lower) The realization of the part in the dashed-dotted line with the modified C-path, merging gates, together with erasers and single-qubit gates.

XPM operations including those in the QND modules for all C-path gates. Together with those of the merging gates, the total number of XPM operations should be $18n - 20$, scaling linearly with the number of the involved qubits. Note that erasers are not necessary in this case. With the QND modules in the merging gates for preserving ancilla photons, in principle, only one ancilla photon will be required for implementing the searching algorithm.

Another example is Shor's factoring algorithm [26]. Quantum Fourier transformation (QFT) shown in the upper part of Fig. 3 is the crucial part of the algorithm. A QFT circuit consists of a series of qubit rotations R_i controlled by other qubits. It can be realized with a regular combination of C-path and merging gates; see the lower part of Fig. 3. A general QFT circuit involving n qubits consists of $n(n-1)/2$ controlled rotations. Therefore, it demands $n(n-1)/2$ C-path gates, $(n-1)(n-2)/2$ erasers, and $n-1$ merging gates for the realization in our architecture. Meanwhile, two points should be paid attention to in constructing such a circuit:

(1) Before implementing a two-qubit control gate, the correlation between the target photon and the previous control photon should be erased.

(2) If no further control operation is to be performed on a photon, its spatial modes should be merged with a merging gate.

Following these two rules, it is convenient to construct any quantum circuit with the modified C-path and merging gates. If one adopts the routine decomposition strategy into two-qubit and single-qubit gates for the realization of a quantum circuit (see, e.g., Ref. [27]), the decomposition is generally irregular, and the design could be rather complicated. Our architecture, following the simple rules of combining C-path, merging gates, as well as erasers, considerably reduces such complexity.

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