Convergent conductivity corrections to the Casimir force via exponential basis functions

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A closed-form finite conductivity correction factor for the ideal Casimir force is proposed, based on exponential basis functions. Our method can facilitate experimental verifications of theories in the study of the Casimir force. A theoretical analysis is given to explain why our method is accurate at both large and small separation gaps. Numerical computations have been performed to confirm that our method is accurate in various experimental configurations. Our approach is widely applicable to various Casimir force interactions between metals and dielectrics. Our study can be extended to the study of the repulsive Casimir force as well.

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I. INTRODUCTION

The Casimir force was first predicted to exist between two electrically neutral conductive planes placed in a vacuum [1]. It has since attracted strong interest from both theoreticians and experimentalists in elementary-particle physics, gravitation and cosmology, and atomic and condensed-matter physics [2]. It has also found many applications in microelectromechanical systems (MEMS) such as separation gap sensors [3,4]. The Casimir force results from the reduced density of the electromagnetic fields between two conductive planes. In Casimir's work, it was calculated that the Casimir force between two parallel, perfectly conductive planes is attractive and its strength is inversely proportional to the fourth power of the separation gap between them [1]. In practice, real materials are not perfectly conductive. To calculate the Casimir force between two real materials, Lifshitz theory is developed and the calculation is based on the dielectric properties of the interacting materials [5]. In the past decade, extensive experiments have been carried out to verify theories and hypotheses related to the Casimir force [6]. In order to obtain precise agreement between theories and experimental results, real experimental conditions like finite conductivity and surface roughness must be accounted for [7].

To facilitate the reconciliation of experimental data with theory, it is necessary to include nonideal correction factors due to the finite conductivity and surface roughness as multiplication factors to the ideal Casimir force in closed forms [8]. This is especially critical in dynamic Casimir force measurements [3,9], as computational loads increase tremendously. Compared to static force measurements, dynamic force measurements are robust to noise and drift and are considered to be a better choice in weak force measurements at nanometer scale [10]. Thus, dynamic force measurements are expected to play an important role in measuring the repulsive Casimir force, which is much weaker than the attractive Casimir force between metals [11]. While it is desirable to obtain correction factors in closed forms, it is impossible to obtain explicit expressions for the nonideal finite conductivity correction factor from Lifshitz theory [12]. To resolve this dilemma, polynomial-based perturbation approaches have been applied to approximate the implicit conductivity correction factor to the ideal Casimir force between the same metallic surfaces at zero temperature [13]. The results are simple and have been extended to the conductivity correction factor when the interacting metals are different and when the temperature is nonzero [14,15]. This perturbation approach has also been applied to calculate the conductivity correction factor to the ideal Casimir force between a metallic surface and a dielectric surface [16]. Although the perturbation approaches are highly accurate when the separation between interacting materials is larger than the plasma wavelength of the metals, which is in the range of 100–150 nm for metals like Au [13], discrepancies between the computational results from perturbation approaches and the Lifshitz theory increase dramatically when interacting materials are at small separations (for example, 60 nm). While it is true that the accuracy at small separations can be improved by increasing the order of polynomials [17], the improvement is quite limited and bigger discrepancies will result when the separation between interacting materials continues to decrease. Since many measurements are carried out at separation gaps that are below 100 nm [18,19], there is a strong need to propose an explicit form of the conductivity correction factor with high accuracy for both large and small separation gaps.

In Sec. II, we will present a detailed analysis to explain why discrepancies are significant at small separations when perturbation approaches based on polynomials are applied. Then, in Sec. III, we will present a systematic method to construct explicit conductivity corrections to the Casimir force by using exponential basis functions when interacting materials are of the same metals at zero temperature. In Sec. IV, we give reasons to explain theoretically why approximations based on exponential basis functions are much more accurate. In Sec. V, we demonstrate that our method can be modified to calculate the conductivity factor when the interacting materials are different at zero temperature. As an example, we show in the numerical calculations that our closed-form conductivity corrections to the ideal Casimir force between metallic surfaces are simple and precise in several configurations of experiments. Although the temperature correction on the Casimir force is small when the separation gap a is <1 μm, where most of the Casimir force experiments are carried out, increasingly more interesting Casimir phenomena are predicted at a high temperature and a

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large distance [20]. Our method can be applied to conductivity corrections at a nonzero temperature as well. We will explain briefly in the Sec. VI how one may apply our method to obtain the finite conductivity corrections at a nonzero temperature.

II. THEORETICAL ANALYSIS

In this section, we will explain theoretically why huge errors are produced when computing the finite conductivity correction to the Casimir force by using polynomial-based perturbation approaches, especially when the separation gap is small. As an example, we study the case in which the interacting materials are of the same metal at zero temperature. It can be calculated from Lifshitz theory that the following relationships between the conductivity correction factor r and δ_0/a are valid when interacting materials are of the same metal at zero temperature:

$$\lim_{(\delta_0/a)\to 0} r = 1, \tag{1}$$

$$\lim_{(\delta_0/a)\to +\infty} r = 0, \tag{2}$$

where $\delta_0 = \lambda_p/2\pi$ is the effective penetration depth of the metal and λ_p is the plasma wavelength of the metal. The nth-order conductivity correction by using the conventional perturbation approach is expressed as

$$\hat{r} = 1 + \sum_{i=1}^{n} b_i (\delta_0/a)^i. \tag{3}$$

It is noted in Ref. [13] that the conventional perturbation approach is very accurate when the separation gap a is large. At large separation gap, the higher-order conductivity correction is more accurate than the lower-order one. However, the accuracy of this approach is degraded gradually with increasing δ_0/a. When the separation gap a between Au- or Al-coated materials decreases further to <100 nm, the errors suddenly become large.

The phenomena can be explained by using Taylor series [21,22]. The implicit function $r(\delta_0/a)$ can be expanded by using Taylor series centered at $\delta_0/a = 0$:

$$r = 1 + \sum_{i=1}^{+\infty} b_i (\delta_0/a)^i. \tag{4}$$

Compared to the Taylor-series expansion in Eq. (4), higher-order terms are truncated and only lower-order terms are preserved in the conventional perturbation approach in Eq. (3). When a is large and δ_0/a is small, lower-order terms in Eq. (4) dominate. Higher-order terms are negligible, and thus the conventional perturbation approach is accurate. The computation results will be increasingly more accurate if more higher-order terms are included in Eq. (3). When a is small, δ_0/a is large and the contribution from higher-order terms in Eq. (4) becomes significant. This will result in large computation errors, because these higher-order terms are truncated in Eq. (3). For our finite conductivity problem, the Taylor-series expansion does not converge when δ_0/a is far away from the center $\delta_0/a = 0$. It is easy to verify that $\lim_{(\delta_0/a)\to+\infty} \hat{r} = b_n(\delta_0/a)^n$, which means $\hat{r}$ will approach infinity when a approaches zero. This is completely different from the computational results based on Lifshitz theory [Eq. (2)], and the difference increases with increased order n. This explains why huge errors are produced at small separations. To minimize the residue errors from higher-order terms truncated in closed-form approximations, basis functions other than polynomials are needed.

The rationale for choosing exponential basis functions to replace polynomials is to mimic the true conductivity correction factor calculated from Lifshitz theory. It is observed in Ref. [13] that the relationship between r and δ_0/a, when interacting metals are the same at zero temperature, is similar to an exponential function. Thus, residue errors can be significantly reduced when the finite conductivity is computed based on exponential basis functions.

III. PROPOSAL OF FINITE CONDUCTIVITY CORRECTIONS BASED ON EXPONENTIAL BASIS FUNCTIONS

In this section, we propose an nth-order closed-form approximation for r by using a series of exponential basis functions when the interaction surfaces are of the same metal at zero temperature:

$$\hat{r} = \sum_{i=1}^{n} b_i F_i(\delta_0/a), \tag{5}$$

where $F_i(x) = e^{-c_i x} x^{2i-2}$. $F_i(x)$ is chosen to mimic the implicit r that satisfies the following equations: $\lim_{(\delta_0/a)\to 0} F_i = 1$ and $\lim_{(\delta_0/a)\to+\infty} F_i = 0$. Rather than Taylor-series expansion, the true conductivity correction r can be expressed in the following way:

$$r = \sum_{i=1}^{+\infty} b_i F_i(\delta_0/a). \tag{6}$$

When δ_0/a is large, $F_i(\delta_0/a)$ is small and approaches zero. Thus, the discrepancy between r and $\hat{r}$ from the truncated higher-order terms is much smaller when δ_0/a is large. Although the aim of proposing the alternative basis functions is to reduce the discrepancy when a is either large or small, it is expected that this discrepancy will still grow gradually with increasing δ_0/a. To achieve reduced discrepancy, $b_1 = 1$ is set to satisfy Eq. (1). Other parameters b_n and c_n are determined sequentially to satisfy

$$\left.\frac{\partial^i r}{\partial x^i}\right|_{x=0} = \left.\frac{\partial^i \hat{r}}{\partial x^i}\right|_{x=0} \quad (i = 1,2,\ldots,2n-1), \tag{7}$$

where $x = \delta_0/a$.

IV. FINITE CONDUCTIVITY CORRECTIONS TO THE IDEAL CASIMIR FORCE BETWEEN TWO SAME METALS AT ZERO TEMPERATURE

In this section, we will compute the finite conductivity corrections to the ideal Casimir force between two same metals

at zero temperature by using our proposed basis functions. Various configurations of experiments will be considered. We will also compare the performance of our method with the conventional perturbation approach.

A. Plate-plate configuration

We will first calculate the finite conductivity corrections to the ideal Casimir force between two parallel plates made of the same metal at zero temperature. The ideal Casimir force per unit area between two perfectly reflective metals, when their separation is small as compared to their size, is given as $F_{p\,\mathrm{ideal}}(a) = -(\pi^2\hbar c)/(240a^4)$. The Casimir force between two same materials with dielectric permittivity $\varepsilon(\omega)$ is calculated as follows [13,23]:

$$F_p(a) = \frac{-\hbar c}{32\pi^2 a^4}\int_0^{+\infty} x^3 dx \int_1^{+\infty}\frac{dp}{p^2} \times\left[\left(\frac{(s+p\varepsilon)^2}{(s-p\varepsilon)^2}e^x-1\right)^{-1}+\left(\frac{(s+p)^2}{(s-p)^2}e^x-1\right)^{-1}\right], \tag{8}$$

where $s=\sqrt{\varepsilon-1+p^2}$, and $\varepsilon(i\xi)=\varepsilon(icx/2pa)$ is the dielectric permittivity on the imaginary frequency axis $\omega=i\xi$. The plasma model is often used to describe the dielectric permittivity of the metal [13]:

$$\varepsilon(i\xi) = 1 + c^2/(\delta_0\xi)^2. \tag{9}$$

Then, the true finite conductivity correction r can be calculated as $r = F_p(a)/F_{p\,\mathrm{ideal}}(a)$. Our first-order finite conductivity correction based on exponential basis functions is obtained as

$$\hat{r}_{1p} = \exp\left(-\frac{16\delta_0}{3a}\right). \tag{10}$$

In a similar way, our second-order finite conductivity correction based on exponential basis functions is given by

$$\hat{r}_{2p} = \exp\left(-\frac{16\delta_0}{3a}\right) + \frac{88}{9}\exp\left[-\left(\frac{4688}{693}-\frac{24\pi^2}{539}\right)\frac{\delta_0}{a}\right]\left(\frac{\delta_0}{a}\right)^2. \tag{11}$$

The finite conductivity correction factors from the various approaches are computed for the configuration of two parallel Au plates when the separation gap a is between 60 and 400 nm. $\delta_0 = 137/(2\pi)$ nm is taken for Au [24]. It can be seen in Fig. 1 that the results from the conventional perturbation approach and our approach are close to the true finite conductivity correction obtained from Lifshitz theory when the separation gap a is large. However, the conventional perturbation approach becomes inaccurate when the separation gap a is <100 nm. By contrast, our approximations, especially with our second-order conductivity correction, are very accurate even when $a = 60$ nm. More accurate results can be obtained if higher-order correction terms based on exponential basis functions are used.

In addition to the plasma model to describe the dielectric permittivity of the metal, our method can also be applied when the dielectric permittivity of the metal is described by the Drude model, generalized plasma-type models, or real optical data [24]. To guarantee an accurate approximation, coefficients b_i and c_i need to satisfy Eq. (7).

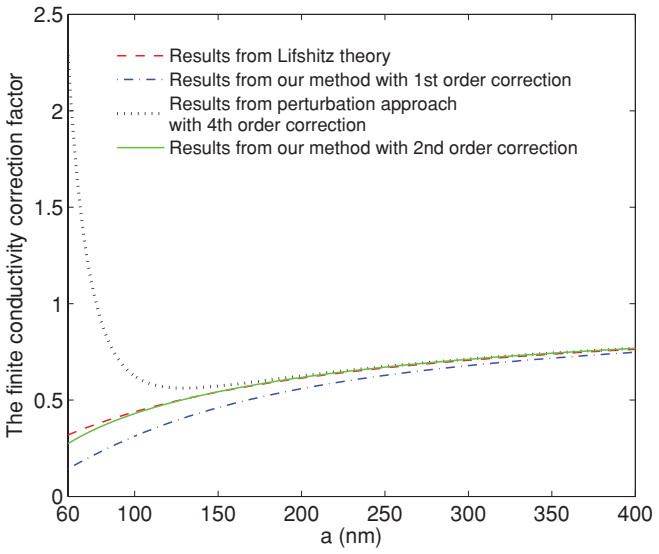

FIG. 1. (Color online) Finite conductivity correction factors with respect to a for two parallel Au plates at zero temperature. Results from Lifshitz theory are shown by the red dashed line. Results from fourth-order correction based on the conventional perturbation approach (Ref. [13]) are shown by black dots. Results from our first-order correction based on exponential basis functions are shown by the blue dash-dotted line. Results from our second-order correction based on exponential basis functions are shown by the green solid line.

B. Sphere-plate configuration

To avoid the difficulty of the plate alignment in the plate-plate configuration, many experiments were carried out in the sphere-plate configuration [25–27]. Thus, it is also of interest and importance to study the finite conductivity correction in this scenario. The ideal Casimir force between a sphere and a plate made of the same material when their separation is small compared to the sphere radius and the plate size is given as $F_{s\,\mathrm{ideal}}(a) = -(\pi^3\hbar c R)/(360a^3)$, where R is the sphere radius. The Casimir force acting between two same real materials with dielectric permittivity $\varepsilon(\omega)$ is calculated as [13]

$$F_s(a) = \frac{\hbar c R}{16\pi a^3}\int_0^{+\infty} x^2 dx \int_1^{+\infty}\frac{dp}{p^2} \times\left[\ln\left(1-\frac{(s-p\varepsilon)^2}{(s+p\varepsilon)^2}e^{-x}\right)+\ln\left(1-\frac{(s-p)^2}{(s+p)^2}e^{-x}\right)\right]. \tag{12}$$

The same plasma model is used for the dielectric permittivity of the metals. $\lambda_p = 137/(2\pi)$ nm is taken for Au as well. Our first- and second-order finite conductivity corrections based on

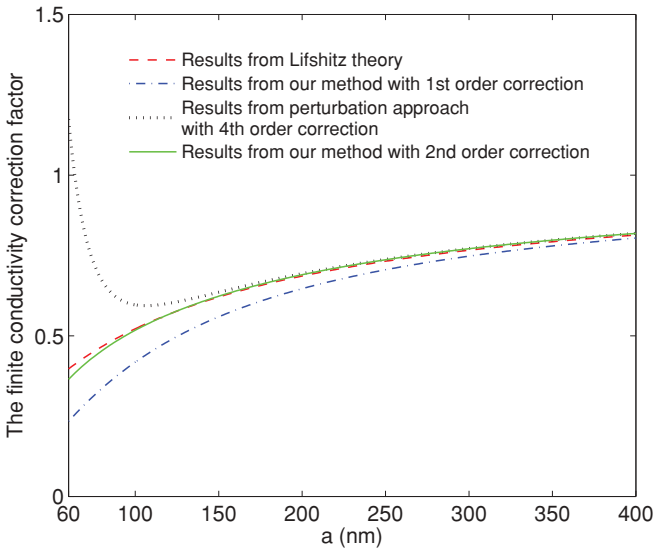

FIG. 2. (Color online) Finite conductivity correction factors with respect to a to the ideal Casimir force between a sphere and a plate made of Au at zero temperature. Results from Lifshitz theory are shown by the red dashed line. Results from fourth-order correction based on the conventional perturbation approach are shown by black dots. Results from our first-order correction based on exponential basis functions are shown by the blue dash-dotted line. Results from our second-order correction based on exponential basis functions are shown by the green solid line.

exponential basis functions are

$$\hat{r}_{1s} = \exp\left(-\frac{4\delta_0}{a}\right), \tag{13}$$

$$\hat{r}_{2s} = \exp\left(-\frac{4\delta_0}{a}\right) + \frac{32}{5}\exp\left[-\left(\frac{115}{21} - \frac{5\pi^2}{147}\right)\frac{\delta_0}{a}\right]\left(\frac{\delta_0}{a}\right)^2. \tag{14}$$

For comparison, we computed the finite conductivity correction factors for the ideal Casimir force between a sphere and a plate made of Au from the various approaches when the separation gap a is between 60 and 400 nm (Fig. 2). Similarly to the previous configuration, results from the conventional perturbation approach and our approach are near to the true finite conductivity correction obtained from Lifshitz theory when the separation gap a is large. The discrepancies increase with decreasing a. The fourth-order conductivity correction with the conventional perturbation approach becomes highly inaccurate when the separation gap a is below 100 nm. Our approximations, especially our second-order conductivity correction, still maintain high accuracy even when $a = 60$ nm.

V. FINITE CONDUCTIVITY CORRECTIONS TO THE IDEAL CASIMIR FORCE BETWEEN DIFFERENT METALS AT ZERO TEMPERATURE

Recently, there has been an increased interest in studying and measuring the Casimir force between two different materials [28–30]. This is important when studying the repulsive Casimir force, as the repulsive Casimir force can only be generated between different materials in a carefully chosen medium [31–33]. Thus, we shall also extend our method to calculate the finite conductivity corrections at zero temperature to the ideal Casimir force between different materials. Our method can also be applied to study the Casimir force interaction between a metal surface and a dielectric surface as well.

A. Plate-plate configuration

Here, we will extend our method to calculate the finite conductivity correction by using exponential basis functions between two parallel plates made of different metals at zero temperature. The Casimir force in this scenario is calculated as

$$F_{dp}(a) = \frac{-\hbar c}{32\pi^2 a^4}\int_0^{+\infty} x^3 dx \int_1^{+\infty}\frac{dp}{p^2} \times \{X_1(p,x) + X_2(p,x)\}, \tag{15}$$

where $X_1(p,x) = (\frac{(s_1+p\varepsilon_1)(s_2+p\varepsilon_2)}{(s_1-p\varepsilon_1)(s_2-p\varepsilon_2)}e^x - 1)^{-1}$ and $X_2(p,x) = (\frac{(s_1+p)(s_2+p)}{(s_1-p)(s_2-p)}e^x - 1)^{-1}$. s_k $(k = 1,2)$ are computed by $s_k = \sqrt{\varepsilon_k - 1 + p^2}$, and ε_k are the dielectric permittivities of the two metals that can be described by the plasma model in Eq. (9). In general, the effective penetration depths of the two metals δ_0 are different. Thus, it is not appropriate to apply our formula in Eq. (5) directly. Instead, our nth-order finite conductivity correction to the ideal Casimir force between two different metals is given as

$$\hat{r} = \sum_{i=1}^{n} b_i F_i(\delta/a), \tag{16}$$

where $\delta = (\delta_1 + \delta_2)/2$, with δ_1 and δ_2 the effective penetration depths of the two different interacting metals. Then, our first- and second-order corrections, when calculated to satisfy Eq. (7), are given as follows:

$$\hat{r}_{1dp} = \exp\left(-\frac{16\delta}{3a}\right), \tag{17}$$

$$\hat{r}_{2dp} = \exp\left(-\frac{16\delta}{3a}\right) + \frac{99}{8}\exp\left\{-\left[\frac{4688}{693} - \frac{24\pi^2}{539}(4 - 12\kappa)\right]\frac{\delta}{a}\right\}\left(\frac{\delta}{a}\right)^2, \tag{18}$$

where $\kappa = \delta_1\delta_2/(\delta_1 + \delta_2)^2$. Numerical simulations are carried out to calculate the finite conductivity correction at zero temperature to the Casimir force between Au and Al parallel plates. $\delta_1 = 137/(2\pi)$ nm is still assumed for Au and $\delta_2 = 100/(2\pi)$ nm is assumed for Al. The finite conductivity correction factors, computed from the various approaches when the separation gap a is between 60 and 400 nm, are plotted in Fig. 3. Similarly to the previous simulation results, our first- and second-order conductivity corrections give very accurate approximations over the whole range, while the fourth-order perturbation approach based on polynomials is accurate only when the separation gap is larger than 100 nm.

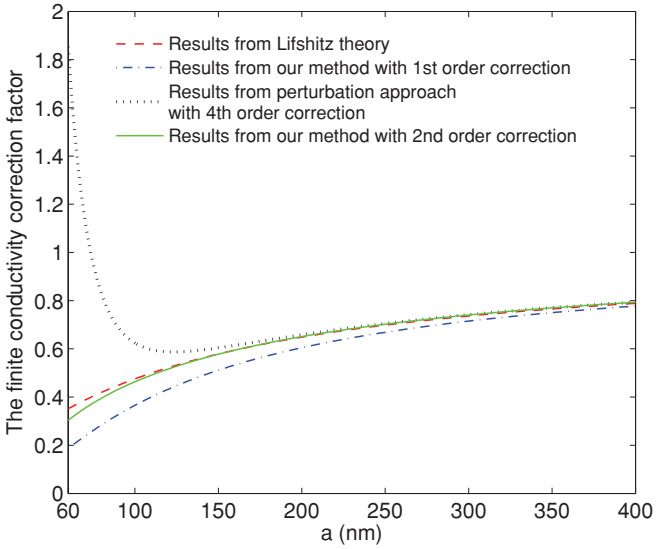

FIG. 3. (Color online) Finite conductivity correction factors with respect to a to the ideal Casimir force between an Au plate and an Al plate at zero temperature. Results from Lifshitz theory are shown by the red dashed line. Results from fourth-order correction based on the conventional perturbation approach are shown by black dots. Results from our first-order correction based on exponential basis functions are shown by the blue dash-dotted line. Results from our second-order correction based on exponential basis functions are shown by the green solid line.

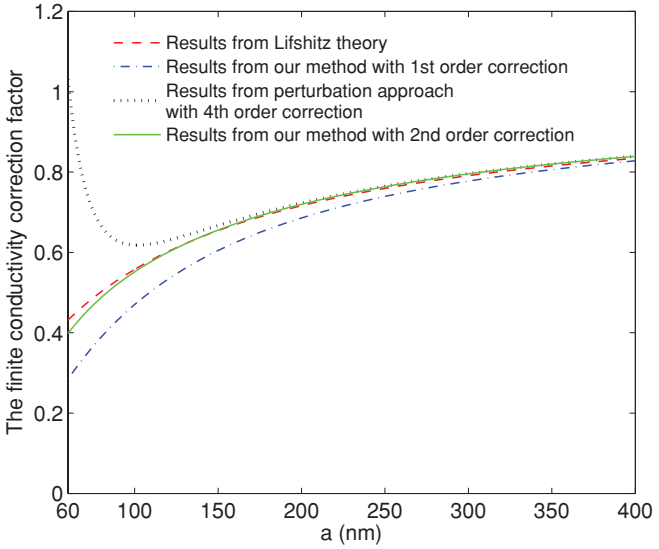

FIG. 4. (Color online) Finite conductivity correction factors with respect to a to the ideal Casimir force between an Au sphere and an Al plate at zero temperature. Results from Lifshitz theory are shown by the red dashed line. Results from fourth-order correction based on conventional perturbation approach are shown by black dots. Results from our first-order correction based on exponential basis functions are shown by the blue dash-dotted line. Results from our second-order correction based on exponential basis functions are shown by the green solid line.

B. Sphere-plate configuration

The Casimir force between a sphere and a plate made of different materials is calculated as

$$F_{ds}(a) = \frac{\hbar c R}{16\pi a^3}\int_0^{+\infty} x^2 dx \int_1^{+\infty}\frac{dp}{p^2} \times \{\ln[Y_1(p,x)] + \ln[Y_2(p,x)]\}, \quad (19)$$

where $Y_1(p,x) = 1 - \frac{(s_1-p\varepsilon_1)(s_2-p\varepsilon_2)}{(s_1+p\varepsilon_1)(s_2+p\varepsilon_2)}e^{-x}$ and $Y_2(p,x) = 1 - \frac{(s_1-p)(s_2-p)}{(s_1+p)(s_2+p)}e^{-x}$. s_k $(k=1,2)$ are computed by $s_k = \sqrt{\varepsilon_k - 1 + p^2}$, and ε_k are the dielectric permittivities of the two metals that can be described by the plasma model in Eq. (9). Our nth-order finite conductivity correction to the ideal Casimir force between two different metals is still in the form of Eq. (16). Our first- and second-order corrections are calculated as

$$\hat{r}_{1ds} = \exp\left(-\frac{4\delta}{a}\right), \quad (20)$$

$$\hat{r}_{2ds} = \exp\left(-\frac{4\delta}{a}\right) + \frac{32}{5}\exp\left\{-\left[\frac{115}{21} - \frac{5\pi^2}{147}(4-12\kappa)\right]\frac{\delta}{a}\right\}\left(\frac{\delta}{a}\right)^2. \quad (21)$$

Numerical simulations are carried out to calculate the finite conductivity correction at zero temperature to the Casimir force between an Au sphere and an Al plate. $\delta_1 = 137/(2\pi)$ nm is assumed for Au and $\delta_2 = 100/(2\pi)$ nm is assumed for Al. The finite conductivity correction factors, computed from various approaches when the separation gap a is between 60 and 400 nm, are plotted in Fig. 4. As expected, our first- and second-order conductivity corrections based on exponential basis functions give very accurate approximations over the whole range. The fourth-order perturbation approach based on polynomials gives unreliable approximations when the separation gap is smaller than 100 nm.

VI. DISCUSSIONS AND CONCLUSIONS

In this paper, we only considered the finite conductivity corrections at zero temperature. The finite conductivity corrections at a nonzero temperature deviate from the ones at zero temperature, but the deviation is very small when the separation gap a is below 1 μm [34]. This makes the measurement of this deviation difficult [35,36]. However, we have noticed strong research interest in the temperature effect recently [37], and our method can be adapted successfully to calculate the finite conductivity at a nonzero temperature as well. In this case, the coefficients b_i and c_i need to be redetermined, and $b_1 \neq 1$ in general. This is because the finite conductivity factor at a nonzero temperature is >1 when a is large enough. Other coefficients can be determined in the same way to satisfy Eq. (7).

The idea of our basis functions can be extended to approximate various kinds of repulsive Casimir forces generated by metamaterials [38] in closed forms. As mentioned, this will be extremely important for the weak repulsive Casimir force measurements. Other sets of basis functions than exponential functions may be needed to mimic the Casimir force over the

whole measurement range, which will be our future research interest.

In conclusion, we have presented an approach to calculating the finite conductivity corrections to the ideal Casimir force at zero temperature, based on exponential basis functions. A theoretical analysis was performed to explain why our approach will give more accurate results, especially when the separation gap a is small. Numerical simulations show that highly precise and convergent finite conductivity corrections are obtained based on our exponential basis functions in plate-plate and sphere-plate experimental configurations between the same and different metals. Furthermore, it is important to note that our method can be readily applied to other experimental configurations of the Casimir interactions between metals [39] and the Casimir interactions between a metal and a dielectric as well.

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