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Addendum to "Behavior of a bipartite system in a cavity"

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This note is an Addendum to our previous article [Phys. Rev. A **81**, 053820 (2010)]. We show that under the assumption of a Bose-Einstein distribution for the thermal reservoir, zero-temperature properties of the entangled states considered there are not changed by heating, for temperatures up to the order of room temperatures. In this case, the system is dissipative in free space and presents stability for a small cavity, both for $T = 0$ and for finite temperature.

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In this note we consider how heating could affect the properties of the bipartite system studied in [1]. This bipartite system consists of two dressed atoms inside a reflecting cavity that do not interact with each other, only with an environmental field. Our approach to this problem makes use of the notion of *dressed thermal states* [2], in the context of a model already employed in the literature, of atoms—or, more generally, material particles—in the harmonic approximation, coupled to an environment modeled by an infinite set of pointlike harmonic oscillators (the field modes). The dressed thermal state approach is an extension of the dressed (zero-temperature) formalism that was introduced earlier. The reader is referred to [1] and [2] for details of our formalism.

Here we consider entanglement as a pure quantum effect, a characteristic of quantum mechanics, which is also nonlocal, in the sense that distant and noninteracting systems may be entangled. This is due to the physical meaning attributed to superposed states, a concept with no correspondence in classical physics, and not to the interaction between the (in our case, dressed) atoms. Indeed, such properties of entanglement of noninteracting systems have been used in the realm of teleportation and quantum information theory and to conceive quantum communication devices. We think that investigation the measure to which the properties of such systems are affected by heating is important, particularly in the case of room temperatures. The possibility of constructing such devices at temperatures of everyday life would be an interesting matter.

In this note we remain on theoretical grounds and perform a study of thermal effects on our bipartite system, assuming that the dressing fields of the atoms obey a Bose-Einstein distribution and that the first-level excited dressed atomic states evolve in the same way as in [1]. Under these assumptions, it results that the reduced density matrix does not depend on temperature. In contrast, employing methods and relying on results obtained in [2] and [3], one finds that the individual dressed atoms in the bipartite system are very little affected by heating for temperatures of about 300 K. The overall conclusion is that the methods and results from [1] are valid if we consider the system at room temperatures.

Our bipartite system is composed of two subsystems, $\mathcal{A}$ and $\mathcal{B}$; the subsystems consist of dressed atoms A and B, respectively, and the whole system is contained in a perfectly reflecting sphere of radius R in thermal equilibrium with an environment (field) at a temperature β^{-1}. We consider each atom to carry its own dressing field cloud, independently of each other. This means that we are considering the behavior of a noninteracting bipartite system. Also, we take a specific type of reservoir, in the thermal Bose-Einstein state, where the excitation distribution is diagonal in the number basis.

We start, at time $t = 0$, from a family of superposed states of the bipartite system given by

$$\begin{aligned} |\Psi_{\mathcal{AB}}\rangle &= \sqrt{\xi}\left|\Gamma_{10}^{(AB)}(0)\right\rangle + \sqrt{1-\xi}\,e^{i\phi}\left|\Gamma_{01}^{(AB)}(0)\right\rangle \\ &\equiv \sqrt{\xi}|1_A 0_B\rangle + \sqrt{1-\xi}\,e^{i\phi}|0_A 1_B\rangle, \end{aligned} \tag{1}$$

where $0 \leqslant \xi \leqslant 1$.

In Eq. (1), $|\Gamma_{10}^{(AB)}(0)\rangle \equiv |1_A 0_B\rangle$ and $|\Gamma_{01}^{(AB)}(0)\rangle \equiv |0_A 1_B\rangle$ stand, respectively, for the $t = 0$ states in which the dressed atom A (B) is at the first level, and the dressed atom B (A) in the ground state. They are

$$\left|\Gamma_{10}^{(AB)}(0)\right\rangle = \left|\Gamma_{1}^{A}(0)\right\rangle \otimes \left|\Gamma_{0}^{B}(0)\right\rangle \tag{2}$$

and

$$\left|\Gamma_{01}^{(AB)}(0)\right\rangle = \left|\Gamma_{0}^{A}(0)\right\rangle \otimes \left|\Gamma_{1}^{B}(0)\right\rangle. \tag{3}$$

To take into account temperature effects, we consider the density matrix,

$$\rho_{\mathcal{AB}}(t,\beta) = |\Psi_{\mathcal{AB}}\rangle\langle\Psi_{\mathcal{AB}}| \otimes \rho_{\mathcal{A}}(\beta) \otimes \rho_{\mathcal{B}}(\beta),$$

in terms of dressed objects. We take the density matrix for the thermal bath obeying the Bose-Einstein distribution,

$$\begin{aligned} \rho_{\mathcal{A}}(\beta) &= \prod_k \frac{1}{Z_k}\, e^{-\hbar\beta\omega_k a_k^{\dagger} a_k} \\ &= \prod_k \frac{1}{Z_k} \sum_{n_k=0}^{\infty} e^{-\hbar\beta\omega_k n_k} |n_k\rangle\langle n_k|, \end{aligned} \tag{4}$$

where $n_k = 0,1,2,\ldots$ correspond to the occupation numbers of the dressed field modes k, and Z_k is obtained by imposing the condition of unit trace for $\rho_{\mathcal{A}}(\beta)$; this gives

$$Z_k = \frac{1}{1 - e^{-\hbar\beta\omega_k n_k}}, \tag{5}$$

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and so,

$$\begin{aligned}\rho_{\mathcal{A}}(\beta) &= \left(\frac{1}{Z_1}\sum_{n_1} e^{-\hbar\beta\omega_1 n_1}|n_1\rangle\langle n_1|\right)\\ &\quad\otimes\left(\frac{1}{Z_2}\sum_{n_2} e^{-\hbar\beta\omega_2 n_2}|n_2\rangle\langle n_2|\right)\otimes\cdots\\ &= \sum_{n_1,n_2,\ldots}\frac{e^{-\hbar\beta(\omega_1 n_1+\omega_2 n_2+\cdots)}}{Z_1 Z_2\cdots}|n_1\rangle\langle n_1|\otimes|n_2\rangle\langle n_2|\otimes\cdots.\end{aligned}\tag{6}$$

For the part containing only the atoms, we have

$$\begin{aligned}|\Psi_{\mathcal{AB}}\rangle\langle\Psi_{\mathcal{AB}}| = {}& \xi[|1_A(t)\rangle\langle 1_A(t)|\otimes|0_B\rangle\langle 0_B|]\\ &+(1-\xi)[|0_A\rangle\langle 0_A|\otimes|1_B(t)\rangle\langle 1_B(t)|]\\ &+\sqrt{\xi(1-\xi)}e^{i\phi}[|0_A\rangle\langle 1_A(t)|\otimes|1_B(t)\rangle\langle 0_B|]\\ &+\sqrt{\xi(1-\xi)}e^{-i\phi}[|1_A(t)\rangle\langle 0_A|\otimes|0_B\rangle\langle 1_B(t)|],\end{aligned}\tag{7}$$

where we have changed the notation, $|\Gamma_1^A(t)\rangle \equiv |1_A(t)\rangle$, and similarly for the other states.

Putting together Eqs. (6) and (7), the reduced density matrix is obtained by taking the trace over the field modes,

$$\begin{aligned}\rho_{p_Ap_B}^{r_Ar_B}(t,\beta) = {}& \xi\sum_{\{n_i\}}\frac{e^{-\hbar\beta\sum\omega_i n_i}}{\prod Z_i}\sum_{\{k_i\}=1}^{\infty}\langle p_A,k_1,\ldots|1_A(t);n_1,\ldots\rangle\langle 1_A(t);n_1,\ldots|r_A,k_1,\ldots\rangle\\ &\times\sum_{\{m_i\}}\frac{e^{-\hbar\beta\sum\omega_i m_i}}{\prod Z_i}\sum_{\{q_i\}=1}^{\infty}\langle p_B,q_1,\ldots|0_B;m_1,\ldots\rangle\langle 0_B;m_1,\ldots|r_B,q_1,\ldots\rangle\\ &+(1-\xi)\sum_{\{n_i\}}\frac{e^{-\hbar\beta\sum\omega_i n_i}}{\prod Z_i}\sum_{\{k_i=1\}}^{\infty}\langle p_A,k_1,\ldots|0_A;n_1,\ldots\rangle\langle 0_A;n_1,\ldots|r_A,k_1,\ldots\rangle\\ &\times\sum_{\{m_i\}}\frac{e^{-\hbar\beta\sum\omega_i m_i}}{\prod Z_i}\sum_{\{q_i\}=1}^{\infty}\langle p_B,q_1,\ldots|1_B(t);m_1,\ldots\rangle\langle 1_B(t);m_1,\ldots|r_B,q_1,\ldots\rangle\\ &+\sqrt{\xi(1-\xi)}e^{i\phi}\sum_{\{n_i\}}\frac{e^{-\hbar\beta\sum\omega_i n_i}}{\prod Z_i}\sum_{\{k_i\}=1}^{\infty}\langle p_A,k_1,\ldots|0_A;n_1,\ldots\rangle\langle 1_A(t);n_1,\ldots|r_A,k_1,\ldots\rangle\\ &\times\sum_{\{m_i\}}\frac{e^{-\hbar\beta\sum\omega_i m_i}}{\prod Z_i}\sum_{\{q_i\}=1}^{\infty}\langle p_B,q_1,\ldots|1_B(t);m_1,\ldots\rangle\langle 0_B;m_1,\ldots|r_B,q_1,\ldots\rangle\\ &+\sqrt{\xi(1-\xi)}e^{-i\phi}\sum_{\{n_i\}}\frac{e^{-\hbar\beta\sum\omega_i n_i}}{\prod Z_i}\sum_{\{k_i\}=1}^{\infty}\langle p_A,k_1,\ldots|1_A(t);n_1,\ldots\rangle\langle 0_A;n_1,\ldots|r_A,k_1,\ldots\rangle\\ &\times\sum_{\{m_i\}}\frac{e^{-\hbar\beta\sum\omega_i m_i}}{\prod Z_i}\sum_{\{q_i\}=1}^{\infty}\langle p_B,q_1,\ldots|0_B;m_1,\ldots\rangle\langle 1_B(t);m_1,\ldots|r_B,q_1,\ldots\rangle.\end{aligned}\tag{8}$$

To calculate the preceding matrix elements, we use the time evolution of the states involved [1],

$$|1_A(t);n_1,n_2,\ldots\rangle = \sum_{\nu} f_{A\nu}(t)|1_\nu(0);n_1,n_2,\ldots\rangle,\tag{9}$$

where the coefficients $f_{A\nu}(t)$ are

$$f_{A\nu}(t) = \sum_s t_\mu^s t_\nu^s e^{-i\Omega_s t},\qquad \sum_\nu |f_{A\nu}(t)|^2 = 1.\tag{10}$$

A similar equation holds for $|1_B(t);n_1,n_2,\ldots\rangle$. Inserting Eq. (9) and its equivalent for atom B into Eq. (8), we obtain, after some rather long manipulations,

$$\begin{aligned}\rho_{p_Ap_B}^{r_Ar_B}(t) = {}& \left(\sum_{\{n_i\}}\frac{e^{-\hbar\beta\sum\omega_i n_i}}{\prod Z_i}\right)\left(\sum_{\{m_i\}}\frac{e^{-\hbar\beta\sum\omega_i m_i}}{\prod Z_i}\right)\\ &\times\Bigg\{\xi\left[|f_{AA}(t)|^2\delta_{p_A1}\delta_{r_A1}+\sum_{i=1}^{\infty}|f_{Ai}(t)|^2\delta_{p_A0}\delta_{r_A0}\right]\\ &\times\delta_{p_B0}\delta_{r_B0}+(1-\xi)\delta_{p_A0}\delta_{r_A0}\\ &\times\left[|f_{BB}(t)|^2\delta_{p_B1}\delta_{r_B1}+\sum_{i=1}^{\infty}|f_{Bi}(t)|^2\delta_{p_B0}\delta_{r_B0}\right]\\ &+\sqrt{\xi(1-\xi)}e^{i\phi}f_{AA}^*(t)f_{BB}(t)\delta_{p_A0}\delta_{r_A1}\delta_{p_B1}\delta_{r_B0}\\ &+\sqrt{\xi(1-\xi)}e^{-i\phi}f_{AA}(t)f_{BB}^*(t)\delta_{p_A1}\delta_{r_A0}\delta_{p_B0}\delta_{r_B1}\Bigg\}.\end{aligned}$$

But

$$\frac{1}{Z_i}\sum_{n_i} e^{-\hbar\beta\omega_i n_i} = 1, \quad \forall i, \tag{11}$$

and therefore, the temperature-dependent terms disappear, leaving the reduced density matrix with exactly the same form as in [1]. For identical atoms we have $f_{AA}(t) = f_{BB}(t) \equiv f_{00}(t)$, and the nonvanishing matrix elements of ρ are

$$\begin{aligned}
\rho_{00}^{00}(t) &= 1 - |f_{00}(t)|^2,\\
\rho_{01}^{01}(t) &= (1-\xi)|f_{00}(t)|^2,\\
\rho_{10}^{10}(t) &= \xi|f_{00}(t)|^2,\\
\rho_{01}^{10}(t) &= \sqrt{\xi(1-\xi)}e^{i\phi}|f_{00}(t)|^2,\\
\rho_{10}^{01}(t) &= \sqrt{\xi(1-\xi)}e^{-i\phi}|f_{00}(t)|^2.
\end{aligned} \tag{12}$$

Note that this property is not specific to the Bose-Einstein distribution we have employed, but it is common to any normalized distribution which is diagonal in the number basis. This may not happen for other types of reservoirs, like coherent squeezed thermal states, where factorization of the temperature-dependent terms may not occur.

At finite temperature, all the results that depend only on the reduced density matrix remain the same as in the zero-temperature case [1]. This conclusion is valid for our choice of Bose-Einstein distribution to the heated environment, using Eq. (9) for the time evolution of the first-level excited atomic states. The choice of the Bose-Einstein distribution to the field modes can be justified: in the case of an arbitrarily large cavity, the dressed field modes coincide with the bare ones [2], and in the limit of vanishing coupling with the atom, these modes follow the Bose-Einstein distribution exactly. Strictly speaking, this is not the case for the coupled atom-field system in a finite cavity. Nevertheless, in many situations this approximation is acceptable in the weak-coupling regime [4]. The use of Eq. (9) for the time evolution of first-level excited atomic states, as we will see, is a good approximation for room temperatures.

In this case, heating does affect each atom individually. In [2] and in [3], some of us studied the time evolution of the temperature-dependent occupation number of a single dressed atom, $n_0'(t,\beta)$, for a very large cavity (free space) and for confinement in a small cavity, respectively, with the same heating procedure that we have used here. In the case of a large cavity, we have shown that, starting from an initial value $n_0'(t=0,\beta)=1$, $n_0'(t,\beta)$ evolves, for weak coupling, steadily from 1 to an equilibrium value of

$$n_0'(\infty,\beta) = 1/(e^{\hbar\beta\bar{\omega}}-1).$$

For $T = 300$ K, for a frequency $\bar{\omega} = 4.0 \times 10^{14}$/s, we have $n_0'(\infty, T=300) \approx 0.09$, a value slightly higher than the corresponding value for $T=0$, $n_0'(\infty,T=0)=0$. For the same initial condition, $n_0'(t=0,\beta)=1$, and the same value of the emission frequency and temperature as in the large cavity, it is found in [2] that, for a small cavity (radius $R \sim 10^{-6}$ m), the time evolution of the occupation number shows an oscillating behavior. Raising the temperature increases the average value of the occupation number, but this increase is significant only for high laboratory temperatures. For instance, for $T = 10^5$ K ($\sim$8.4 eV; a value lower than the ionization temperature of 13.6 eV for the hydrogen atom) and a frequency $\bar{\omega} = 4.0 \times 10^{14}$/s, as before, one obtains that the average occupation number is about 3.5 times higher than the zero-temperature value, $n_0'(t,T=0)\approx 1$. In contrast, this effect is negligible for room temperatures; in this case, taking, for instance, a temperature $T \gtrsim 300$ K, it is found that the occupation number is given by $n_0'(t, T \gtrsim 300) \gtrsim 1$, which will remain very close to the zero-temperature value. Therefore, if we consider the system at room temperatures, individual atoms are very little affected by heating and the use of Eq. (9) is justified.

Our system is composed of two noninteracting atoms, each one carrying its own dressing field. The superposition principle dictates the existence of correlations between the parts of the system (the two atoms) even if they do not interact. By introducing temperature in the environment by means of the Bose-Einstein distribution, the result is that the entanglement properties of the bipartite system considered are not affected by heating up to room temperatures. In particular, considering two identical atoms, the concurrence, the entanglement of formation, and the negativity depend only on the matrix elements in Eq. (12) and, thus, cannot depend on the temperature. However, the temperature of the environment cannot be so high that the atoms dissociate or such that the adopted atomic model becomes invalid. More precisely, from a physical point of view, our model applies for room temperatures. As we have shown here, for temperatures of the order of $\sim$300 K, or not much higher, the harmonic approximation we used in [1] remains valid and the time evolution of the thermal occupation number of the atoms is very close to the zero-temperature case. This establishes the range of validity of the present study.

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[4] In [2] we considered a cavity of radius $R \approx 10^{-6}$ m, which is $\sim 10^4$ times larger than the size of a hydrogen atom (the Bohr radius). In this case the atom "sees" the cavity to be a very large one, and in the weak-coupling regime, the approximation that the field modes follow the Bose-Einstein distribution is justified. Note that the size for a cavity of the order of micrometers is of the order of magnitude of the cavity used in experiments to investigate confinement effects of atoms within metallic cavities. See E. A. Hinds, K. S. Lail, and M. Schnell, Philos. Trans. R. Soc. London A **355**, 2353 (1997).