Generation of hyperentangled states between remote noninteracting atomic ions

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We propose a scheme of generating four-qubit hyperentangled states between a pair of remote noninteracting atomic ions with a Λ configuration that are confined in Paul traps. These hyperentangled states, different from the normal entangled states that are entangled in a single degree of freedom, are entangled in both spin and motion degrees of freedom. In our proposal, the entanglement is first generated in spin degrees of freedom using linear optics and then transferred to the motion degree of freedom using a sequence of laser pluses, including the stimulated Raman carrier transitions and sideband transitions. The proposal is completed with regenerating entanglement in spin degrees of freedom using linear optics.

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As key resource for quantum-information processing (QIP), entangled states have attracted much attention as they have been introduced into quantum-information science [1,2]. To date, most of the effort that has been devoted to the research of entangled states has focused on states that are entangled in a single degree of freedom, for example, polarization, spin, or momentum direction. In recent years, the concept of hyperentangled states (HESs) [3], which are entangled in several independent degrees of freedom, has been proposed and studied [3–20]. Several recent works have reported the experimental realization of photon HESs [3–9] and the utilization of these states in Bell state analysis [3,10–13], entanglement purification [13], entangled state preparation [14], quantum computation [15], quantum error-correcting [16], and superdense coding and quantum teleportation [10], which shows the potential advantage of HESs in QIP.

The generation of entangled states in a single degree of freedom has already been demonstrated in different systems with interaction. Meanwhile, schemes for generating entangled states in noninteracting systems by a consequence of measuring photons propagating along multiple quantum paths have been proposed [21–24] and realized between trapped atoms [25] and between disordered clouds of atoms [26]. On the other hand, the technique of laser cooling enables us to prepare certain states in the motion degrees of freedom of trapped atoms or trapped ions [27–33]. Moreover, Jost *et al.* [34] has experimentally demonstrated generation of entangled states of the stretch model in the ground state and the first excited state of two mechanical oscillators using a sequence of laser cooling and stimulated Raman carrier transitions and sideband transitions [30–38].

The previous works on HESs all concern systems of photons. It is interesting to generate HESs in other systems, for example, in trapped atoms or ions. Motivated by these previous works, we propose in this brief report a scheme for generating HESs in both spin and motion degrees of freedom of two remote trapped noninteracting atomic ions. In our protocol, we utilize two atomic ions with a Λ configuration confined in harmonic potential wells. First, in order to generate spin-entangled states between atoms, we excite the internal ground state to its excited state and then measure the photons emitted spontaneously from the ions [21–24]. Next, we transfer the entanglement from spin degrees of freedom to motion degrees of freedom using a sequence of laser cooling and sideband transitions. Finally, we repeat the first step to generate the entanglement in the spin degrees of freedom, and thus we obtain the desired HESs.

Taking advantage of an enlarged Hilbert space, a HES of a system entangling in N degrees of freedom can be expressed by the product of N Bell states $\mathcal{B}$ [2,7,16–18],

$$|\Psi_N\rangle = |\mathcal{B}_1\rangle \otimes |\mathcal{B}_2\rangle \otimes \cdots \otimes |\mathcal{B}_N\rangle, \tag{1}$$

one of each degree of freedom.

Using the entropy of entanglement as the entanglement measurement, the entanglement of $|\Psi_N\rangle$ is

$$E(|\Psi_N\rangle) = S[\mathrm{tr}_A(|\Psi_N\rangle\langle\Psi_N|)] = S[\mathrm{tr}_B(|\Psi_N\rangle\langle\Psi_N|)], \tag{2}$$

where S is standard von Neumann entropy, defined as $S(\rho) = \mathrm{tr}(\rho\log_2\rho)$. It is easy to demonstrate that $E(|\Psi_N\rangle) = N$, independent of any local operation. Some applications of this type of HESs in QIP have been discussed [3,10,11,16]. Nowadays HESs of photons have already been demonstrated, for example, for photons entangled in polarization spatial mode and time-energy [4], time-bin and polarization [6], and polarization, linear momentum, and time-energy [7]. In the following, we propose a scheme for generating hyperentanglement in the system of atomic ions.

We consider a system consisting of two atomic ions with a Λ configuration moving along one direction in the Paul trap. The two ions are separated, that is, the coupling between two ions can be neglected. Experimental techniques now enable a single ion to be confined in three dimensions and its vibrational motion restricted effectively to one dimension, and the ion can be cooled to the vibrational ground state with a probability greater than 99% [39,40]. Such a trapped ion can be treated as a single harmonic oscillator [34,41]. Itano *et al.* [41] reported an experiment of laser cooling a single ion in a Paul trap to the ground ($n = 0$) quantum harmonic oscillator state with greater than 90% probability. In this scheme, we deal with the two separated ions as one-dimensional harmonic oscillators

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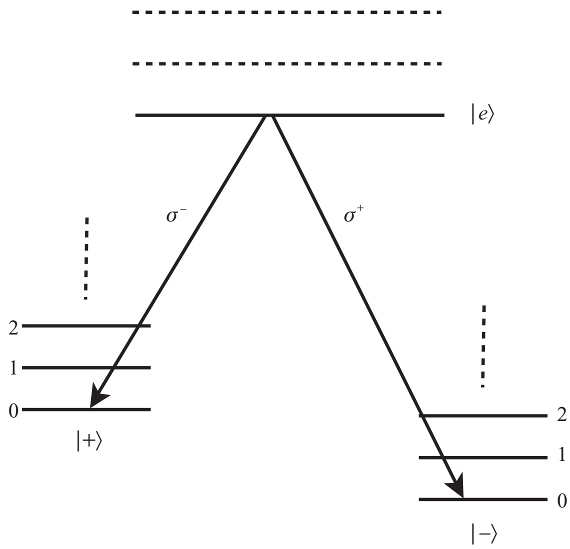

FIG. 1. Internal and motional energy levels of ion. The $|+\rangle$ and $|-\rangle$ states are ground states of deexcited state of ions, and each internal state can exist in a ladder of vibrational energy states $|n\rangle$, where $n = 0,1,2,\ldots$.

moving along the x direction, and the Hamiltonian of each oscillator is

$$H_i = \sum_i \hbar\omega_i a_i^\dagger a_i + \frac{\hbar\omega_0}{2}\sigma_{zi}, \tag{3}$$

where subscript $i = A$ and B denotes the two traps, $a_i^\dagger$ and a_i are creation and annihilation operators for the oscillator mode, with ω_i being the corresponding frequency, and σ_{zi} is the Pauli matrix. The eigenstate for such a system can be denoted as $|j,n\rangle = |j\rangle \otimes |n\rangle$, where $|j\rangle$ denotes the internal spin state and $|n\rangle$ denotes the motional state. In this brief report we are interested in the ground state $|n_g\rangle = |n = 0\rangle \equiv |0\rangle$ and the first excited state $|n_e\rangle = |n = 1\rangle \equiv |1\rangle$ of the motional state.

In our scheme, the two atomic ions are initially held in two individual Paul traps and the two traps are separated such that the coupling between them can be neglected. First, the ions are excited by π-polarized laser pulse to the internal excited state $|e\rangle$ in spin degrees of freedom. As the laser cooling does not affect the spin states [28,42], the ions are simultaneously cooled with laser pulse to ground state in motion degrees of freedom. The ions in excited state $|e\rangle$ then decay along two possible channels, $|e\rangle \to |+\rangle$ and $|e\rangle \to |-\rangle$, accompanied by the spontaneous emission of a σ^- or σ^+ polarized photon, respectively (Fig. 1). The states $|+\rangle$ and $|-\rangle$ correspond to the Zeeman sublevels of the ions in the ground-state manifold. Thus for a single decaying ion, the polarization state of the emitted photon is entangled with the corresponding ground state of the deexcited atom [21–24,43,44]. As long as these emission processes are indistinguishable in all other degrees of freedom, we obtain the following maximally entangled state:

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|+\rangle|\sigma^-\rangle + |-\rangle|\sigma^+\rangle). \tag{4}$$

The emitted photons are collected by single-mode optical fibers and interfere at a polarization beam splitter (PBS), with

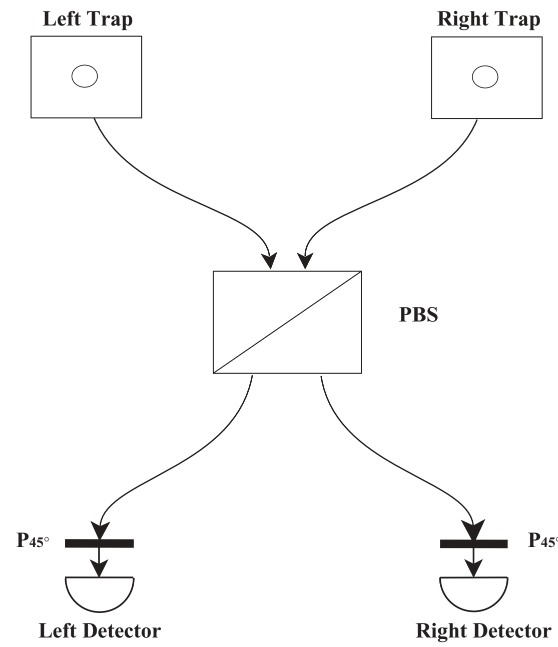

FIG. 2. Schematic setup to generate entanglement between two atomic ions confined in separated Paul traps using linear optics.

the outputs detected by two single-photon detectors after 45° polarizers (Fig. 2).

In a successful measurement cycle, each atom emits a single photon and each detector registers one photon, the two atomic ions are prepared in the maximally entangled state [23] in the spin degree of freedom, $|\Psi^+\rangle = (|+-\rangle + |-+\rangle)/\sqrt{2}$, and the total state of the system is

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle_L|-\rangle_R + |-\rangle_L|+\rangle_R) \otimes |n=0\rangle_L|n=0\rangle_R. \tag{5}$$

A laser provides control of the ions' motion and internal states through laser cooling and stimulated Raman carrier transitions or sideband transitions [30–38]. Since the sideband transitions affect the motional state, such transitions can be use to couple the spin and motion and produce entanglement between the ions' spin and their motion [32,34]. Several recent works report the experimental realization of such sideband transitions in different systems [32,34,37,38].Using a sequence of sideband transitions and stimulated Raman carrier transitions, one can transfer the entanglement from the internal spin state to the external motional state of the ions [30,34], and Ref. [34] reports an experiment that transfers the entanglement from ions' internal states to the motion of the separated mechanical oscillators.

We transfer the entanglement from the spin to the motion with a sequence of laser pulses including the stimulated Raman carrier transitions and sideband transitions. The carrier transition only affects the spin states and can be described as generalized rotation [34],

$$\Omega_k(\theta,\phi) = \begin{pmatrix} \cos\frac{\theta}{2} & -ie^{-i\phi}\sin\frac{\theta}{2} \\ -ie^{i\phi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}, \tag{6}$$

where $k \in \{L,R\}$ denotes the left or right traps. The rotation angle θ is proportional to the intensity and duration of the pulses, and the phase ϕ is determined by the phase difference between the two optical Raman fields [34]. Carrier transitions correspond to rotations in the basis

$$\Omega_k(\theta,\phi): |-\rangle \Rightarrow |+\rangle, \quad \Omega_k(\theta,\phi): |+\rangle \Rightarrow |-\rangle. \tag{7}$$

The sideband transitions can couple the spin and the motion, and the corresponding Hamiltonian can be described as $H = Ja^\dagger\sigma_- + \text{h.c.}$ or $H = Ja\sigma_+ + \text{h.c.}$, where Pauli operators are defined by $\sigma_+ = |+\rangle\langle-|, \sigma_- = |-\rangle\langle+|$ [36]. These can be described as rotation

$$T_k(\theta,\phi) = \Omega_k^s(\theta,\phi) \otimes \Omega_k^m(\theta,\phi), \tag{8}$$

where $\Omega_k^s(\theta,\phi)$ denotes rotation in the spin state and $\Omega_k^m(\theta,\phi)$ denotes rotation in the motional state. Sideband transitions correspond to rotations between $|+\rangle|0\rangle$ and $|-\rangle|1\rangle$ or to rotations between $|+\rangle|1\rangle$ and $|-\rangle|0\rangle$ [31,34,37],

$$|+\rangle|0\rangle \overset{T_k(\theta,\phi)}{\longleftrightarrow} |-\rangle|1\rangle, \quad |+\rangle|1\rangle \overset{T_k(\theta,\phi)}{\longleftrightarrow} |-\rangle|0\rangle. \tag{9}$$

In order to transfer the entanglement from the spin to the motion, we first apply $T_L(\pi,0)$ to state (5) in the left trap, creating the state

$$|\Psi\rangle_1 = \frac{1}{\sqrt{2}}|+\rangle_L(|0\rangle_L|-\rangle_R - i|1\rangle_L|+\rangle_R)|0\rangle_R. \tag{10}$$

Then we apply $T_R(\pi,0)$ in the right trap, creating the state

$$|\Psi\rangle_2 = \frac{1}{\sqrt{2}}|+\rangle_L|+\rangle_R(|0\rangle_L|1\rangle_R - |1\rangle_L|0\rangle_R). \tag{11}$$

In the experiment, after the first spin $\rightarrow$ motion transfer on the ion in the left trap, the spin of the ion in the right trap may be sensitive to decoherence from fluctuating magnetic fields. In order to minimize this effect, a spin-echo pulse, $\Omega_R(\pi,0)$, can be applied to the ion in the right trap after the first spin $\rightarrow$ motion transfer [34], creating the state

$$|\Psi\rangle_3 = \frac{1}{\sqrt{2}}|+\rangle_L(|0\rangle_L|+\rangle_R - i|1\rangle_L|-\rangle_R)|0\rangle_R. \tag{12}$$

Then we apply $T_R(\pi,0)$ on the right trap, creating the state

$$|\Psi\rangle_4 = \frac{1}{\sqrt{2}}|+\rangle_L|+\rangle_R(|0\rangle_L|0\rangle_R - |1\rangle_L|1\rangle_R). \tag{13}$$

The states (11) and (13) are entangled superpositions of stretch modes in ground and excited states; thus we have transfer the entanglement from spin to motion degrees of freedom.

In order to create the desired HESs, we should repeat the first step to generate entanglement in spin degrees of freedom. Exciting ions in Eq. (11) or (13) to the internal excited state and performing a successful measurement on the spontaneously emitted photons creates the maximally entangled state $|\Psi^+\rangle$ in spin degrees of freedom, and thus we finally obtain the HESs in both spin and motion degrees of freedom of two remote trapped atomic ions:

$$|\Phi\rangle = \tfrac{1}{2}(|+\rangle_L|-\rangle_R + |-\rangle_L|+\rangle_R) \otimes (|0\rangle_L|1\rangle_R - |1\rangle_L|0\rangle_R), \tag{14}$$

or

$$|\Phi\rangle = \tfrac{1}{2}(|+\rangle_L|-\rangle_R + |-\rangle_L|+\rangle_R) \otimes (|0\rangle_L|0\rangle_R - |1\rangle_L|1\rangle_R). \tag{15}$$

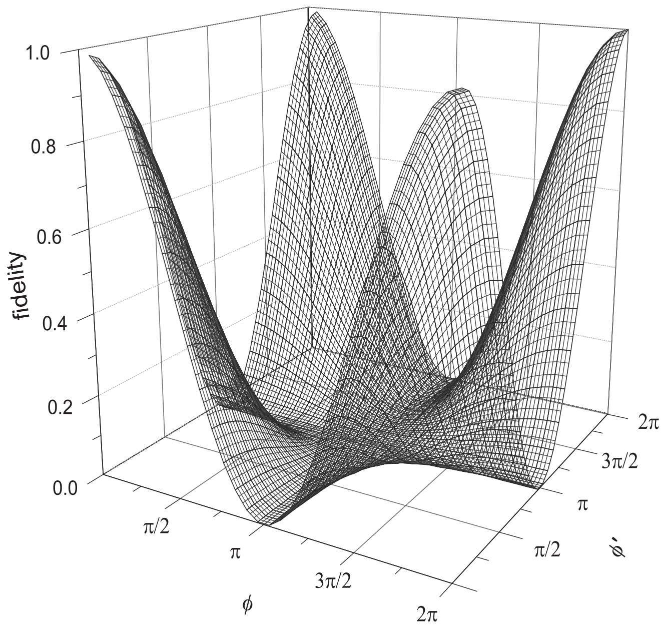

FIG. 3. Variation of fidelity $\mathcal{F}$ with phases ϕ and ϕ' due to imperfections (assuming that phase ϕ and ϕ' can take any value). Phases ϕ and ϕ' are introduced in different steps in the protocol.

The aforementioned proposal provides a feasible method for generating HESs. In this scheme, we make use of the fact that the spin of the deexcited ion is entangled with the polarization of the photon spontaneously emitted from the decaying ion. In experiment, the entanglement created between the ionic and photonic qubits is not perfect, because of the intensity of the radiation patterns for laser pulses and for some other experimental reasons [43]. Thus in experiment we may not obtain a maximally entangled state in Eq. (4) in spin degrees of freedom. As a result, the final obtained HESs may not be strict in the form of Eq. (14) or (15). The precise forms of the final HESs depend on the concrete experiment and we think it is meaningful to check this in detail experimentally.

Finally, let us estimate the fidelity of our scheme. The fidelity of a quantum state $\rho = |\varphi\rangle\langle\varphi|$ can be defined as its overlap with an appropriate target state $|\psi_{\text{tag}}\rangle$, $\mathcal{F} = \langle\psi_{\text{tag}}|\rho|\psi_{\text{tag}}\rangle$ [43]. In our scheme, several factors will affect the fidelity of the generation of the expected states, including spontaneous photon scattering [24,43], imperfect rotations of the spin states, and motional decoherence [34]. Figure 3 shows the variation of fidelity $\mathcal{F}$ with the phase that accumulates through the protocol due to the difference in magnetic field between the left trap and the right trap and imperfect rotations, and it indicates that for the ideal case, that is, all phase tends to 0, we can exactly obtain the expected state, which is consistent with the experiment. Furthermore, considering all the aforementioned factors, we estimate a bound on the fidelity to be $\mathcal{F} \geqslant 87.85\%$ for the generation of the expected state.

In conclusion, we consider a system of two remote noninteracting atomic ions with a Λ configuration. We propose a scheme for generating four-qubit HESs between these two ions that are entangled in spin and motion degrees of freedom, different from former schemes that use photons. In our scheme, we generate entanglement in spin degrees of freedom using the linear optics tools and then transfer the entanglement to the motion degrees of freedom utilizing the sideband transitions. We realize that it may be difficult to extend this scheme to generate HESs in systems consisting of N atoms or ions or to hyperentangled quantum networks because of the use of sideband transitions. Thus an experimentally feasible scheme for generating HESs for systems consisting of N atoms or ions deserves further study. Schemes for HESs of solid quantum systems in more degrees of freedom, such as energy-time or orbital angular momentum, are also worth further investigation, and this may require considering systems that have internal interactions.

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