

Cavity quantum electrodynamics with a Rydberg-blocked atomic ensemble

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The realization of a Jaynes-Cummings model in the optical domain is proposed for an atomic ensemble. The scheme exploits the collective coupling of the atoms to a quantized cavity mode and the nonlinearity introduced by coupling to high-lying Rydberg states. A two-photon transition resonantly couples the single-atom ground state $|g\rangle$ to a Rydberg state $|e\rangle$ via a nonresonant intermediate state $|i\rangle$, but due to the interaction between Rydberg atoms only a single atom can be resonantly excited in the ensemble. This restricts the state space of the ensemble to the collective ground state $|G\rangle$ and the collectively excited state $|E\rangle$ with a single Rydberg excitation distributed evenly on all atoms. The collectively enhanced coupling of all atoms to the cavity field with coherent coupling strengths which are much larger than the decay rates in the system leads to the strong coupling regime of the resulting effective Jaynes-Cummings model. We use numerical simulations to show that the cavity transmission can be used to reveal detailed properties of the Jaynes-Cummings ladder of excited states and that the atomic nonlinearity gives rise to highly nontrivial photon emission from the cavity. Finally, we suggest that the absence of interactions between remote Rydberg atoms may, due to a combinatorial effect, induce a cavity-assisted excitation blockade whose range is larger than the typical Rydberg dipole-dipole interaction length.

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I. INTRODUCTION

The Jaynes-Cummings model (JCM) [1] provides the general framework to describe the interaction of a two-level system, such as an atom, with a quantum harmonic oscillator, e.g., a quantized cavity mode in cavity quantum electrodynamics (CQED). It allows us to explain very specific behaviors of such systems as, for example, the collapses and revivals of Rabi oscillations [2,3]. The JCM was also used to describe various physical situations outside the field of CQED, such as the coupling of the internal states to the center-of-mass vibrational levels of a trapped ion subject to a laser beam in the Lamb-Dicke regime (for a review, see Ref. [4]) or, more recently, a laser-driven electron floating on liquid helium [5]. Precisely because of this universality, the JCM also appears now as one of the key ingredients for applications in quantum information processing.

While many successful CQED experiments have been carried out with microwave cavities, resonant with transitions between atomic Rydberg excited states [6,7] and with superconducting “artificial atoms” [8,9], it is harder to reach the strong coupling regime with single atoms and optical cavities. This regime is, however, of considerable practical interest, as it makes it possible for the cavity to work as an interface among optical photons and atoms, the flying and stationary qubits, in quantum computing and communication networks [10]. A viable path to obtain a highly coherent system in the optical regime is to use a collection of N atoms whose coupling strength to an optical field mode is magnified by the factor \sqrt{N} relative to the single-atom case [11–21]. For large N , the collective excitation degree of freedom of an atomic ensemble is approximately equivalent to a quantum harmonic oscillator and the system atoms-cavity therefore tends to be well described by a simple quadratic Hamiltonian in the raising

and lowering operators for the atomic and field excitations. Concepts to squeeze and entangle the atomic and field degrees of freedom by this Hamiltonian have been developed but the systems stay within the restricted family of Gaussian states [22,23].

In this article, we propose an implementation of the Jaynes-Cummings model in the optical domain using an atomic ensemble. The ensemble atoms are placed in an optical high-finesse cavity and a nonlinearity is introduced by the interaction between atoms resonantly coupled to a high-lying Rydberg state. In relatively small ensembles, dipole-dipole interactions significantly shift the energy of states with two or more Rydberg excitations [24]. When states with two excited atoms are shifted far away from the optical resonance we observe the so-called Rydberg blockade phenomenon [25–31], where the atomic ensemble effectively behaves as a single two-level system. The entire atoms-cavity system is therefore well described by the Jaynes-Cummings model, and the collective enhancement factor \sqrt{N} compared to the single atom case may then lead to the strong coupling regime.

We will give an outline of the idea and analyze a specific implementation with realistic physical parameters in Sec. II. In Sec. III the expected signature of the strongly coupled ensemble on the optical transmission of the cavity is discussed and investigated numerically by use of Monte Carlo simulations. In particular, we show that both the average signal and the fluctuations present interesting, and perhaps surprising, features linked with the eigenstates of the Jaynes-Cummings model. In Sec. IV, we suggest a new mechanism by which the cavity can extend the range of the Rydberg blockade beyond the dipole-dipole interaction length. In Sec. V, we conclude and discuss briefly the vast range of possible investigations that can be made with the system and a few possible extensions of the theoretical model.

II. PHYSICAL IMPLEMENTATION

The principle of our proposed experiment is depicted in Fig. 1. A Bose-Einstein condensate consisting of $N \gtrsim 1000$ ^{87}Rb atoms is placed in an ultrahigh finesse cavity [17] and is transversely illuminated by a homogeneous classical laser field. The $5S_{1/2}$ ground atomic state denoted by $|g\rangle$ is assumed to be resonantly coupled to a Rydberg level $|e\rangle$ through a two-photon process, via the $5P_{3/2}$ intermediate state $|i\rangle$. The transition $|g\rangle \rightarrow |i\rangle$, of frequency $\omega_{g \rightarrow i}$, is nonresonantly coupled with the coupling strength g_0 to a single quantized cavity mode with annihilation operator \hat{a} and frequency ω_c . The cavity mode is detuned by the amount $\Delta = \omega_c - \omega_{g \rightarrow i}$, while the transition $|i\rangle \rightarrow |e\rangle$ of frequency $\omega_{i \rightarrow e}$ is nonresonantly driven by the laser field with the Rabi frequency Ω and frequency ω_l detuned by the amount $\omega_l - \omega_{i \rightarrow e} = -\Delta$. Omitting at first the interatomic interactions, one can describe the physical situation by the following Hamiltonian

$$\begin{aligned} \tilde{H} = & -\hbar\Delta \sum_{j=1}^N |i_j\rangle\langle i_j| + \left[\hbar g_0 \hat{a}^\dagger \sum_{j=1}^N |g_j\rangle\langle i_j| + \text{H.c.} \right] \\ & + \left[\hbar\Omega \sum_{j=1}^N |e_j\rangle\langle i_j| + \text{H.c.} \right], \end{aligned} \quad (1)$$

written in the interaction picture with respect to $H_0 = \hbar\omega_c(\hat{a}^\dagger\hat{a} + \sum_{j=1}^N |i_j\rangle\langle i_j|) + \hbar(\omega_c + \omega_l) \sum_{j=1}^N |e_j\rangle\langle e_j|$, and in the rotating wave approximation. Dispersive shifts on both transitions have been neglected for simplicity. Assuming a large detuning from the intermediate state, $\Delta \gg \Gamma$, where Γ^{-1} is the lifetime of $|i\rangle$, we can adiabatically eliminate the unpopulated intermediate state, which leads to the effective Hamiltonian

$$\begin{aligned} \tilde{H}_{\text{eff}} = & \hbar \frac{g_0^* \Omega}{\Delta} \hat{a} \sum_{j=1}^N |e_j\rangle\langle g_j| + \text{H.c.} \\ = & \hbar \sqrt{N} \frac{g_0^* \Omega}{\Delta} \hat{a} \hat{S}^\dagger + \text{H.c.}, \end{aligned} \quad (2)$$

where the collective mode atomic excitation is described by $\hat{S}^\dagger = \frac{1}{\sqrt{N}} \sum_{j=1}^N |e_j\rangle\langle g_j|$.

Equation (1) is written under the assumption that all atoms have the same coupling coefficients Ω , g_0 to the laser and cavity field. This describes the situation of a Bose-Einstein condensate, when neglecting the photon recoil. Alternatively, one can consider a real space where the coupling strengths depend on the value of the mode functions at the location of the individual atoms, and $g_0^* \Omega$ should be replaced by different coefficients $g_j^* \Omega_j$ on each term in the sum in (2). It turns out, however, that in the limit of many atoms, one can define a collective operator like \hat{S}^\dagger which is a weighted sum of the individual atomic raising operators and which also obeys the oscillator-like commutator relations to a good approximation. As long as the atoms do not move appreciably and change their coupling strengths on the time scale of interest for the experiment, this weighted collective atomic mode plays the same role as the ideal symmetric mode and its coupling is enhanced by the same factor \sqrt{N} and an extra mode-dependent factor of order unity; see, e.g., Ref. [32]. For low Rydberg excitation numbers, $[\hat{S}, \hat{S}^\dagger] \simeq 1$, and the Hamiltonian Eq. (2)

approximately describes the linear exchange of excitations between two degenerate oscillators.

The dipole-dipole interactions we have omitted so far are strong only between neighboring Rydberg atoms. Their effect on an atomic sample is to considerably shift the energy of multiply Rydberg excited states within a sample size at most of the order of the length l_R at which the dipole-dipole interaction equals the atom-light interaction. Driving the transition $|g\rangle \rightarrow |e\rangle$ as described above, one can thus resonantly couple the collective ground state $|G\rangle \equiv |g_1, \dots, g_N\rangle$ to the collective symmetric state with a single Rydberg excitation $|E\rangle \equiv \hat{S}^\dagger |G\rangle$ only, as higher excited levels are too far detuned. This constitutes the Rydberg blockade phenomenon [25,26]. Restricting the Hamiltonian Eq. (2) to the physically relevant atomic subspace $\{|G\rangle, |E\rangle\}$ we get $\tilde{H}'_{\text{eff}} = \hbar g_{\text{eff}} \hat{a} |E\rangle\langle G| + \text{H.c.}$, which corresponds to the interaction part of the Jaynes-Cummings Hamiltonian describing the resonant interaction of the cavity field with a fictitious two-level ‘‘superatom’’ [33], with the effective coupling constant $g_{\text{eff}} \equiv \sqrt{N} g_0^* \Omega / \Delta$. We finally pass to a new rotating frame defined by the unitary state transformation $\exp\{-it\omega_c(\hat{a}^\dagger\hat{a} + |E\rangle\langle E|)\}$ to obtain the full Jaynes-Cummings Hamiltonian

$$\tilde{H}_{\text{JC}} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar\omega_c |E\rangle\langle E| + [\hbar g_{\text{eff}} \hat{a} |E\rangle\langle G| + \text{H.c.}] \quad (3)$$

with an effectively equal excitation energy of the field and ensemble atomic states.

The parameters of this Jaynes-Cummings model implementation can be tuned over a wide range. The collective coupling on the first transition $\sqrt{N} g_0$ scales indeed as the square root of the atom number and can thus be varied in the experiment up to the GHz range [17]. One possible scheme would be to excite the $70s$ Rydberg state from a 1000-atom Bose-Einstein condensate (BEC), with a cavity-intermediate state detuning of $\Delta = 2\pi \times 900$ MHz, a single atom resonant coupling on that transition of $g_0 = 2\pi \times 10$ MHz, and a blue laser Rabi coupling $\Omega = 2\pi \times 30$ MHz. Excitation of the intermediate atomic state $|i\rangle$ can be neglected here since the cavity detuning Δ is large compared to both the atomic radiative decay rate $\Gamma = 2\pi \times 3$ MHz from the intermediate state and the one-photon collective coupling strength $\sqrt{N} g_0 \sim 2\pi \times 320$ MHz. The resulting two-photon coupling $g_{\text{eff}} \sim 2\pi \times 10$ MHz is higher than both decay rates of the cavity $\kappa = 2\pi \times 1.3$ MHz and of the Rydberg state $\gamma = 2\pi \times 0.55$ kHz. This system reaches the strong coupling regime on the ground-to-Rydberg state two-photon transition. Within the Thomas-Fermi radius of the BEC $r_{\text{TF}} \sim 2 \mu\text{m}$ corresponding to an isotropic trapping frequency $2\pi \times 180$ Hz, the minimum Rydberg interaction shift over the atomic sample is $\Delta_{\text{RI}} \sim 2\pi \times 200$ MHz, which is much bigger than the two-photon coupling g_{eff} . The states with two or more Rydberg excited atoms are thus spectrally well separated from the singly excited ones. Accordingly, note that the size of the sample $\sim 4 \mu\text{m}$ is indeed below the blockade radius calculated with these parameters $\ell_R \sim 7 \mu\text{m}$.

III. THE DRIVEN JAYNES-CUMMINGS MODEL

In the previous section, we arrived at an effective description of the ensemble-light interaction in terms of a two-level superatom whose Hilbert space consists of the

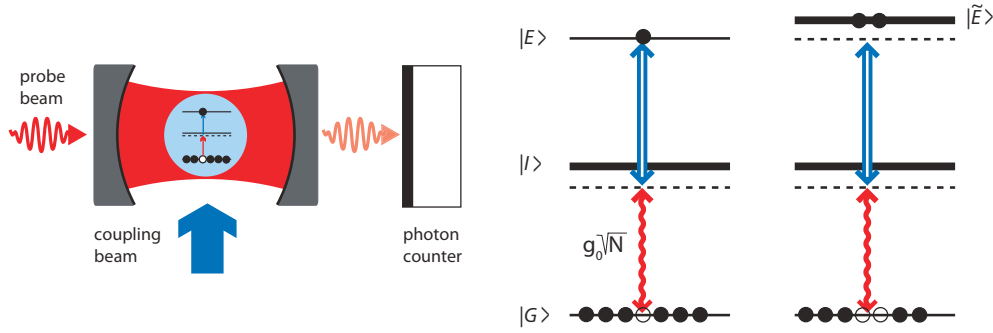


FIG. 1. (Color online) (Left) A Bose-Einstein condensate inside an ultrahigh finesse optical cavity is coupled in a two-photon scheme to a Rydberg state, with the first photon being provided by a single quantized cavity mode and the second photon by a classical coupling beam. The system is interrogated by monitoring the cavity transmission of a weak probe beam on a photon counter. (Middle) The quantized cavity mode has a frequency ν_1 and couples the collective atomic ground state $|G\rangle$ to an intermediate level $|I\rangle$ with a detuning Δ . The collective vacuum Rabi coupling is given by $g_0\sqrt{N}$. A blue photon of frequency ν_2 drives the excitation to a Rydberg state $|E\rangle$ with a classical Rabi coupling Ω . (Right) The interaction between two Rydberg atoms leads to an energy shifted and broadened state $|\tilde{E}\rangle$ containing two excitations. This allows selectively addressing the single excitation state. The resulting situation can be described by a Jaynes-Cummings Hamiltonian.

collective ground and singly Rydberg excited states of the ensemble $|G\rangle, |E\rangle$. The associated effective Jaynes-Cummings Hamiltonian Eq. (3) is block diagonal, coupling only pairs of states $(|G, n\rangle, |E, n-1\rangle)$ with the same total number n of either photonic or atomic excitations. The frequencies $\omega_{n,\pm}$ of the dressed eigenstates $|n, \pm\rangle$, obtained by direct diagonalization of \hat{H}_{JC} , are given by

$$\omega_{n,\pm} = n\omega_c \pm g_{\text{eff}}\sqrt{n}. \quad (4)$$

The resulting nonlinearity is depicted in Fig. 2.

This system can be driven by a classical probe field, which either illuminates the atoms on the g - i transition or feeds the cavity via one of the cavity mirrors. In the following we shall consider the latter solution. Due to the cavity and atomic

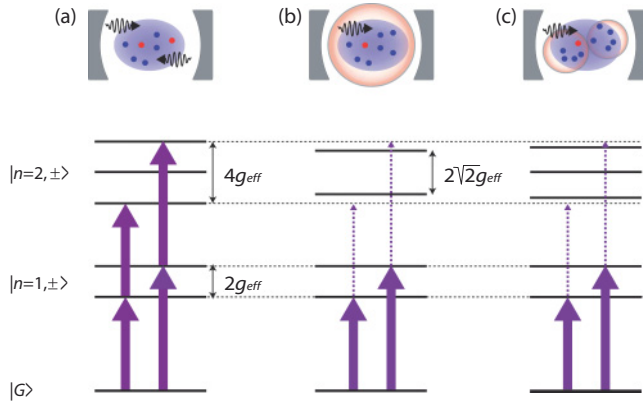


FIG. 2. (Color online) Dressed ladder states of the atoms-cavity system: (a) without interatomic interaction, the system would be formally equivalent to two coupled oscillators and transition frequencies between the multiplicities are degenerate; (b) when the interaction between excited atoms is strong enough for the atomic sample to be fully dipole-blocked, the system is described by the JCM, and the nonlinearity prevents simultaneous excitation of several multiplicities; (c) for larger atomic samples or weaker dipole interaction, several Rydberg “bubbles” can appear, and the system’s spectrum is described by “combinatorially dressed states,” still preserving a nonlinearity for small enough bubble numbers.

decay, the combined field-atom system will reach a steady state after a few relaxation times, i.e., a few microseconds for the specific system under consideration. If the driving field is weak, this state only slightly differs from the ground state $|G, n=0\rangle$ and, depending on the driving frequency, we may estimate the probability amplitude with which the system populates the excited eigenstates $|n, \pm\rangle$ of the unperturbed Jaynes-Cummings model.

A. One-photon resonance

To excite the system, the driving frequency should match one of the two dressed state components at $\omega_c \pm g_{\text{eff}}$, shifted by the so-called vacuum Rabi splitting. Directly measuring the absorption or transmission of the cavity as a function of the driving frequency, thus reveals the collectively enhanced coupling strength g_{eff} . The width of the resonances is mainly governed by the cavity decay (as noted above the atomic decay rate γ is negligible compared to the cavity decay rate κ). With $g_{\text{eff}} \sim 2\pi \times 10$ MHz and $\kappa \sim 2\pi \times 1$ MHz, the two resonance lines are then clearly split, and it is possible to selectively excite one of the dressed states $|n=1, \pm\rangle \equiv (|G, 1\rangle \pm |E, 0\rangle)/\sqrt{2}$. If the probe field couples to the cavity mode according to the Hamiltonian

$$V = \alpha^* e^{i\omega t} \hat{a} + \alpha e^{-i\omega t} \hat{a}^\dagger, \quad (5)$$

the coupling strength of the ground state to either of the dressed states is the same, given by $\beta_1 = \langle n=1, \pm | V | G, 0 \rangle = \alpha^*/\sqrt{2}$. We therefore expect these states to be excited with a probability $p_1 \sim |\beta_1|^2/(\delta^2 + \kappa^2/4)$, where δ denotes the probe frequency detuning with respect to the dressed state eigenfrequency. The dressed state populated is a superposition of the atomic and field excited state, and the mean photon number inside the cavity is expected to be $\langle n \rangle = \frac{1}{2} p_1$.

To analyze the problem theoretically, we have carried out numerical Monte Carlo simulations of the dynamics of the driven atom + cavity system. The cavity decay occurs by emission of photons [34], and we simulate the detection of these emission events by a photon counter in the Monte Carlo wave function (MCWF) formalism [35,36]. Such simulations

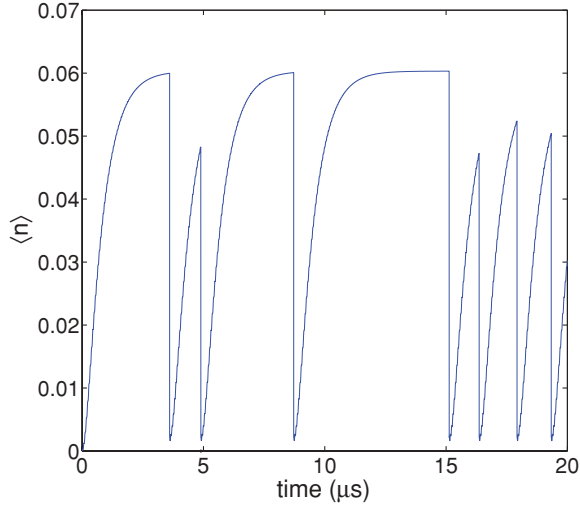


FIG. 3. (Color online) Detection record for one-photon resonance ($\alpha = 2\pi \times 0.15$ MHz).

do not converge to a steady state; they rather present a stochastic dynamics with click events followed by transient evolution until the next click event is detected. When averaged over many independent realizations of this stochastic process, one recovers the predictions of the master equation. Moreover, the individual stochastic simulations are also useful, as they provide typical records of the randomly selected detection events which is, indeed, information similar to the one obtained in a real transmission experiment. Experimental trajectories similar to Monte Carlo simulated trajectories, agreeing in mean value with the master equation evolution, have for example been observed, in a different system, in [34,37].

Figure 3 shows the result of a simulation, where we plot the mean photon number in the cavity $\langle n \rangle$ as a function of time. The graph shows characteristic oscillations at the Rabi frequency of the transition, interrupted by sudden jumps corresponding to the detection of a photon. At jumps the intracavity photon number drops to nearly zero, while the state of the system, in the dressed state superposition of $|G,1\rangle$ and $|E,0\rangle$ just before the measurement, collapses to the ground state $|G,0\rangle$. One also observes an immediate consequence of these jumps: The system is unable to emit another photon until it has been re-excited, hence the clicks on the detector are predicted to arrive at a rate given by the mean excited state population and to be antibunched on the time scale of $1/\beta_1$.

B. Two-photon resonance

Recently [38], Schuster *et al.* studied the Jaynes-Cummings model of a single atom in an optical cavity, and they identified a peak in the transmitted power, when the cavity is driven at one of the frequencies $\omega_c \pm g_{\text{eff}}/\sqrt{2}$. Indeed, the driving field is then resonant—by two-photon absorption—with one of the dressed states $|n = 2, \pm\rangle$ with respective energies $\omega_{2,\pm} = 2\omega_c \pm g_{\text{eff}}\sqrt{2}$. This is a difficult experiment, both because this resonance is only a few decay widths away from the single photon resonance and because the light atom interaction may cause heating of the atomic motion in the frequency range giving the optimal resolution of the resonance.

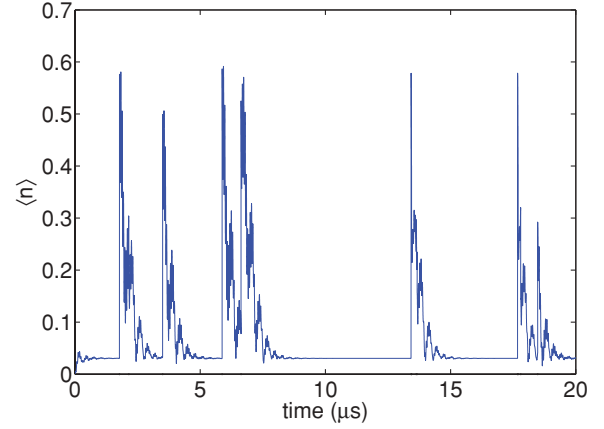


FIG. 4. (Color online) Detection record for two-photon resonance ($\alpha = 2\pi \times 1.5$ MHz).

With our parameters, the splitting is larger, and due to the long lifetime of the atomic Rydberg excited states, the light induced heating will be much reduced. The two-photon transition between the ground state $|G,0\rangle$ and the dressed states $|n = 2, \pm\rangle$ is detuned from the intermediate states $|n = 1, \pm\rangle$ by the amount $\omega_c \pm g_{\text{eff}}\sqrt{2} - (\omega_c \pm g_{\text{eff}}/\sqrt{2}) = \pm g_{\text{eff}}(1 - 1/\sqrt{2})$. The two-photon coupling strength is hence estimated to be $\beta_2 \sim \pm(\alpha/\sqrt{2})\alpha(1 + \sqrt{2})/[g_{\text{eff}}(1 - 1/\sqrt{2})] \sim \pm 3\alpha^2/g_{\text{eff}}$. Accordingly, the steady-state population of the doubly excited dressed state is $p_2 \sim |\beta_2|^2/(\delta^2 + \kappa'^2)$, where κ' denotes the coherence decay rate of the photonic components of the dressed states. Since the dressed state is composed of equal weight components with a single and two photons, the mean photon number is expected to be $\langle n \rangle = \frac{3}{2}p_2$ and $\kappa' = 3\kappa/2$ in this case.

Again, we carried out simulations which confirmed the existence of the expected resonance. This time, however, the simulation record differs significantly from the results of the previous section, as shown in Fig. 4. The mean photon number is low, as is the average transmitted flux, but every time a photon is detected, we now see a drastic *increase* in the mean photon number in the cavity. This has a simple explanation, since the state prior to the detection event is a superposition of the ground state of the system and a doubly excited dressed state. The back action of the detection of a single photon, the quantum jump, is implemented by the action of the field annihilation operator \hat{a} , which replaces the state before the detection event by

$$\begin{aligned} |\Psi\rangle_{\text{jump}} &\propto \hat{a}\{|G,0\rangle + \sqrt{p_2}(|G,2\rangle \pm |E,1\rangle)\} \\ &\propto \sqrt{2}|G,1\rangle \pm |E,0\rangle. \end{aligned} \quad (6)$$

The mean photon number in this state is $2/3$, which is much larger than the potentially nearly infinitesimal time averaged photon number in the cavity. The figure shows the mean photon number evaluated with the stochastic wave function method, and the transient peaks confirm our simple analysis. The detection record also shows a strong bunching effect: A majority fraction of the jumps are followed by a second jump within the cavity lifetime. A dedicated experiment should be able to verify these nonclassical intensity correlations, and in a future perspective, one may even imagine the possibility to

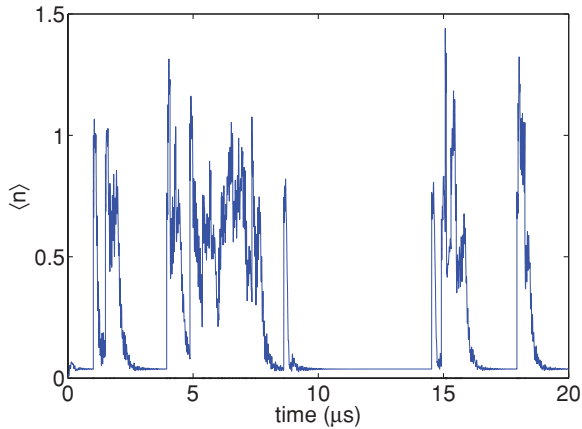


FIG. 5. (Color online) Detection record for three-photon resonance ($\alpha = 2\pi \times 1.25$ MHz).

apply feedback and modify the driving field immediately after the first click event and thus prepare a variety of other quantum states of the system [39].

C. Three-photon resonance

More generally, the spectrum Eq. (4) predicts the existence of n -photon resonances at the driving frequencies $\omega_n = (n\omega_c \pm g_{\text{eff}}\sqrt{n})/n = \omega_c \pm g_{\text{eff}}/\sqrt{n}$. For the specific experimental setup considered here, these resonances are well distinguishable, even for $n > 2$: for example, the three-photon resonance is well separated by a few line widths from the two- and four-photon resonances. Of course, the three-photon excitation is a higher-order process with intermediate nonresonant one- and two-photon virtual excitations of the system, and it competes with the nonresonant excitation of the system by lower-order processes. All these processes are included in the full numerical simulations of the system.

Figure 5 shows a simulated detection record for a driving field resonant with the triply excited dressed state of the atoms+cavity system. This time, our simple analysis suggests that clicks cause a quantum jump of the state from a superposition of the ground state and $(|G,3\rangle \pm |E,2\rangle)$ toward $\sqrt{3}|G,2\rangle \pm \sqrt{2}|E,1\rangle$, which has an average photon number of $8/5$. Within the cavity lifetime, one should thus expect bursts of two to three photon detection events.

Such bunching of detection events is clearly seen in Fig. 5. But when we zoom in on the detection record (see Fig. 6), we observe somewhat surprising ‘‘bursts’’ of three and four and even up to seven detection events, which contradict our simple picture of the dynamics. Looking more closely into the state conditioned on the subsequent detection events, we have identified two mechanisms which seem to contribute to the observed effect: (i) Not only does the system populate the ground and triply excited dressed states but also a small off-resonant excitation of higher-lying states occurs. The corresponding population is then amplified by the action of the annihilation operators and the \sqrt{n} factors associated with the detections of the first, second, and third photons. (ii) The system evolves between clicks, due to both the driving field and the states of

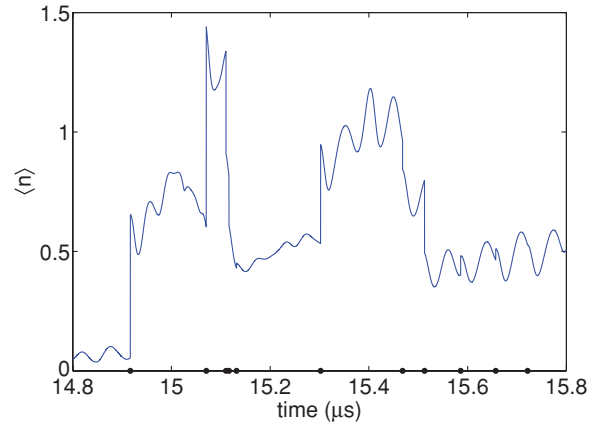


FIG. 6. (Color online) Detection record for three-photon resonance (magnified view). Each click is indicated by a bullet symbol on the time axis ($\alpha = 2\pi \times 2$ MHz).

the form $\sqrt{3}|G,2\rangle \pm \sqrt{2}|E,1\rangle$ resulting from the first click not being eigenstates of the atoms-cavity coupling. Also, the small probability amplitude in excited states, which is being magnified by a quantum jump, contributes coherently to the buildup of further excited-state population by the coherent probe field. We have qualitatively verified the last point by setting the probe amplitude α to zero after the first click in our simulations and observed a reduction of the number of transient detection events.

This multiphoton burst phenomenon constitutes an interesting experimental effect, but we will not investigate it further at this point.

IV. COMBINATORIAL EXTENSION OF DIPOLE BLOCKADE MECHANISM

In the previous sections, we suggested to take advantage of the full Rydberg blockade in a small atomic ensemble to effectively implement the JCM in the strong coupling regime and we studied the transmission properties of such an ensemble-cavity system. In this section, we shall see how the coupling of the atoms to the cavity can actually be used to extend the range of the blockade mechanism beyond the limit fixed by the length ℓ_R defined in Sec. II.

In the absence of interatomic interactions, i.e., for ensembles with interatomic separations larger than ℓ_R , the atomic ensemble and the cavity mode behave like two coupled oscillators leading to two atom-field eigenmodes with frequencies $\omega_c \pm g_{\text{eff}}$ [Fig. 2(a)]. Driving the cavity with a classical probe field tuned on one of these frequencies, one resonantly excites a coherent state of the corresponding eigenmode equivalent to product of coherent states of the field and the collective atomic oscillator comprising all the Rydberg excitation number states. In the opposite limit, i.e., for ensembles whose linear dimension is of the order of ℓ_R , the dipole-dipole interactions between Rydberg atoms are so strong that they shift the multiply Rydberg excited states out of resonance: The ensemble is equivalent to a two-level atom, and the entire system is described by the JCM, Eq. (3) [Fig. 2(b)].

Let us now turn to the intermediate regime where the extent of the ensemble is larger than ℓ_R but the Rydberg dipole-dipole interactions still play an important role in the system. In that case, the interaction does not shift all the multiply excited states out of resonance and we may identify an unshifted doubly excited collective state

$$|E_2\rangle \equiv \frac{1}{\sqrt{\mathcal{A}}} \left(\sum_{|\vec{r}_j - \vec{r}_k| > \ell_R} |e_j, e_k\rangle \langle g_j, g_k| \right) |G\rangle \quad (7)$$

in which the excited atoms (j, k) are always too far apart from each other to interact strongly. The normalization constant \mathcal{A} can be evaluated by counting the number of states $|e_j, e_k\rangle$ which contribute to $|E_2\rangle$, i.e., the number of pairs of atoms in the ensemble which are separated by a distance larger than ℓ_R . This state is thus resonantly accessible from the first excited state $|E\rangle$ via the two-photon absorption from the cavity and the blue laser classical field. The Hamiltonian Eq. (3) must therefore be complemented by the term

$$2\hbar\omega_c |E_2\rangle \langle E_2| + \hbar g_{\text{eff}} \langle E_2 | \hat{S}^\dagger | E \rangle |E_2\rangle \langle E | \hat{a} + \text{H.c.}$$

More generally, due to the size of the sample, the resonant coupling by the cavity mode and laser fields to higher multiply excited states $|E_3\rangle, |E_4\rangle, \dots$ in which $n_e = 3, 4, \dots$ Rydberg atoms are separated by more than ℓ_R , may be possible. We can visualize the system as if the Rydberg blockade phenomenon decomposes the ensemble into a number of “bubbles” of radius ℓ_R [40]. In each bubble only a single Rydberg excitation is possible and the number of atoms which can be simultaneously excited in the ensemble coincides with the number of bubbles, $n_b \simeq \frac{V}{\frac{4}{3}\pi\ell_R^3}$. For a mesoscopic sample which can thus accommodate n_b Rydberg excitations, the Hamiltonian Eq. (3) must therefore be complemented by $(n_b - 1)$ terms coupling $|E_0\rangle \equiv |G\rangle$ to $|E_1\rangle \equiv |E\rangle$, $|E_1\rangle$ to $|E_2\rangle$, $|E_2\rangle$ to $|E_3\rangle$, \dots , $|E_{n_b-1}\rangle$ to $|E_{n_b}\rangle$.

The Rydberg interaction falls off in a continuous manner, and hence the bubbles represent a rather crude description of the multiply excited states, and while $|G\rangle$ and $|E_1\rangle$ constitute a closed two-state system under the interaction with the cavity field, states with more than a single excitation may populate higher-dimensional subspaces than indicated by the states $|E_{k>1}\rangle$. The purpose of the current analysis, however, is not to analyze the dynamics of higher excited states but to identify a mechanism that prevents the system from exploring states above $|E_1\rangle$. This mechanism relies on our driving of the cavity on the resonance frequency of one of the first excited dressed states ($|G, 1\rangle \pm |E_1, 0\rangle$)/ $\sqrt{2}$. A second photon from the driving field will be resonantly absorbed if the excited state manifold of two excitations contains a state at twice the driving field frequency. This indeed occurs if the atoms do not interact at all, while any suppression of excitation reduces the coupling between $|E_1, 1\rangle$ and collectively excited states with two atomic excitations and hence causes the doubly excited dressed state energies to lie at very different values. To make a simple estimate of this mechanism, we apply the bubble picture, but we recall that our calculation will only be used to quantify to which extent the excitation terminates at a single excitation.

In this simple analysis, the ensemble-cavity dynamics is thus governed by the Hamiltonian

$$H = \hbar\omega_c \hat{a}^\dagger \hat{a} + \hbar\omega_c \left(\sum_{k=1}^{n_b} k |E_k\rangle \langle E_k| \right) + \hat{a} \left(\sum_{k=1}^{n_b} g_k |E_k\rangle \langle E_{k-1}| \right) + \text{H.c.} \quad (8)$$

The coupling strengths g_k can be easily evaluated in the “bubble” picture, since each bubble provides an effective two-level atom and the states $|E_k\rangle$ are the symmetrically excited states of these effective two-level systems with precisely k excitations. These states, in turn, are equivalent to effective angular momentum states $|j, m\rangle$ with the quantum numbers $j = n_b/2$ and $m = -n_b/2 + k$.

The coupling strengths g_k are given by the well-known matrix elements of the angular momentum raising operator multiplied by the effective two-state coupling of each bubble, $\langle k | \hat{j}^\dagger | k-1 \rangle$, and we get

$$\begin{aligned} g_k &= \sqrt{(2j - k + 1)k} \sqrt{\frac{N}{n_b} \frac{g_0^* \Omega}{\Delta}} \\ &= \sqrt{(n_b - k + 1)k} \sqrt{\frac{N}{n_b} \frac{g_0^* \Omega}{\Delta}}. \end{aligned} \quad (9)$$

Setting $k = 1$, we recover the expected collective coupling g_{eff} to the first excited states, while

$$g_2 = \sqrt{1 - \frac{1}{n_b}} \sqrt{2} \sqrt{N} \frac{g_0^* \Omega}{\Delta}, \quad (10)$$

which vanishes for $n_b = 1$ as it should, and which approaches the expected $\sqrt{2}g_{\text{eff}}$ in the nonblocked oscillator limit of large n_b .

Any finite value, and in practice a not too large value, of n_b thus breaks the coupled oscillator picture, and hence the excitation spectrum of the coupled system will not constitute an equidistant ladder of states. In particular, the resonant excitation of the lower component $|1, -\rangle$ is red detuned with respect to the next excitation step, cf. Fig. 2(c), and the system thus behaves as if the Rydberg blockade extends over the entire system across different “bubbles.” Assuming that the Rydberg interaction energy is positive for all pairs of excited atoms (a property which is fulfilled for a wide range of Förster resonant states [41]), the field is also significantly red detuned from any interacting pair of excited atoms, and hence only the single excitation dressed state $|1, -\rangle$ is resonantly excited by the laser field. The absence of simultaneous excitation of several bubbles is not due to their mutual interaction but due to the coupling strength to higher excited states which is modified because of the number of pairs that contribute to this excitation. The blockade behavior is thus due to a combinatorial effect. Diagonalizing the Hamiltonian (8), within the space of two excitations, we find the eigenvalues $0, \pm \sqrt{2 - \frac{1}{n_b}} \hbar \sqrt{2} g_{\text{eff}}$, and hence the field resonant with the

first dressed state $|1, -\rangle$ is red detuned from the two-photon resonance by $-2(1 - \sqrt{1 - \frac{1}{2n_b} g_{\text{eff}}}) \sim \frac{g_{\text{eff}}}{2n_b}$.

This analysis suggests that we can extend the blockade over only a few, say fewer than 10, “bubbles.” We intend to perform more detailed numerical studies of the collectively excited atoms. The picture with a given fixed set of bubbles of atoms is not an exact one as any atom has the same amplitude to be excited and hence form the center of a bubble of excluded excitations around it, and the full many-body state is a superposition of correspondingly different bubble configurations [42–44]. We do expect, however, that our combinatorial argument is robust to finer details of the description and that any two-atom collective component will not be excited at the same frequency as the dressed states with a single collective excitation.

V. DISCUSSION

We have suggested to use the collective coupling of ~ 1000 atoms to a cavity field combined with the Rydberg blockade effect to restrict the atomic ensemble to an effective two-level system. Physical parameters realistically reach the JCM of strong coupling CQED, and we have shown that exploration of higher excited states of the dressed states ladder of this model should be possible by optical transmission experiments.

We have analyzed the system dynamics with Monte Carlo wave functions, and we note that the transients following the simulated detection of a transmitted photon represent the evolution happening in a real experiment conditioned on photon detection. These transients explore states of the light field which may be highly nonclassical and which may be further explored or manipulated by a suitable feedback operation, e.g., along the lines of [45].

In a longer perspective, many more possibilities can be investigated. One may take advantage of the atomic sublevel structure to significantly enrich the phenomenology with collective qubit encoding [46], multimode atomic storage states, STIRAP processes [47], and several quantum field components may couple to Rydberg states with different pairwise interaction properties [48] to accommodate effective optical nonlinearities [49].

In this work, we disregarded the role of atomic motion, but we note that even though the blockade regime precludes strong interatomic forces as only single atoms are excited, the position-dependent collective field-atom coupling may induce mechanical motion entangled with the field and atomic internal state degrees of freedom.

Another interesting direction of research, involves the examination of larger systems. In such systems, the Rydberg blockade is only efficient in blockade spheres around each atom, and remote atoms may be simultaneously excited. We have proposed that the combinatorics of the atoms-cavity coupling may distort the ladder of energies of singly and multiply occupied Rydberg states and thus maintain the blockade effect over a larger atomic ensemble. This should be more quantitatively addressed in future work.

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