Periodic revival of entanglement of two strongly driven qubits in a dissipative cavity

Marcin Dukalski and Ya. M. Blanter
Kavli Institute of Nanoscience, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands

(Received 8 July 2010; revised manuscript received 11 August 2010; published 24 November 2010)

We present a phenomenon of an asymptotic revival of bipartite entanglement between two stable solid-state qubits interacting via a single-mode cavity subject to dissipation. This effect is achievable in cavity quantum electrodynamics systems, assisted with strong classical pumping detuned from the cavity eigenmode, under the assumption of short system-bosonic reservoir correlation times. Moreover, we prove that this effect is independent of the initial cavity state and that all initially prepared Bell states experience the same qualitative effects. We present a method that can be used to generalize an arbitrary number of solid-state qubits.

DOI: 10.1103/PhysRevA.82.052330 PACS number(s): 03.67.Mn, 03.65.Yz, 03.67.Bg, 78.47.jp

I. INTRODUCTION

Entanglement is a resource essential for successful implementation of quantum information processing [1], and engineering sustainable entanglement is a necessary requirement for any physical realization of a quantum computer. Since a number of solid-state qubits have been successfully realized and single qubit operations have been demonstrated [2], the attention of experimentalists has now shifted toward interactions between qubits. It is therefore of vital importance to study processes that can generate or alter entanglement through an interaction between two or more qubits.

In many realizations it is easier to use cavity quantum electrodynamics (CQED) systems [3] and couple solid-state qubits via an optical or a microwave cavity rather than directly. However, the dissipation inherent to the cavity is detrimental for the entanglement. It is known that qubit or cavity dissipation can very quickly lead to disentanglement of two qubits [4–8]. For carefully chosen initial conditions and system parameters, a period of disentanglement can be followed by a finite period of the entanglement revival; however, the predicted degree of entanglement after revival is typically about 1%–10% of its initial value [9]. Additionally, a small amount of entanglement can be created to later undergo a decay [10]. Moreover, the concept of entanglement revival has been studied outside the Markovian regime [11–13].

In this article we prove that there are systems where entanglement is robust against Markovian dissipation. We find an entanglement behavior where two initially entangled qubits experience periodic entanglement drops and revivals to asymptotically recover its initial value. This phenomenon can be achieved with two qubits driven by a classical ac off-resonant field and coupled via a dissipative cavity so that the direct interaction between the qubits is negligible. The dissipation of the qubits must be sufficiently weak. Lastly we show that this result is universal with regard to the initial amplitude of the coherent state of the cavity. Our theory can be experimentally realized in superconducting qubits coupled to a microwave cavity [14] or a nitrogen-vacancy (NV) center in diamond strongly coupled to an optical cavity [15].

This paper is structured as follows. In Sec. II we present the derivation of the effective multiqubit Hamiltonian in the interaction picture. Afterward we find the solutions for a problem of a single qubit interacting with a coherent mode of radiation in a dissipative cavity, which is followed by an extension of this problem to the two-qubit case. In Sec. III we analyze the temporal dynamics of entanglement of this system. We state our conclusions in Sec. IV.

II. THE MODEL

The interaction of a qubit and a cavity is commonly described in terms of the Jaynes-Cummings model (JCM) [16], which is one of the few interacting quantum systems admitting closed-form solutions. The JCM and its several variants have become a textbook tool to discuss coupled qubit and photon systems. Recently, it has been realized that the qubit-field interaction with an additional strong driving also can be solved analytically [17–19] even if the cavity dissipation is also included in the system. Moreover, in Refs. [20,21] it has been proven that the solutions to the equations of motion for strongly driven qubits interacting through a cavity vacuum field can be extended to an unlimited number of qubits, which cannot be achieved in the simple JCM Hamiltonian.

To keep the treatment general, we consider a Hamiltonian of a system of N identical qubits coupled to a single-mode cavity and additionally driven by a classical electromagnetic field [19],

$$\hat{H} = \frac{\Omega}{2}\sum_{j=1}^{N}\sigma_j^z + \omega\hat{a}^\dagger\hat{a} + A\sum_{j=1}^{N}(e^{-i\omega_c t}\sigma_j^+ + e^{i\omega_c t}\sigma_j^-) + \sum_{j=1}^{N} g_j(\sigma_j^+\hat{a} + \sigma_j^-\hat{a}^\dagger), \tag{1}$$

where Ω is the level spacing of the qubits, ω is the frequency of the eigenmode of the cavity, A and ω_c are the amplitude and the frequency of the classical field, and g_j is the coupling strength between the jth qubit and the cavity mode. In addition to that σ^z and $\sigma^\pm = \frac{1}{2}(\sigma^x \pm i\sigma^y)$ are the (linear combinations of) Pauli matrices and $\hat{a}$ ($\hat{a}^\dagger$) is the annihilation (creation) operator of the quantum field modes. See Fig. 1. Throughout the article we set $\hbar = 1$. We assume that the qubits are driven strongly and that they are very stable and moderately coupled to the cavity mode, $A \gg \omega, \omega_c, |\delta| \gg g \gg \gamma$, where $\delta = \omega - \omega_c$ is the cavity mode-driving field detunning and γ stands for the qubit decay rates. Therefore, we can ignore the qubit dephasing or decoherence rates as well as the energy-violating ("counterrotating") Rabi Hamiltonian terms [19], $\sigma^+\hat{a}^\dagger$ and

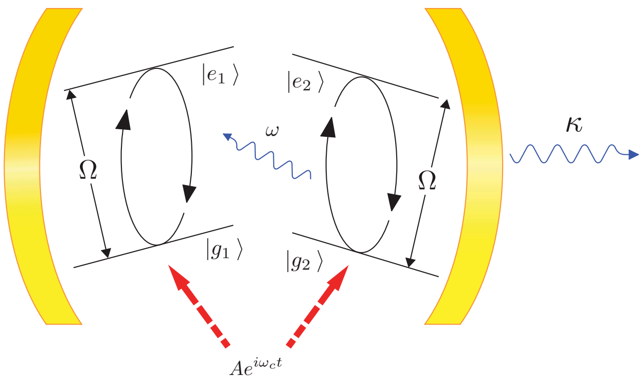

FIG. 1. (Color online) Schematic representation of the setup.

$\sigma^-\hat{a}$. Additionally, we take the classical field to be sufficiently off-resonant, $\omega_c \neq \omega$, so that we can ignore the classical field-cavity coupling. In previous works [20,22] the coupling strengths of the two qubits were taken to be identical; however, here we insist on keeping them different. We recall that the coupling strength is proportional to the scalar product of the dipole moment and the polarization of the cavity eigenmode, so it is very unlikely that two qubits will couple in exactly the same way. We begin by applying an entanglement-preserving time-local unitary transformation

$$\hat{H} \to \hat{H}' = \hat{U}^\dagger \hat{H}\hat{U} - i\hat{U}^\dagger \partial_t \hat{U},$$
$$|\psi\rangle \to |\psi'\rangle = \hat{U}|\psi\rangle,$$

with $\hat{U} = \exp(-i\omega_c t\hat{a}^\dagger\hat{a} - i\omega_c t\sum_j \sigma_j^z/2)$. The resulting Hamiltonian now takes the form

$$\hat{H} = \hat{H}_o + \hat{H}_I,$$
$$\hat{H}_o = \frac{1}{2}\Delta\sum_{j=1}^N \sigma_j^z + \delta\hat{a}^\dagger\hat{a} + A\sum_{j=1}^N(\sigma_j^+ + \sigma_j^-),$$
$$\hat{H}_I = \sum_{j=1}^N g_j(\sigma_j^+\hat{a} + \sigma_j^-\hat{a}^\dagger),$$

with $\Delta = \Omega - \omega_c$. The interaction picture Hamiltonian $\mathcal{V} = e^{-i\hat{H}_o t}\hat{H}_I e^{i\hat{H}_o t}$ upon setting the qubits in resonance with the classical field $\Delta = 0$ yields

$$\mathcal{V} = \sum_{j=1}^N \frac{1}{2} g_j(|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j| + e^{2iAt}|+_j\rangle\langle -_j| - e^{-2iAt}|-_j\rangle\langle +_j|)\hat{a}e^{-i\delta t} + \text{H.c.},$$

where $|\pm_j\rangle = \frac{1}{\sqrt{2}}(|e_j\rangle \pm |g_j\rangle)$ are the eigenstates of the Pauli σ^x matrix in the jth qubit space. Disregarding the quickly rotating terms and redefining $\frac{1}{2}g_j \to g_j$, we obtain

$$\mathcal{V} = \sum_{j=1}^N g_j\sigma_j^x(\hat{a}e^{-i\delta t} + \hat{a}^\dagger e^{i\delta t}) = \sum_{j=1}^N \mathcal{V}_j. \tag{2}$$

A. Single qubit master equation

Let us first focus on an interaction between a single qubit $N = 1$ and a coherent state of the cavity α. The evolution of this system in a dissipative cavity is driven by the Lindblad-type master equation,

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[\hat{\mathcal{V}},\rho] + \kappa\mathcal{D}(\rho), \tag{3}$$

where $\mathcal{D}(\rho) = 2\hat{a}\rho\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\rho - \rho\hat{a}^\dagger\hat{a} \equiv 2\mathcal{M}(\rho) - \mathcal{R}(\rho) - \mathcal{L}(\rho)$ is the so-called dissipation operator and κ represents the cavity decay rate.

Using the interaction picture Hamiltonian and expressing the qubit density matrix in the $|\pm\rangle$ basis as $\rho(t) = p_{kl}(t)|k\rangle\langle l|$, $k,l = \pm$, one can write the equations of motion (3) for individual density-matrix entries,

$$\dot{p}_{++}(t)|\alpha\rangle\langle\alpha| = -ig[\hat{a}e^{-i\delta t} + \hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|]p_{++}(t) + p_{++}(t)\kappa\mathcal{D}(|\alpha\rangle\langle\alpha|), \tag{4}$$

$$\dot{p}_{+-}(t)|\alpha\rangle\langle\alpha| = -ig\{\hat{a}e^{-i\delta t} + \hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|\}p_{+-}(t) + p_{+-}(t)\kappa\mathcal{D}(|\alpha\rangle\langle\alpha|). \tag{5}$$

Here $[\cdot,\cdot]$ ($\{\cdot,\cdot\}$) denote (anti)commutator brackets. Additionally, equations for $p_{--}(t)$ and $p_{-+}(t)$ are obtained by substituting $g \to -g$ in Eqs. (4) and (5), respectively.

These decoupled equations can be solved using the super-operator method [21,23] assuming that the cavity is initiated in the coherent state $|\alpha\rangle$,

$$p_{++}(t) = e^{g\mathcal{A}/\delta + \kappa\mathcal{D}t}|\alpha\rangle\langle\alpha|p_{++}(0), \tag{6}$$
$$p_{+-}(t) = e^{g\mathcal{B}/\delta + \kappa\mathcal{D}t}|\alpha\rangle\langle\alpha|p_{+-}(0), \tag{7}$$

where we define

$$\mathcal{A} = c_-(t)\,\hat{a}(\cdot) - c_+(t)\,\hat{a}^\dagger(\cdot) - c_-(t)(\cdot)\,\hat{a} + c_+(t)(\cdot)\,\hat{a}^\dagger,$$
$$\mathcal{B} = \mathcal{X} + \mathcal{Y},$$
$$\mathcal{X} = 2[c_-(t)\,\hat{a}(\cdot) - c_+(t)(\cdot)\,\hat{a}^\dagger],$$
$$\mathcal{Y} = c_-(t)(\cdot)\,\hat{a} - c_-(t)\,\hat{a}(\cdot) - c_+(t)\,\hat{a}^\dagger(\cdot) + c_+(t)(\cdot)\,\hat{a}^\dagger,$$
$$c_\pm(t) = \pm i\delta\int_0^t e^{\pm i\delta t'}dt' = e^{\pm i\delta t} - 1.$$

Next, using the commutation relations (A1) to apply the Baker-Campbell-Hausdorff formula to decompose expressions in (6) and (7), we study the effects of the operators on the coherent state (see Appendix A for details) to obtain

$$p_{++}(t) = e^{\frac{g}{\delta\kappa t}(1-e^{-\kappa t})\mathcal{A}}e^{\kappa\mathcal{D}t}p_{++}(0)|\alpha\rangle\langle\alpha| = p_{++}(0)|\alpha e^{-\kappa t} - f(t)c_+(t)\rangle\langle\alpha e^{-\kappa t} - f(t)c_+(t)|, \tag{8}$$

$$p_{+-}(t) = e^{h_1(t)}e^{\frac{g}{\delta t\kappa}(1-e^{-\kappa t})\mathcal{Y}}e^{\kappa\mathcal{D}t}e^{-\frac{g}{\delta t\kappa}(1-e^{-\kappa t})\mathcal{X}}|\alpha\rangle\langle\alpha|p_{+-}(0) = e^{h_1(t)+h_2(t)}p_{+-}(0)|\alpha e^{-\kappa t} - f(t)c_+(t)\rangle\langle\alpha e^{-\kappa t} + f(t)c_+(t)|. \tag{9}$$

In the above we defined

$$f(t) = \frac{g}{\delta t\kappa}(1 - e^{-\kappa t}),$$
$$h_1(t) = -(1-\cos\delta t)\left(\frac{8g^2}{\delta^2 t^2\kappa^2}(e^{-\kappa t} - 1 + \kappa t) + 4f(t)^2\right),$$
$$h_2(t) = -2if(t)(2 - e^{-\kappa t})[\text{Im}(\alpha)\alpha(\cos\delta t - 1) - \text{Re}(\alpha)\sin\delta t].$$

B. Two qubits interacting via a cavity

Extending this treatment to two qubits in a single cavity requires taking another copy of the interaction picture Hamiltonian (2). The only difference is that now there will be more Hamiltonians $\hat{\mathcal{V}}_i$ acting separately on different qubit states and jointly on the same cavity state. Working in the eigenbasis of the $\sigma_1^x \otimes \sigma_2^x$ operator one obtains a set of decoupled equations. The solutions are obtained analogously to a single qubit case (see Appendix B).

Tracing out the cavity leads to a two-qubits reduced density matrix

$$\rho_{q_1,q_2} = \begin{pmatrix} \rho_{++;++}(0) & \varrho^-(g_2)\rho_{++;+-}(0) & \varrho^-(g_1)\rho_{++;-+}(0) & \varrho^-(g_1+g_2)\rho_{++;--}(0) \\ \varrho^+(g_2)\rho_{+-;++}(0) & \rho_{+-;+-}(0) & \varrho^-(g_1-g_2)\rho_{+-;-+}(0) & \varrho^-(g_1)\rho_{+-;--}(0) \\ \varrho^+(g_1)\rho_{-+;++}(0) & \varrho^+(g_1-g_2)\rho_{-+;+-}(0) & \rho_{-+;-+}(0) & \varrho^-(g_2)\rho_{-+;--}(0) \\ \varrho^+(g_1+g_2)\rho_{--;++}(0) & \varrho^+(g_1)\rho_{--;+-}(0) & \varrho^+(g_2)\rho_{--;-+}(0) & \rho_{--;--}(0) \end{pmatrix},$$

where we define

$$\varrho^{\pm}(\xi) = \exp\left(-\frac{8\xi^2(1-\cos\delta t)}{\delta^2 t^2 k^2}(e^{-kt} - 1 + kt)\right) \times \exp\left(-\frac{4i\xi}{\delta t k}(1-e^{-kt})^2[\mathrm{Im}(\alpha)(\cos\delta t - 1) - \mathrm{Re}(\alpha)\sin\delta t]\right).$$

In the above expression we see that the amplitude of the initial coherent state enters as an argument of the trigonometric functions of the imaginary part of the exponent. Effectively it only acts as a phase present in every element independently. Note that this phase is missing if the cavity is initiated in the ground state and that this phase will be inconsequential to our follow-up results.

III. ENTANGLEMENT EVOLUTION

Using the approach proposed in Ref. [24], we can now quantify the degree of entanglement of a 2×2 system by means of concurrence, defined as

$$C = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}),$$

where λ_i are the descending eigenvalues of the real matrix $R = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)\rho$. We will assume that the qubit pair is initialized in either of the generalized sets of Bell states

$$\Psi = \cos\theta|++\rangle + e^{i\phi}\sin\theta|--\rangle,$$
$$\Phi = \cos\theta|+-\rangle + e^{i\phi}\sin\theta|-+\rangle,$$

and afterward it is evolving according to the dynamics given by (3). As a result, the concurrence is a nontrivial function of time

$$C = \sin 2\theta e^{-8(1-\cos\delta t)(g_1 \pm g_2)^2(\kappa t + e^{-\kappa t} - 1)/(\kappa^2\delta^2 t^2)}, \qquad (10)$$

where the upper (lower) sign is used to denote the entanglement evolution of the Ψ (Φ) states. The graph for the evolution of both of these is plotted in Figs. 2 and 3. By approximating each of the peaks with a Gaussian we find that every consecutive maximum will have the form

$$C_n(t) = \sin 2\theta \exp\left(-\frac{(t - 2\pi n/\delta)^2}{\tau_n^2}\right),$$

where we used the standard deviation τ_n to be a measure of every consecutive revival time given by

$$\tau_n = \frac{2\sqrt{2}\kappa n\pi}{g_1 \pm g_2}[-2\kappa\pi\delta n + \delta^2(1 - e^{-\frac{2\kappa n\pi}{\delta}})]^{-\frac{1}{2}}. \qquad (11)$$

Equation (10) displays a number of striking properties. First, after an initial sharp decrease the concurrence periodically recovers its initial value $\sin 2\theta$, never exceeding it throughout. This confirms the previous result that qubit-qubit entanglement enhancement is not possible in this system [20,22], and complements the work in Ref. [23] where a qubit-cavity entanglement creation and partial revival have been observed in a dispersive regime. Second, the entanglement exhibits oscillatory behavior showing periodic revivals at $\delta t = 2n\pi$, with the revival time intervals $\tau_n \to \infty$ as $n,t \to \infty$. Third, the greater the rate of cavity decay κ (Fig. 4) or the degree of detuning δ (Fig. 5), the quicker is the recovery of the initially entangled state. The reason is that with greater κ the cavity eigenmode field depletes quicker and so the chance for qubits to interact with the quantum field decreases. This effect is enhanced if the qubits are detuned from the quantum

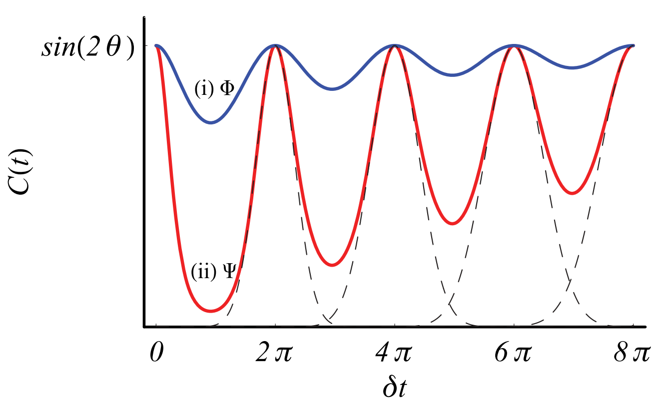

FIG. 2. (Color online) Concurrence for the Bell (i) Φ (blue) and (ii) Ψ (red) states. We see that the amplitude of variation is significantly smaller and the entanglement recovery speed is greater in the case of the latter states. Dashed lines are Gaussians with standard deviations given by Eq. (11). Plots were made for $g_1 t = 2g_2 t = 1$ and $\kappa t = 1$.

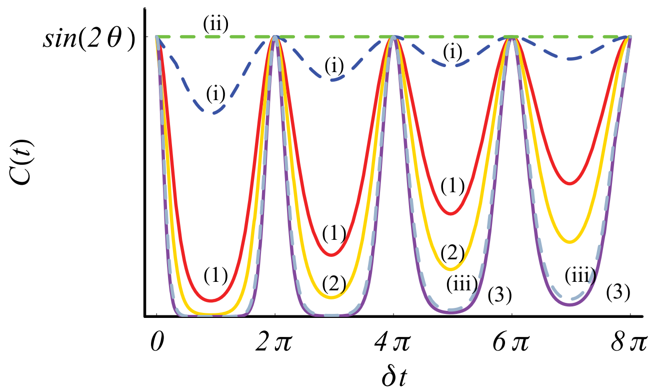

FIG. 3. (Color online) Concurrence for the Bell Ψ [red (1), gold (2), and violet (3) solid lines] and [blue (i), green (ii), and gray (iii) dashed lines] states. Plots were made for $\kappa t = 1$, and for (1) and (i) $g_1 t = 2g_2 t = 1$, for (2) and (ii) $g_1 t = g_2 t = 1$, and for (iii) and (3) $g_1 t = 30 g_2 t = 3$. If the coupling strengths are the same, (ii) Φ will experience no changes. If the relative coupling strength is large, (iii) and (3), the concurrences for Φ and Ψ are very similar.

eigenmode inhibiting interaction. As a result both of these effects lead to a decreased opportunity of disentanglement. Moreover, unlike Refs. [20,22], we have chosen to work with an arbitrary initial coherent state amplitude $\alpha(0) \neq 0$ to observe that its value plays no role in the qubit-qubit entanglement evolution, thus making this result universal for all cavities.

Finally we also find, in line with Ref. [20], that qubits initialized to different Bell states respond differently in this system. In the aforementioned reference the authors claimed that the concurrence of the Φ type states is unaffected by cavity dissipation. We find that this is only true if the qubits are

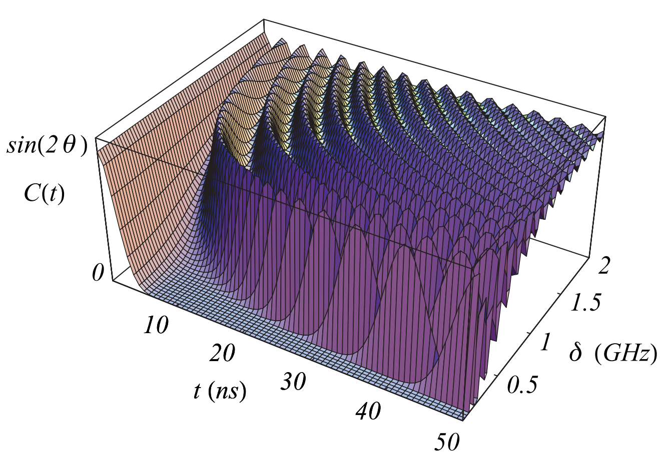

FIG. 4. (Color online) A plot of concurrence of Ψ as a function of time and detuning. For detunings $\delta \ll 1$ GHz we observe the entanglement behavior already noted by [22]. However, this is not what is known as entanglement sudden death as the two qubits do not disentangle in a finite time; here the $\delta = 0$ contour corresponds to a hyperexponential entanglement decay. For increasing values of detuning, the concurrence function reaches the steady-state maximum value quicker. Plots were made for $g_1 = 2g_2 = 0.2$ GHz and $\kappa = 0.1$ GHz.

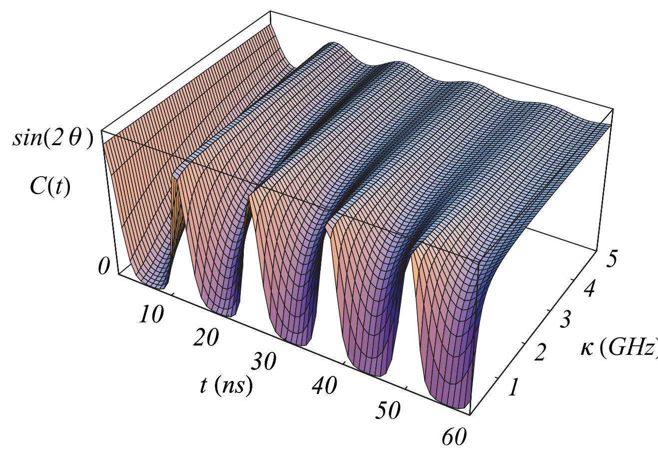

FIG. 5. (Color online) A three-dimensional plot of concurrence of Ψ as a function of the cavity decay rate and time. For increasing decay rates we observe that less entanglement is lost and its asymptotic recovery is quicker. Plots were made for $g_1 = 3g_2 = 0.3$ GHz and $\delta = 0.5$ GHz.

equally coupled to the cavity vacuum field. As a result all Bell states formed with unequally coupled qubits will decay and be revived, depending on the values of δ and κ; however, the Φ states will do it at a slower rate and the value of concurrence will drop to a lesser extent (see Fig. 2).

To simplify calculations, we have chosen the regime $A \gg \omega, |\delta| \gg g \gg \gamma$. These conditions can be realized in two types of solid-state qubits. In a superconducting qubit coupled to a microwave cavity [14], one can achieve $\omega \approx 5$ GHz, $g \approx 100$ MHz, and $\gamma \approx 1$ MHz. The critical parameter here is the qubit dissipation rate γ, which in our analytical study is assumed to be zero. We realize that this is an idealized case and we expect that the imposition of the condition $g \gg \gamma$ will not qualitatively affect the results; however, the quantitative aspect merits a separate investigation. Another system is a NV center in diamond strongly coupled to an optical cavity [15] and used as a spin qubit. For this realization, the coupling strength is the crucial parameter to observe the entanglement revival.

IV. CONCLUSIONS

We have presented a system exhibiting non-trivial entanglement dynamics, with asymptotic entanglement recovery being the most striking feature. Additionally, we show that the result is independent of the cavity coherent state chosen, and that no Bell states are strictly protected; however, they can be better secured by skillful adjustment of the detuning parameter.

We have disregarded the decay of the qubits, and all dissipation in our system originates from the cavity. It was shown in the literature that two directly interacting qubits can become entangled via a spontaneous decay [9,10]. However, in our case the entanglement between the qubits must be generated externally and does not result from the interaction with the cavity.

This study can be extended by considering the dynamics of a tripartite system composed of any Bell state and the coherent state of the cavity. Here one can consider the evolution of entanglement between any selected pair of subsystems and study entanglement creation between the cavity and the qubits during the qubit-qubit disentanglement phase. These results will be published elsewhere [25].

Moreover, by virtue of extendibility of this model to an arbitrary number of qubits as well as cavities, using this framework one could study multipartite entanglement and how, depending on the conditions and parameters choice (i.e., coupling strengths and decay rates), entanglement could be exchanged or transferred between different subsystems. Additionally, even with three qubits in the cavity, one could try to find more differences in evolution between two maximally entangled classes: the GHZ and the W states.

ACKNOWLEDGMENTS

The authors wish to thank Antonio Borras Lopez, Pol Forn-Diaz, and Toeno van der Sar for useful discussions. This work was supported by the Netherlands Foundation for Fundamental Research on Matter (FOM).

APPENDIX A: SINGLE QUBIT MASTER EQUATIONS AND DECOMPOSITION INTO INDIVIDUAL OPERATORS

Here we present in detail how expressions (8) and (9) are obtained from (6) and (7). These results are later used in Appendix B to find the two-qubit interaction solutions.

In order to obtain (8) from (6) we have to use the Bakker-Hausdorf-Cambell formula

$$e^{t(X+Y)} = e^{tX}e^{tY}e^{-\frac{t^2}{2!}[X,Y]}e^{\frac{t^3}{3!}([X,[X,Y]])}e^{\frac{-t^4}{4!}[[[X,Y],X],X]}\ldots$$

and the fundamental commutation relations

$$[\hat{a}\cdot,\hat{a}^\dagger\cdot]=1,\quad [\cdot\hat{a},\cdot\hat{a}^\dagger]=-1,$$

which finally lead to

$$\begin{aligned}[\mathcal{D},\mathcal{A}]=-\mathcal{A},\quad [\mathcal{D},\mathcal{X}]=\mathcal{X},\quad [\mathcal{D},\mathcal{Y}]=-\mathcal{Y},\\ [\mathcal{A},\mathcal{B}]=0,\quad [\mathcal{X},\mathcal{Y}]=-8(1-\cos\delta t).\end{aligned} \tag{A1}$$

As a result of the first commutator we get

$$\begin{aligned}e^{(\frac{g}{\delta}\mathcal{A}+\kappa\mathcal{D})t} &= e^{\kappa\mathcal{D}t}e^{\frac{g}{\delta}\mathcal{A}}e^{\frac{t^2}{2!}\frac{g}{\delta}\kappa\mathcal{A}}e^{\frac{t^3}{3!}\frac{g}{\delta}\kappa^2\mathcal{A}}e^{\frac{t^4}{4!}\frac{g}{\delta}\kappa^3\mathcal{A}}\ldots\\ &= e^{\kappa\mathcal{D}t}e^{\frac{g}{\delta\kappa}(e^{\kappa t}-1)\mathcal{A}}.\end{aligned}$$

We can also rearrange the order of exponents appearing in the last line of the above expression by

$$e^{a\mathcal{D}}e^{b\mathcal{A}}e^{-a\mathcal{D}}=e^{be^{-a}\mathcal{A}},$$

where a and b are arbitrary real functions. As a result we can write

$$\begin{aligned}e^{(\frac{g}{\delta}\mathcal{A}+\kappa\mathcal{D})t} &= e^{\kappa\mathcal{D}t}e^{\frac{g}{\delta\kappa}(e^{\kappa t}-1)\mathcal{A}}e^{-\kappa\mathcal{D}t}e^{\kappa\mathcal{D}t}\\ &= e^{\frac{g}{\delta\kappa}(1-e^{-\kappa t})\mathcal{A}}e^{\kappa\mathcal{D}t}.\end{aligned}$$

We are now in a position to study the action of the individual operators on the coherent state present in the cavity. From the Heuristic derivation of the cavity dissipation term found in [3] we see that the dissipation operator is responsible for transforming a coherent state $|\alpha\rangle$ into $|\alpha e^{-\kappa t}\rangle$. The same result can be obtained by further using the commutation relations

$$[\mathcal{L},\mathcal{M}]=[\mathcal{R},\mathcal{M}]=-\mathcal{M},\quad [\mathcal{R},\mathcal{L}]=0.$$

We find that

$$\begin{aligned}e^{\kappa t\mathcal{D}} &= e^{-\kappa(\mathcal{R}+\mathcal{L})t}e^{2\mathcal{M}\kappa t-\frac{(\kappa t)^2}{2!}4\mathcal{M}+\frac{(\kappa t)^3}{3!}8\mathcal{M}-\cdots}\\ &= e^{-\kappa t\mathcal{R}}e^{-\kappa t\mathcal{L}}e^{(1-e^{-2\kappa t})\mathcal{M}},\end{aligned}$$

where in the last step we have used the fact that right and left multiplication by creation and/or annihilation operators commute. Now by using

$$e^{c\hat{a}^\dagger\hat{a}}|\alpha\rangle = e^{|\alpha|^2(e^{2c}-1)/2}|\alpha e^c\rangle,$$

we can find the action of the operators separately,

$$\begin{aligned}e^{f(t)\mathcal{M}}|\alpha\rangle\langle\alpha| &= \sum_{n=0}^{\infty}\frac{f(t)^n}{n!}\hat{a}^n|\alpha\rangle\langle\alpha|(\hat{a}^\dagger)^n\\ &= e^{f(t)|\alpha|^2}|\alpha\rangle\langle\alpha|,\\ e^{f(t)\mathcal{R}}|\alpha\rangle\langle\alpha| &= e^{|\alpha|^2(e^{f(t)^2}-1)/2}|\alpha e^{f(t)}\rangle\langle\alpha|,\\ e^{f(t)\mathcal{L}}|\alpha\rangle\langle\alpha| &= e^{|\alpha|^2(e^{f(t)^2}-1)/2}|\alpha\rangle\langle\alpha e^{f(t)}|,\end{aligned}$$

where $f(t)$ is an arbitrary function, and by combining the results we obtain

$$\begin{aligned}e^{\kappa t\mathcal{D}}|\alpha\rangle\langle\alpha| &= e^{(1-e^{-2\kappa t}|\alpha|^2)}e^{-\kappa t(\mathcal{L}+\mathcal{R})}|\alpha\rangle\langle\alpha|\\ &= |\alpha e^{-\kappa t}\rangle\langle\alpha e^{-\kappa t}|.\end{aligned}$$

Let us postpone the action of the $e^{b\mathcal{A}}$ operator on the coherent states and now find the solution to Eq. (7). The fact that $\mathcal{X}$ and $\mathcal{Y}$ do not commute is the reason for decoherence entering into the equations. When we attempt to break up the exponent of the operators into exponents of an operator we first decompose operator $\alpha\mathcal{X}$ from the sum of operators $a\mathcal{Y}+b\mathcal{D}$ (a and b arbitrary) so that

$$\begin{aligned}e^{a(\mathcal{X}+\mathcal{Y})+b\mathcal{D}} &= e^{a\mathcal{Y}+b\mathcal{D}}e^{a\mathcal{X}}e^{-\frac{1}{2}[a\mathcal{Y}+b\mathcal{D},a\mathcal{X}]}\\ &\quad\times e^{\frac{1}{3!}([\mathcal{X},[a\mathcal{Y}+b\mathcal{D},a\mathcal{X}]]+[\mathcal{Y}+b\mathcal{D},[\mathcal{Y}+\beta\mathcal{D},a\mathcal{X}]])}\ldots.\end{aligned}$$

Since a double appearance of $\mathcal{X}$ in the commutator leads to a commutator between a constant from $[\mathcal{X},\mathcal{Y}]$ or another $\mathcal{X}$ from $[\mathcal{D},\mathcal{X}]=\mathcal{X}$, with another $\mathcal{X}$, we only consider the singly appearing $\mathcal{X}$ in the nested commutators. Additionally all of the constants generated by the $[\mathcal{X},\mathcal{Y}]$ commutator will be vanishing throughout, unless they appear in the last stage. Based on that we can write

$$\begin{aligned}&[a\mathcal{Y}+b\mathcal{D},[a\mathcal{Y}+b\mathcal{D},[\cdots,\mathcal{X}]]]_n\\ &\quad=[a\mathcal{Y}+b\mathcal{D},b^{n-1}\mathcal{X}-8(1-\cos\delta t)b^{n-2}a]\\ &\quad=b^n\mathcal{X}+8(1-\cos\delta t)b^{n-1}a,\end{aligned}$$

where $[\cdot,[\cdot,[\cdots,\mathcal{X}]]]_n$ denotes n-fold commutation with consecutive commutators coming with the same operator from the left. As a result we get

$$\begin{aligned}&e^{a(\mathcal{X}+\mathcal{Y})+b\mathcal{D}}\\ &\quad=e^{a\mathcal{Y}+b\mathcal{D}}e^{a\mathcal{X}}e^{-\frac{1}{2}(ab\mathcal{X}-4a^2)}e^{\frac{1}{3!}(ab^2\mathcal{X}-4a^2b)}e^{-\frac{1}{4!}(ab^3\mathcal{X}-4a^2b^2)}\ldots\\ &\quad=e^{a\mathcal{Y}+b\mathcal{D}}e^{\frac{a}{b}(e^{-b}-1)\mathcal{X}}e^{\frac{4a^2}{b^2}(e^{-b}-1+b)}.\end{aligned}$$

Following the same steps as we did when considering Eq. (8), we cannot separate the other exponents and get

$$e^{\alpha\mathcal{Y}+\beta\mathcal{D}}=e^{\beta\mathcal{D}}e^{\frac{\alpha}{\beta}(e^{\beta}-1)\mathcal{Y}}.$$

By using the commutation relations found before, and by noticing that the ones between $\mathcal{D}$ and $\mathcal{A}$ are of the same kind as that of $\mathcal{D}$ and $\mathcal{Y}$, we quickly find that

$$e^{kt\mathcal{D}}e^{f\mathcal{Y}}e^{-kt\mathcal{D}} = e^{fe^{-kt}\mathcal{Y}}.$$

Thus finally we can write

$$e^{\frac{g}{\delta}(\mathcal{X}+\mathcal{Y})+\kappa t\mathcal{D}} = e^{\frac{-8g^2(1-\cos\delta t)}{\delta^2 t^2\kappa^2}(e^{-\kappa t}-1+\kappa t)}e^{\frac{g}{\delta t\kappa}(1-e^{-\kappa t})\mathcal{Y}}e^{\kappa t\mathcal{D}} \times e^{\frac{g}{\delta t\kappa}(e^{-\kappa t}-1)\mathcal{X}}. \quad \text{(A2)}$$

By invoking the definition of a coherent state and its expansion in terms of Fock states we can show that

$$\begin{aligned} e^{fe^{it\delta}\hat{a}^\dagger}|\alpha\rangle &= \exp(fe^{it\delta}\hat{a}^\dagger)\exp(-|\alpha|^2/2)\exp(\alpha\hat{a}^\dagger)|0\rangle \\ &= e^{\frac{1}{2}|f|^2+f(\alpha e^{-it\delta}+\alpha^* e^{it\delta})}|\alpha+fe^{it\delta}\rangle. \end{aligned}$$

Next, realizing that the ordering of the algebra of superoperators behaves such that $(\cdot\hat{a})(\cdot\hat{a}^\dagger)\rho = \rho\hat{a}^\dagger\hat{a}$, we find that

$$\begin{aligned} e^{f(t)\mathcal{A}}|\alpha\rangle\langle\alpha| &= e^{-f(t)^2 c_+ c_-}e^{-f(t)c_+\hat{a}^\dagger\cdot}e^{f(t)c_-\hat{a}\cdot}|\alpha\rangle\langle\alpha|e^{c_+ f(t)\cdot\hat{a}^\dagger}e^{-c_- f(t)\cdot\hat{a}} \\ &= |\alpha+c_- f(t)\rangle\langle\alpha+c_- f(t)|, \\ e^{f(t)\mathcal{Y}}|\alpha\rangle\langle\alpha| &= e^{f(t)^2 c_+ c_-}e^{-f(t)c_+\hat{a}^\dagger\cdot}e^{-f(t)c_-\hat{a}}e^{f(t)c_+\cdot\hat{a}^\dagger}e^{f(t)c_-\cdot\hat{a}}|\alpha\rangle\langle\alpha| \\ &= e^{4f(t)^2(1-\cos\delta t)+2if(t)[\Re(\alpha)\sin\delta t+\Im(\alpha t)(1-\cos\delta t)]}|\alpha-f(t)c_+\rangle\langle\alpha+f(t)c_+|. \end{aligned}$$

Alternatively, we can say that the action of the $e^{f\mathcal{A}}$ on the coherent state density matrix upon decomposition into left- and right-acting creation and annihilation operators leaves left- and right-acting coherent state creation operators $D(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$ with the coefficient $\alpha = -\frac{g}{\delta kt}(e^{i\delta t}-1)(1-e^{-kt})$, which leads to the solution (8). It should not be surprising that these states do not leave any additional state prefactors behind, as these these expressions are in our model characteristic to the diagonal elements and as such should be trace preserving. The action of $e^{f\mathcal{Y}}$, on the other hand, brings about oscillations and decays in overall coherence.

APPENDIX B: EQUATIONS AND SOLUTIONS TO A TWO-QUBIT MASTER EQUATION

Here we present the equation and solutions to Eq. (3) for a two-qubit case. The density matrix of the two-qubits states is labeled by $p_{ij;kl,\alpha}(t)$, where $i,j=\pm$ and $k,l=\pm$ refer to the first and second qubits, respectively. Equations for individual atomic density-matrix entries read

$$\dot{p}_{++;++}|\alpha\rangle\langle\alpha| = p_{++;++}\left(\frac{g_1+g_2}{i}[\hat{a}e^{-i\delta t}+\hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|]+\kappa\mathcal{D}(|\alpha\rangle\langle\alpha|)\right),$$

$$\dot{p}_{++;+-}|\alpha\rangle\langle\alpha| = p_{++;+-}\left(\frac{g_1}{i}[\hat{a}e^{-i\delta t}+\hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|]+\frac{g_2}{i}\{\hat{a}e^{-i\delta t}+\hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|\}+\kappa\mathcal{D}(|\alpha\rangle\langle\alpha|)\right),$$

$$\dot{p}_{++;-+}|\alpha\rangle\langle\alpha| = p_{++;-+}\left(\frac{g_1}{i}[\hat{a}e^{-i\delta t}+\hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|]-\frac{g_2}{i}\{\hat{a}e^{-i\delta t}+\hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|\}+\kappa\mathcal{D}(|\alpha\rangle\langle\alpha|)\right),$$

$$\dot{p}_{++;--}|\alpha\rangle\langle\alpha| = p_{++;--}\left(\frac{g_1+g_2}{i}\{\hat{a}e^{-i\delta t}+\hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|\}+\kappa\mathcal{D}(|\alpha\rangle\langle\alpha|)\right),$$

$$\dot{p}_{+-;+-}|\alpha\rangle\langle\alpha| = p_{+-;+-}\left(\frac{g_1-g_2}{i}[\hat{a}e^{-i\delta t}+\hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|]+\kappa\mathcal{D}(|\alpha\rangle\langle\alpha|)\right),$$

$$\dot{p}_{+-;-+}|\alpha\rangle\langle\alpha| = p_{+-;-+}\left(\frac{g_1-g_2}{i}\{\hat{a}e^{-i\delta t}+\hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|\}+\kappa\mathcal{D}(|\alpha\rangle\langle\alpha|)\right),$$

$$\dot{p}_{+-;--}|\alpha\rangle\langle\alpha| = p_{+-;--}\left(\frac{g_1}{i}\{\hat{a}e^{-i\delta t}+\hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|\}-\frac{g_2}{i}[\hat{a}e^{-i\delta t}+\hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|]+\kappa\mathcal{D}(|\alpha\rangle\langle\alpha|)\right),$$

$$\dot{p}_{-+;-+}|\alpha\rangle\langle\alpha| = p_{-+;-+}\left(\frac{-g_1+g_2}{i}[\hat{a}e^{-i\delta t}+\hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|]+\kappa\mathcal{D}(|\alpha\rangle\langle\alpha|)\right),$$

$$\dot{p}_{-+;--}|\alpha\rangle\langle\alpha| = p_{-+;--}\left(\frac{-g_1}{i}[\hat{a}e^{-i\delta t}+\hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|]+\frac{g_2}{i}\{\hat{a}e^{-i\delta t}+\hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|\}+\kappa\mathcal{D}(|\alpha\rangle\langle\alpha|)\right),$$

$$\dot{p}_{--;--}|\alpha\rangle\langle\alpha| = p_{--;--}\left(-\frac{g_1+g_2}{i}[\hat{a}e^{-i\delta t}+\hat{a}^\dagger e^{i\delta t},|\alpha\rangle\langle\alpha|]+\kappa\mathcal{D}(|\alpha\rangle\langle\alpha|)\right).$$

From the equations above the solutions immediately follow:

$$p_{++;++,\alpha} = p_{++;++}(0)e^{(g_1+g_2)\mathcal{A}/\delta+\kappa\mathcal{D}t}|\alpha\rangle\langle\alpha|, \quad \text{(B1)}$$

$$p_{++;+-,\alpha} = p_{++;+-}(0)e^{g_1\mathcal{A}/\delta+g_2\mathcal{B}/\delta+\kappa\mathcal{D}t}|\alpha\rangle\langle\alpha|,$$
$$p_{++;-+,\alpha} = p_{++;-+}(0)e^{g_2\mathcal{A}/\delta+g_1\mathcal{B}/\delta+\kappa\mathcal{D}t}|\alpha\rangle\langle\alpha|,$$

$$p_{++;--,\alpha} = p_{++;--}(0)e^{(g_1+g_2)\mathcal{B}/\delta+\kappa\mathcal{D}t}|\alpha\rangle\langle\alpha|, \quad \text{(B2)}$$

$$p_{+-;+-,\alpha} = p_{+-;+-}(0)e^{(g_1-g_2)\mathcal{A}/\delta+\kappa\mathcal{D}t}|\alpha\rangle\langle\alpha|, \quad \text{(B3)}$$

$$p_{+-;-+,\alpha} = p_{+-;-+}(0)e^{(g_1-g_2)\mathcal{B}/\delta+\kappa\mathcal{D}t}|\alpha\rangle\langle\alpha|, \quad \text{(B4)}$$

$$p_{+-;--,\alpha} = p_{+-;--}(0)e^{-g_2\mathcal{A}/\delta+g_1\mathcal{B}/\delta+\kappa\mathcal{D}t}|\alpha\rangle\langle\alpha|,$$
$$p_{-+;-+,\alpha} = p_{-+;-+}(0)e^{-(g_1-g_2)\mathcal{A}/\delta+\kappa\mathcal{D}t}|\alpha\rangle\langle\alpha|, \quad \text{(B5)}$$

$$p_{-+;--,\alpha} = p_{-+;--}(0)e^{-g_1\mathcal{A}/\delta+g_2\mathcal{B}/\delta+\kappa\mathcal{D}t}|\alpha\rangle\langle\alpha|,$$
$$p_{--;--,\alpha} = p_{--;--}(0)e^{-(g_1+g_2)\mathcal{A}/\delta+\kappa\mathcal{D}t}|\alpha\rangle\langle\alpha|. \quad \text{(B6)}$$

From the equations above we can see that Eqs. (B1), (B3), (B5), and (B6) are of the form (8) and Eqs. (B2) and (B4) are of the form (9) and can be solved by means presented in Appendix A. The other equations involve both operators $\mathcal{A}$ and $\mathcal{B}$ which can be decomposed into

$$e^{(\frac{g}{\delta}\mathcal{B}+\kappa\mathcal{D})t+\mathcal{A}} = e^{(\frac{g}{\delta}\mathcal{B}+\kappa t\mathcal{D})}e^{\frac{g}{\delta t\kappa}(e^{\kappa t}-1)\mathcal{A}}.$$

We can now follow the single qubit case steps to further decompose the exponent of $\frac{g}{\delta}\mathcal{B}+\kappa\mathcal{D}t$ into separate exponents, see Eq. (A2). Finally, we obtain the following set of solutions:

$$p_{++;++}(t) = p_{++;++}(0)|\alpha e^{-\kappa t} - g_1 f c_+ - g_2 f c_+\rangle\langle\alpha e^{-\kappa t} - g_1 f c_+ - g_2 f c_+|,$$
$$p_{++;+-}(t) = p_{++;+-}(0)e^{x(g_2,t)}|\alpha e^{-\kappa t} - g_1 f c_+ - g_2 f c_+\rangle\langle\alpha e^{-\kappa t} - g_1 f c_+ + g_2 f c_+|,$$
$$p_{++;-+}(t) = p_{++;-+}(0)e^{x(g_1,t)}|\alpha e^{-\kappa t} - g_1 f c_+ - g_2 f c_+\rangle\langle\alpha e^{-\kappa t} + g_1 f c_+ - g_2 f c_+|,$$
$$p_{++;--}(t) = p_{++;--}(0)e^{x(g_1+g_2,t)}|\alpha e^{-\kappa t} - g_1 f c_+ - g_2 f c_+\rangle\langle\alpha e^{-\kappa t} + g_1 f c_+ + g_2 f c_+|,$$
$$p_{+-;+-}(t) = p_{+-;+-}(0)|\alpha e^{-\kappa t} - g_1 f c_+ + g_2 f c_+\rangle\langle\alpha e^{-\kappa t} - g_1 f c_+ + g_2 f c_+|,$$
$$p_{+-;-+}(t) = p_{+-;-+}(0)e^{x(g_1-g_2,t)}|\alpha e^{-\kappa t} - g_1 f c_+ + g_2 f c_+\rangle\langle\alpha e^{-\kappa t} + g_1 f c_+ - g_2 f c_+|,$$
$$p_{+-;--}(t) = p_{+-;--}(0)e^{x(g_1,t)}|\alpha e^{-\kappa t} - g_1 f c_+ + g_2 f c_+\rangle\langle\alpha e^{-\kappa t} + g_1 f c_+ + g_2 f c_+|,$$
$$p_{-+;-+}(t) = p_{-+;-+}(0)|\alpha e^{-\kappa t} - g_1 f c_+ + g_2 f c_+\rangle\langle\alpha e^{-\kappa t} - g_1 f c_+ + g_2 f c_+|,$$
$$p_{-+;--}(t) = p_{-+;--}(0)e^{x(g_2,t)}|\alpha e^{-\kappa t} - g_1 f c_+ - g_2 f c_+\rangle\langle\alpha e^{-\kappa t} - g_1 f c_+ + g_2 f c_+|,$$
$$p_{--;--}(t) = p_{--;--}(0)|\alpha e^{-\kappa t} - g_1 f c_+ - g_2 f c_+\rangle\langle\alpha e^{-\kappa t} - g_1 f c_+ - g_2 f c_+|,$$

where we have defined

$$x(\xi,t) = -\frac{8\xi^2(1-\cos\delta t)}{\delta^2\kappa^2 t^2}(e^{-\kappa t}-1+\kappa t) + 4\xi^2 f(t)^2(\cos\delta t - 1) - 2i\xi f(t)(2-e^{-\kappa t}) \times [\mathrm{Im}(\alpha)(\cos\delta t - 1) - \mathrm{Re}(\alpha)\sin\delta t],$$

and where f and c_+ were defined before.

These solutions can be used to extend the treatment to more than one unequally coupled qubit and to study multipartite entanglement in this system.

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
[2] Y. V. Nazarov and Y. M. Blanter, *Quantum Transport: Introduction to Nanoscience* (Cambridge University Press, Cambridge, 2009).
[3] S. M. Dutra, *Cavity Quantum Electrodynamics: The Strange Theory of Light in a Box* (Wiley Interscience, New York, 2004).
[4] J. H. Eberly and T. Yu, Science **316**, 555 (2007).
[5] T. Yu and J. H. Eberly, Phys. Rev. B **66**, 193306 (2002).
[6] T. Yu and J. H. Eberly, Phys. Rev. Lett. **93**, 140404 (2004).
[7] J. Laurat, K. S. Choi, H. Deng, C. W. Chou, and H. J. Kimble, Phys. Rev. Lett. **99**, 180504 (2007).
[8] M. P. Almeida, F. de Melo, M. Hor-Meyll, A. Salles, S. P. Walborn, P. H. S. Ribeiro, and L. Davidovich, Science **316**, 579 (2007).
[9] Z. Ficek and R. Tanaś, Phys. Rev. A **74**, 024304 (2006).
[10] Z. Ficek and R. Tanaś, Phys. Rev. A **77**, 054301 (2008).
[11] F. F. Fanchini and R. d. J. Napolitano, Phys. Rev. A **76**, 062306 (2007).
[12] S. Maniscalco, F. Francica, R. L. Zaffino, N. Lo Gullo, and F. Plastina, Phys. Rev. Lett. **100**, 090503 (2008).
[13] G. Gordon, Europhys. Lett. **83**, 30009 (2008).
[14] R. Bianchetti, S. Filipp, M. Baur, J. M. Fink, M. Goppl, P. J. Leek, L Steffen, A. Blais, and A. Wallraff, Phys. Rev. A **80**, 043840 (2009).

[15] L. Robledo, H. Bernien, I. van Weperen, and R. Hanson, Phys. Rev. Lett. **105**, 177403 (2010).
[16] E. Jaynes and F. Cummings, Proc. IEEE **51**, 89 (1963).
[17] I. V. Jyotsna and G. S. Agarwal, Opt. Commun. **99**, 344 (1993).
[18] P. Lougovski, F. Casagrande, A. Lulli, B.-G. Englert, E. Solano, and H. Walther, Phys. Rev. A **69**, 023812 (2004).
[19] E. Solano, G. S. Agarwal, and H. Walther, Phys. Rev. Lett. **90**, 027903 (2003).
[20] F. C. M. Bina and A. Lulli, Eur. Phys. J. D **49**, 257 (2008).
[21] J.-S. Zhang, Can. J. Phys. **87**, 1031 (2009).
[22] M. Bina, F. Casagrande, A. Lulli, and E. Solano, Phys. Rev. A **77**, 033839 (2008).
[23] J.-S. Zhang, Opt. Commun. **282**, 2543 (2009).
[24] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
[25] M. Dukalski and Y. Blanter (unpublished).