Universal quantum computer from a quantum magnet

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We show that a local Hamiltonian of spin-$\frac{3}{2}$ particles with only two-body nearest-neighbor Affleck-Kennedy-Lieb-Tasaki and exchange-type interactions has a unique ground state, which can be used to implement universal quantum computation merely with single-spin measurements. We prove that the Hamiltonian is gapped, independent of the system size. Our result provides a further step toward utilizing systems with condensed-matter-type interactions for measurement-based quantum computation.

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I. INTRODUCTION

Quantum computers are believed to be more powerful than their classical counterpart, resulting in tremendous efforts to implement quantum computation with different physical systems. The model of a one-way quantum computer [1,2] has opened an approach toward the possible experimental realization of quantum computation. In the one-way model, quantum computation starts by preparing certain universal resource states, namely, the cluster states [3], and is achieved by merely performing single-qubit measurements on these states. The principal task turns out to be the preparation of these highly entangled resource states. One straightforward way is to apply entangling gates to couple a lattice of qubits [4]. It is, however, clearly appealing if there exists a universal resource such as the unique ground state of a naturally occurring gapped two-body local Hamiltonian for the advantage of flexible-state preparation as well as the stability against the local perturbations.

Recall that the cluster states cannot occur as a nondegenerate ground state of any two-local spin-$\frac{1}{2}$ Hamiltonian [5] and have singular entanglement features, e.g., vanishing two-point correlation functions [6]. Perturbative Hamiltonians have been proposed with (encoded) graph states as the approximate ground states [7], which, however, require a highly precise control over system parameters and for which the spectral gap would get significantly smaller according to the order of the perturbation. Recently, a gapped two-body Hamiltonian of six-level particles [8] has been constructed with the so-called tricluster state as its unique ground state, which is universal for measurement-based quantum computation. Here, we construct a gapped local Hamiltonian, with the constitutional two-body Affleck-Kennedy-Lieb-Tasaki (AKLT) [9] and exchange-type interactions acting on four-level particles. The simplicity of the Hamiltonian might allow one to identify possible realizations in models of condensed matter systems or in quantum optical setups. The present approach differs from the previously used methods to construct a parent Hamiltonian for projected entangled pair states by finding the local support subspace [8,10], but it can be utilized to construct a family of local Hamiltonians with the same properties.

In this paper, motivated by the usage of the gapped ground state of the one-dimensional (1D) AKLT spin-1 chain as a universal quantum wire [11] (cf. [12,13]), we start from 1D *quasichain* of spin-$\frac{3}{2}$ particles as a building block. From there, we construct a full two-dimensional (2D) resource that allows a deterministic decoupling of the 1D quantum-wire structure (in a similar way to the cluster state). After proving that the 1D quasichain is gapped with a unique ground state, we construct a gapped two-body Hamiltonian on an octagonal 2D lattice, which is transitionally invariant and consists of only nearest-neighbor AKLT together with exchange-type interactions. We demonstrate how to implement universal quantum computation by simply making single-spin measurements on the individual four-level particles of its unique ground state. This Hamiltonian provides an example that can be utilized as a complete measurement-based ground-code quantum computer without the demand of dynamical coupling [11]. Extensions of our approach to the other geometric configurations, e.g., 2D lattice, or other Hamiltonians are also possible.

II. 1D AKLT QUASICHAIN

We first consider the 1D AKLT model defined on the quasichain, as in Fig. 1, and show that the model is gapped and has a unique ground state. The quasichain consists of spin-$\frac{3}{2}$ (A) and spin-$\frac{1}{2}$ particles (b) coupled with nearest-neighbor two-body interactions:

$$H = J\left(\sum_{i=1}^{N-1} P^3_{A_i,A_{i+1}} + \sum_{i=1}^{N} P^2_{A_i,b_i} + P^2_{A_1,b_0} + P^2_{A_N,b_{N+1}}\right), \tag{1}$$

where $P^S_{m,n}$ represents the projector onto the spin-S irreducible representation of the total spin for particles m and n (cf. Refs. [9,14]). As the spin per particle equals half of the local coordination number (i.e., the number of the bonds from a particle), the 1D AKLT quasichain in Eq. (1) has a unique ground state $|\mathcal{G}\rangle$ [15], which can be obtained via a projector which maps the symmetric part of three spin-$\frac{1}{2}$ virtual qubits of maximally entangled pairs into one spin-$\frac{3}{2}$ physical particle.

To prove that the 1D AKLT quasichain is gapped in the thermodynamic limit, we first regroup the Hamiltonian in Eq. (1) into blocks as $H = \sum_i \Pi_{i,i+1}$ (see Fig. 1), where

$$\Pi_{i,i+1} = P^3_{A_i,A_{i+1}} + \tfrac{1}{2}P^2_{A_i,b_i} + \tfrac{1}{2}P^2_{A_{i+1},b_{i+1}} \tag{2}$$

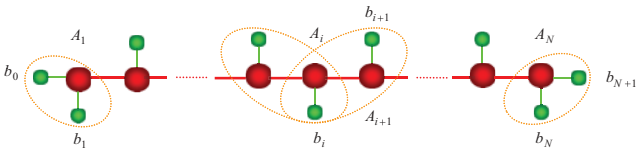

FIG. 1. (Color online) Configuration of a 1D AKLT quasichain, which consists of spin-$\frac{3}{2}$ (red, large circles) and spin-$\frac{1}{2}$ (green, small circles) particles. The Hamiltonian is regrouped into blocks, each of which is marked by a dotted circle.

is the sum of the AKLT interactions among the particles inside the block. We consider the ground state $|\mathcal{G}\rangle$ and denote the support subspace of the reduced density matrix corresponding to each block as $\mathcal{S}_{i,i+1}$ and the projector onto the orthogonal subspace as $\mathcal{S}^{\perp}_{i,i+1}$. From $\mathcal{S}_{i,i+1} = \ker \Pi_{i,i+1}$ it follows that $|\mathcal{G}\rangle$ is also the unique ground state of the projective Hamiltonian $H_p = \sum_{i=0}^{N} \mathcal{S}^{\perp}_{i,i+1}$. One can directly calculate the energy gap γ of $\Pi_{i,i+1}$ and find $\gamma \geqslant 0.3518$, which leads to

$$\Pi_{i,i+1} \geqslant J\gamma \mathcal{S}^{\perp}_{i,i+1}. \tag{3}$$

We further write the block Hamiltonian from H_p on $n+1$ units as $h_{n,i} = \sum_{j=i}^{i+n-1} \mathcal{S}^{\perp}_{j,j+1}$. If $h^2_{n,i} \geqslant \varepsilon h_{n,i}$, the results in Ref. [16] imply that

$$H_p^2 \geqslant \frac{n}{n-1}\left(\varepsilon - \frac{1}{n}\right) H_p. \tag{4}$$

Our calculations show that $\varepsilon = 0.4132$ for $n = 4$. We thus conclude that H_p is gapped as $\varepsilon > \frac{1}{4}$, where better bounds can, in principle, be obtained for higher n. Therefore, the energy gap of the 1D AKLT quasichain is lower bounded by $\Delta E \geqslant 0.0766J$.

III. 2D GAPPED HAMILTONIAN BY MERGING 1D QUASICHAINS

With the results for the 1D AKLT quasichain established here, we will show how to obtain a model on an octagonal lattice with two-body interactions that is gapped and has a unique ground state. We start from a number of independent 1D AKLT quasichains and introduce the unitary transformation U, which maps two spin-$\frac{1}{2}$ particles into one spin-$\frac{3}{2}$ particle by

$$U = \sum_{m_1,m_2=\pm\frac{1}{2}} \left|\frac{3}{2}, m_1+2m_2\right\rangle\left\langle\frac{1}{2}, m_1\right|\left\langle\frac{1}{2}, m_2\right|, \tag{5}$$

where the label $|j,m\rangle$ denotes $|\mathbf{S},\mathbf{S}_z\rangle$. Notice that, in principle, *any* unitary operation can be used at this stage, each leading to a gapped model with a unique ground state that can be used for universal measurement-based computation. Using such a transformation, we can merge a number of 1D quasichains and get a 2D Hamiltonian as $H_{2d} = \mathcal{U}\sum_i H^{(i)}\mathcal{U}^{\dagger}$ with $\mathcal{U} = \bigotimes_{\langle b_k,b_l\rangle\in E} U(b_k,b_l)$, where $\langle b_k,b_l\rangle \in E$ denotes two neighboring spin-$\frac{1}{2}$ particles in the same merging circle

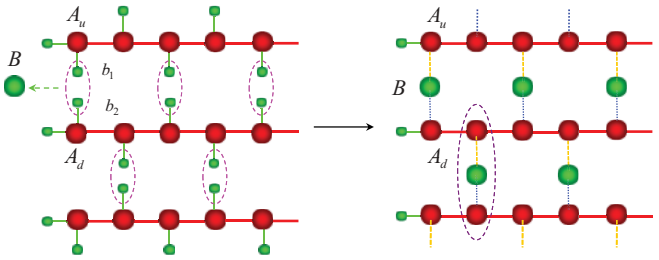

FIG. 2. (Color online) A 2D gapped Hamiltonian from 1D AKLT quasichains. Two spin-$\frac{1}{2}$ particles (b_1 and b_2) on two neighboring chains are mapped into one spin-$\frac{3}{2}$ particle (B). The nearest-neighbor interactions consist of two-body couplings as E_a(red, thick solid), E_u (yellow, dashed), E_d (blue, dotted), E_b (green, thin solid).

(see Fig. 2). Thus,

$$H_{2d} = \sum_{t=a,b,u,d} \sum_{\langle m,n\rangle\in E_t} \Pi^t_{m,n}, \tag{6}$$

where $\Pi^a_{m,n} = P^3_{m,n}$, $\Pi^b_{m,n} = P^2_{m,n}$, $\Pi^u_{m,n} = U(P^2_{A,b}\otimes\mathbb{I})U^{\dagger}$, and $\Pi^d_{m,n} = U(\mathbb{I}\otimes P^2_{b,A})U^{\dagger}$ and E_a, E_u, E_d, and E_b represent different types of couplings (see Fig. 2). The couplings between A- and B-type particles,

$$\Pi^u = \tfrac{1}{2}\mathbf{S}_{A_u}\cdot\mathbf{S}'_B + \tfrac{5}{8}\mathbb{I}, \quad \Pi^d = \tfrac{1}{2}\mathbf{S}''_B\cdot\mathbf{S}_{A_d} + \tfrac{5}{8}\mathbb{I}, \tag{7}$$

are effectively exchange-type interactions with $\mathbf{S}' = \mathbf{s}(-\frac{3}{2}, -\frac{1}{2}) \oplus \mathbf{s}(+\frac{1}{2}, +\frac{3}{2})$ and $\mathbf{S}'' = \mathbf{s}(-\frac{3}{2}, +\frac{1}{2}) \oplus \mathbf{s}(-\frac{1}{2}, +\frac{3}{2})$, where $\mathbf{s}(\alpha,\beta)$ is the effective spin-$\frac{1}{2}$ operator defined on two levels, $\mathbf{S}_z = \alpha,\beta$. It is easy to verify that $\mathbf{S}'$ and $\mathbf{S}''$ satisfy the commutation relations analogous to the spin angular momentum.

As H_{2d} is equivalent up to a unitary transformation to N independent AKLT quasichains, the spectrum and the corresponding eigenvalues can be easily obtained from the spectrum and eigenvalues of H. In particular, it follows that H_{2d} is gapped (with the same constant energy gap ΔE as H) and has the unique ground state $|\Psi\rangle = \mathcal{U}(|\mathcal{G}\rangle\otimes\cdots\otimes|\mathcal{G}\rangle)$.

IV. MEASUREMENT-BASED QUANTUM COMPUTATION

We will demonstrate how to use the resource state $|\Psi\rangle$ for measurement-based quantum computation by following the notation and scheme of Refs. [12,13,17–19]. The state $|\Psi\rangle$ can be represented as a projected entangled pair state [17,20], in which a number of maximally entangled pairs $1/\sqrt{2}(|00\rangle + |11\rangle)$ of virtual qubits are mapped into physical particles; see Fig. 3 for one computational block. The corresponding tensor matrices for the physical site A_u are

$$A_u\left[+\tfrac{3}{2}\right] = |1\rangle_r\langle 0|_l\otimes\langle 1|_d, \tag{8}$$

$$A_u\left[-\tfrac{3}{2}\right] = |0\rangle_r\langle 1|_l\otimes\langle 0|_d, \tag{9}$$

$$A_u\left[+\tfrac{1}{2}\right] = -1/\sqrt{3}(Z\otimes\langle 1|_d + |1\rangle_r\langle 0|_l\otimes\langle 0|_d), \tag{10}$$

$$A_u\left[-\tfrac{1}{2}\right] = 1/\sqrt{3}(Z\otimes\langle 0|_d - |0\rangle_r\langle 1|_l\otimes\langle 1|_d). \tag{11}$$

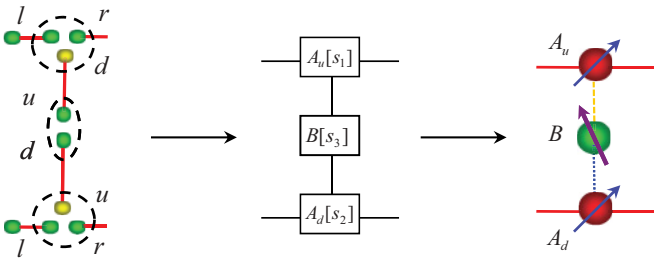

FIG. 3. (Color online) Computational tensor network per single block. Entangled pairs of virtual qubits (left), which carry logical quantum information, are mapped into physical particles (right). The measurement on site B in different bases effectively prepares the vertical virtual qubits (yellow, small light circles) at sites A_u and A_d (d and u) into either a product state or an entangled state, which is used to implement decoupling 1D chains (single-qubit rotations) or two-qubit gates, respectively.

The local tensor A_d can be written in a similar way. The tensor matrices for site B are

$$B\left[+\tfrac{3}{2}\right] = |0\rangle_u\langle 1|_d, \tag{12}$$
$$B\left[-\tfrac{3}{2}\right] = -|1\rangle_u\langle 0|_d, \tag{13}$$
$$B\left[+\tfrac{1}{2}\right] = |1\rangle_u\langle 1|_d, \tag{14}$$
$$B\left[-\tfrac{1}{2}\right] = -|0\rangle_u\langle 0|_d. \tag{15}$$

Measurement-based quantum computation on such a resource state can be understood as follows (see [12,13,17–19] for details): The logical information is carried by the virtual qubit, and the measurement in a certain basis on the physical particle will induce either unitary or readout operators on the virtual qubits according to the tensor matrices in Eqs. (8)–(15).

Each logical virtual qubit can be initialized to $|0\rangle$ or $|1\rangle$ by measuring the left end spin-$\frac{1}{2}$ particle in the $\hat{z}$ basis $\{|\pm\tfrac{1}{2}\rangle\}$. Quantum computation then proceeds gradually from left to right by measuring each computational block (A_u, A_d, and B); see Figs. 2 and 3.

Before performing computational measurements on each block, we need to introduce a *prenormalization step* as follows to equalize the different coefficients (1 versus $1/\sqrt{3}$) in the tensor matrices A_u (A_d). We first apply the local filter operation $\{L,\bar{L}\}$ on sites A_u and A_d with

$$L = \mathrm{diag}\{1/\sqrt{3},1,1,1/\sqrt{3}\}, \tag{16}$$
$$\bar{L} = \mathrm{diag}\{\sqrt{2/3},0,0,\sqrt{2/3}\}. \tag{17}$$

An outcome $\bar{L}$ corresponds to an unsuccessful filter attempt and leads, after suitable measurements on sites B, A_u, and A_d, to the transport of quantum information to the next block up to a local Pauli operation X or ZX. The procedure can hence be repeated until it succeeds (i.e., the outcome L occurs). We find that $l \sim O(\log\frac{1}{\epsilon})$ trials lead to an overall success probability of $p_s = 1-(1/3)^l \geqslant 1-\epsilon$, i.e., the process is efficient. The required measurements are given by

$$\{|\beta\rangle\} = 1/\sqrt{2}\{|\mu_0\rangle \pm |\nu_0\rangle, |\mu_1\rangle \pm |\nu_1\rangle\} \tag{18}$$

for site B, where

$$|\mu_s\rangle = 1/\sqrt{2}\left[\left|-\tfrac{3}{2}\right\rangle + (-1)^s\left|+\tfrac{1}{2}\right\rangle\right], \tag{19}$$
$$|\nu_s\rangle = 1/\sqrt{2}\left[\left|-\tfrac{1}{2}\right\rangle + (-1)^s\left|+\tfrac{3}{2}\right\rangle\right], \tag{20}$$

and

$$\{|\alpha\rangle\} = \left\{\left|+\tfrac{3}{2}\right\rangle + \left|-\tfrac{3}{2}\right\rangle, \left|+\tfrac{3}{2}\right\rangle - \left|-\tfrac{3}{2}\right\rangle, \left|+\tfrac{1}{2}\right\rangle, \left|-\tfrac{1}{2}\right\rangle\right\} \tag{21}$$

for sites A_u and A_d.

In the following, we will assume that the filter operation was successful (i.e., the result L was obtained). We show that a suitable choice of the measurement in B allows us either to decouple 1D chains and obtain single-qubit operations or readout or, alternatively, to couple two chains directly and obtain a two-qubit gate. Two chains are decoupled by a measurement of site B in the $\hat{z}$ basis $\{|\pm\tfrac{1}{2}\rangle, |\pm\tfrac{3}{2}\rangle\}$, leading to vertical virtual qubits d and u, corresponding to sites A_u and A_d, respectively (see Fig. 3), to be prepared into either $|0\rangle$ or $|1\rangle$. For example, if the vertical virtual qubit is $|0\rangle$, the effective tensor matrices for A_u are given by

$$A\left[+\tfrac{3}{2}\right] = 0, A\left[+\tfrac{1}{2}\right] = -|1\rangle_r\langle 0|_l,$$
$$A\left[-\tfrac{1}{2}\right] = Z, A\left[-\tfrac{3}{2}\right] = |0\rangle_r\langle 1|_l.$$

For other outcomes and the tensor A_d, the reduced tensor matrices are equivalent up to a local basis change. Without loss of generality, we use the effective 1D tensor matrices A to show how to implement arbitrary single-qubit rotations following a similar protocol as in [11] (note that the effective tensor matrices are not equivalent to the 1D AKLT spin-1 chain) and read out logical quantum information.

A. Readout

The readout is realized by simply measuring site A in the $\hat{z}$ basis $\{|\pm\tfrac{1}{2}\rangle, |\pm\tfrac{3}{2}\rangle\}$. Once we get the outcome $|+\tfrac{1}{2}\rangle$ ($|-\tfrac{3}{2}\rangle$), one can infer that the logical qubit is $|0\rangle(|1\rangle)$. Otherwise, the logical qubit gets a Z by-product operator, and we repeat the above procedure.

B. Single-qubit gates

An arbitrary single-qubit operation can be decomposed into three rotations around the Z and X axes with three Euler angles. In order to implement a Z-rotation $R_z(\theta) = |0\rangle\langle 0| + e^{i\theta}|1\rangle\langle 1|$, we measure site A in the basis

$$\{|\alpha_z(\theta)\rangle\} = \left\{\left|+\tfrac{3}{2}\right\rangle, 1/\sqrt{2}\left(e^{-i\theta}\left|-\tfrac{3}{2}\right\rangle - \left|+\tfrac{1}{2}\right\rangle\right),\right.$$
$$\left. -1/\sqrt{2}\left(e^{-i\theta}\left|-\tfrac{3}{2}\right\rangle + \left|+\tfrac{1}{2}\right\rangle\right), \left|-\tfrac{1}{2}\right\rangle\right\}. \tag{22}$$

The first outcome is not possible, while the fourth induces a by-product operator Z. If, however, the outcome is the second or third one, we implement the rotation $R_z(\theta)$ with a Pauli by-product operator X or XZ. An X-rotation $R_x(\theta) = |+\rangle\langle +| + e^{i\theta}|-\rangle\langle -|$ is implemented in a similar way. We apply the local filter operation $\{L,\bar{L}\}$ at the initial prenormalization step in the $\mathbf{S}_x$ basis $\{|\tfrac{3}{2},\mathbf{S}_x = m\rangle, m = \pm\tfrac{3}{2}, \pm\tfrac{1}{2}\}$. X rotations can then be realized by the same protocol as the Z rotations, except with the exchange of $\mathbf{S}_z$ and $\mathbf{S}_x$ basis, where the measurement basis on site B at the prenormalization and decoupling steps correspondingly changes as $\{|\beta\rangle\} \leftrightarrow \{|\pm\tfrac{1}{2}\rangle, |\pm\tfrac{3}{2}\rangle\}$.

C. Two-qubit gate

The basic idea for the implementation of an entangling gate is to prepare the vertical virtual qubits d and u, which correspond to sites A_u and A_d, into an entangled state by a suitable measurement of B. In order to determine the kind of two-qubit gate that is implemented for different measurement outcomes, we first rewrite the tensor matrices A_u and A_d in the aforementioned basis of $|\mu_s\rangle$ and $|\nu_s\rangle$. We further define another basis for site B as

$$|\mu_s'\rangle = 1/\sqrt{2}\left[\left|-\tfrac{1}{2}\right\rangle + i(-1)^s\left|+\tfrac{1}{2}\right\rangle\right], \tag{23}$$

$$|\nu_s'\rangle = 1/\sqrt{2}\left[\left|-\tfrac{3}{2}\right\rangle + i(-1)^s\left|+\tfrac{3}{2}\right\rangle\right]. \tag{24}$$

The contracted tensor from $A_u[\mu_s,\nu_s]$, $A_d[\mu_s,\nu_s]$, and $B[\mu_s',\nu_s']$ results in one of the two-qubit entangling gates,

$$V_{m,n} = I\otimes I + i\sigma_m\otimes\sigma_n, \tag{25}$$

where $\sigma_{m,n} = X$ or Y. Moreover, any Pauli by-product operator can propagate through the above entangling gates as $V_{m,n}(\pi_1\otimes\pi_2) = (\pi_1'\otimes\pi_2')V_{m,n}$, where both π_i and π_i' are Pauli operators.

The explicit procedure to implement a specific entangling gate is as follows: we first measure sites A_u and A_d in the basis $\{|\mu_s\rangle,|\nu_s\rangle\}$; if the outcomes correspond to the desired entangling gate out of $V_{m,n}$, we proceed to measure site B in the basis $\{|\mu_s'\rangle,|\nu_s'\rangle\}$. Otherwise, if we do not obtain the desired outcomes from the measurements on sites A_u and A_d, we measure site B in the $\hat{z}$ basis $\{|\pm\tfrac{1}{2}\rangle,|\pm\tfrac{3}{2}\rangle\}$, which decouples the chains and leads to transport of quantum information to the next block up to a by-product Pauli operator. In that case, we have to repeat the above procedure in order to obtain the target two-qubit gate, but an arbitrary high-success probability is achievable efficiently as well. The present protocol offers the flexibility to choose a two-qubit gate on demand from a set of entangling gates.

V. GROUND-CODE QUANTUM COMPUTATION

If the bulk Hamiltonian is maintained during the measurement-based computation as proposed in the ground-code scheme [11], the spectral gap appears to provide certain protection against local noises, so that it may make the computer more robust and easier to meet a stringent fault-tolerant error threshold for quantum error correction. However, potential advantages of the gap in protecting quantum information are currently intensively being studied in the context of topological memory [21]. The general question as to what extent the passive Hamiltonian protection is helpful in measurement-based computation, which is far from the equilibrium, is more involved and is addressed elsewhere. Here, we describe the complete scheme with the Hamiltonian present.

We first look at the residual Hamiltonian of 1D AKLT quasichain after measuring the first j particles,

$$H(j) = J\left(\sum_{i=j}^{N-1} P^3_{A_i,A_{i+1}} + \sum_{i=j}^{N} P^2_{A_i,b_i} + P^2_{A_N,b_{N+1}}\right). \tag{26}$$

It is gapped and twofold degenerate, which can encode one logical qubit. We can show that the operators

$$\Sigma_\sigma = \bigotimes_{i=j}^{N}\left\{\left[i\sigma\left(+\frac{3}{2},-\frac{3}{2}\right)\oplus\sigma\left(-\frac{1}{2},+\frac{1}{2}\right)\right]^{(A_i)}\otimes\sigma^{(b_i)}\right\} \otimes\sigma^{(b_{N+1})}, \tag{27}$$

with $\sigma = X,Z$, form the representation of su(2), and the degenerate ground states are connected only by these nonlocal operators. The computation on the 2D resource equips similar robustness, as the Hamiltonian of the 2D model is locally unitary equivalent to N independent chains. To utilize the gap protection, we need to turn off the interactions that couple the computational block (see Fig. 3) to the bulk together with those inside the block, prior to any measurement for this block. As the Hamiltonian H_{2d} is frustration-free, this can be done in a constant time. Also, particles already measured need to stay decoupled from the remaining bulk.

In a potential implementation of H_{2d} with trapped polar molecules in an optical lattice [22], nearest-neighbor interactions can be turned off by changing the potential depth of local wells, which, in turn, suppresses the tunneling rate between two neighboring wells. An alternative method without turning off interactions is to apply fast measurements and remove the particle from the system after the measurement or drive it to a dark state (which no longer interacts with neighboring particles).

VI. EXTENSION TO 2D LATTICES

Our approach can be extended from the octagonal lattice to the 2D square lattice, for instance. The corresponding 1D AKLT quasichain consists of spin-2 particles, each of which is connected with two spin-$\frac{1}{2}$ particles. In a similar way, we can show that the energy gap is lower bounded by $\Delta E \geqslant J\gamma\varepsilon_p = 0.0418J$ with $\gamma = 0.241$ and $\varepsilon_p \geqslant 0.1735$. We can merge a number of such 1D AKLT quasichains into a 2D resource state as well, and the computational protocol is similar.

VII. SUMMARY

We have proposed a translational-invariant gapped Hamiltonian of spin-$\frac{3}{2}$ particles with nearest-neighbor two-body AKLT- and exchange-type interactions. Its unique ground state is proved to be universal for measurement-based quantum computation. The Hamiltonian inherits important properties from the original AKLT model, while at the same time, it has distinct features, e.g., a strictly proved energy gap. Further study on such a Hamiltonian and its order parameter might reveal new aspects of many-body physics regarding computational capability.

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