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Quantum metrology to probe atomic parity nonconservation

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An entangled state prepared in a decoherence-free subspace, together with a Ramsey-type measurement, can probe parity violation in heavy alkali-metal ions such as Ba^+ or Ra^+. Here we propose an experiment with Ba^+ as an example to measure the small parity-violating effect in this system. It has been shown that a measurement on a maximally correlated system will reduce the uncertainty as compared to that on a single ion measurement, and also provides a feasible solution to measure the nuclear-spin-dependent part of the total parity-violating light shift in an ionic system.

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Measurement of atomic parity nonconservation (PNC) in the $6S$-$7S$ transition of atomic Cs has been performed with an uncertainty reaching 0.35% [1–3]. An equally demanding theoretical effort in this atom [4] leads to the evaluation of the weak nuclear charge Q_W, which is a unique low-energy test of the standard electroweak theory. Further improvement in the precision will lead to reducing the limits on the mass of an eventual additional light or heavy boson [5]. Apart from the necessity of improving the PNC measurement in Cs, it would be worthwhile to consider other possible experimental techniques for the measurement of PNC in other systems. Recently, the largest PNC effect has been measured in the $6s^2\,{}^1S_0$-$5d6s\,{}^3D_1$ transition in atomic ytterbium [6] employing the same technique as used in the Cs experiment. The enhancement in this case is caused by degeneracy of atomic levels [7]. Although the measured PNC dipole transition amplitude ($E1_{\rm PNC}$) is 100 times larger than that in atomic cesium, the experimental precision is not sufficient to verify the Standard Model or to predict any physics beyond it.

A proposed method, adopted for the Cs measurement, involved left-right asymmetry of the forbidden transition rate in the $6S$-$7S$ transition [8,9]. This method is presently being pursued for Fr, the heaviest alkali metal [10,11]. Unfortunately, the requirement of a large number of atoms to observe the asymmetry limits this experiment. Recently, it has been proposed to observe a linear Stark shift in an interferometric measurement with a small number of atoms of Fr [12]. The measurement of light shift arising due to the interference between $E1_{\rm PNC}$ and electric quadrupole transition amplitude $E2$ in a heavy ion such as Ba^+ and Ra^+ (proposed by Fortson [13]) seems to be the most promising technique. It can, in principle, achieve a precision of 0.1%. Presently it is being pursued at different experimental laboratories [14,15]. Initial radio frequency (rf) spectroscopy on Ba^+ has also been performed to observe the light shifts of different Zeeman sublevels. The major limitations in these measurements appear to be from magnetic-field noise as well as from laser-frequency noise [16]. In order to finally observe the PNC-induced light shift, it is necessary to achieve an uncertainty well below 1 Hz in the ground-state Larmor frequency since even in the presence of a strong electric field, the shift is only of the order of 0.2 Hz. Although maximally entangled states for quantum metrology have only recently been studied, they have already been implemented in a relatively few cases [17–19]. They have been used to improve the signal-to-noise ratio [20], to efficiently detect quantum states [21], to measure scattering length [22], and to perform spectroscopy in decoherence-free subspace (DFS) [23]. An entangled state prepared in a DFS [24,25] makes any measurement immune to environmental changes, and therefore it can be effectively used to overcome the magnetic-field noise limitation of the single ion experiment to observe the PNC light shift. In the following, we outline this promising technique of such a measurement with high precision.

Parity nonconservation in an atomic system leads to a small mixing between states of opposite parities, resulting in a nonzero probability in the electric dipole transition, which is strictly forbidden by the parity conservation rule. The effect, though scales as Z^3 for heavier atoms [3], is on the order of $10^{-11}ea_0$. It is thus an experimental challenge to measure such a small quantity directly. Instead, experimenters look for interference-like phenomena between $E1_{\rm PNC}$ and a much stronger higher-order electromagnetic transition between the same states. For Ba^+ or Ra^+, such an interference between $E1_{\rm PNC}$ and $E2$ in the $nS_{1/2}$-$(n-1)D_{3/2}$ transition is proposed to measure the vector light shift [13]. In the presence of the electric field of a laser,

$$\mathbf{E}(\mathbf{r},t) = \tfrac{1}{2}\mathbf{E}_0(e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} + \text{c.c.}), \tag{1}$$

the $E1_{\rm PNC}$ and $E2$ couplings between $S_{1/2}$ and $D_{3/2}$ are described in terms of their respective Rabi frequencies as

$$\Omega^{\rm PNC}_{m'm} = \frac{1}{2\hbar}\sum_i \varepsilon^{\rm PNC}_{m'm} E_i(0), \tag{2}$$

$$\Omega^{Q}_{m'm} = \frac{1}{2\hbar}\sum_{i,j} \varepsilon^{Q}_{m'm} \left(\frac{\partial E_i(r)}{\partial x_j}\right)_0, \tag{3}$$

$r = 0$ being the position of the ion in the trap. Here $\varepsilon^{\rm PNC}_{m'm}$ and $\varepsilon^{Q}_{m'm}$ describe $E1_{\rm PNC}$ and $E2$ matrix elements between the m sublevel of $S_{1/2}$ and the m' sublevel of $D_{3/2}$. The resultant Rabi frequency of the m sublevel of $S_{1/2}$ is [13]

$$\Omega_m \approx \Omega^Q_m + \text{Re}\sum_{m'} \left(\Omega^{\rm PNC*}_{m'm}\Omega^Q_{m'm}\right)/\Omega^Q_m, \tag{4}$$

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where $(\Omega_m)^2 = \sum_{m'} |\Omega_{m'm}|^2 = \sum_{m'} |\Omega^Q_{m'm} + \Omega^{\rm PNC}_{m'm}|^2$ and $(\Omega^Q_m)^2 = \sum_{m'} |\Omega^Q_{m'm}|^2$. By considering the Zeeman splitting of the magnetic sublevels to be comparable to the linewidth of the $S_{1/2}$-$D_{3/2}$ laser, the light shift of the m sublevel of the ground state is given by

$$\Delta\omega_m = \delta/2 - \Omega_m, \tag{5}$$

where $\delta = \omega_0 - \omega$ is the detuning of the laser frequency from the atomic transition frequency. It is convenient to drive both the quadrupole and PNC-allowed $S_{1/2}$-$D_{3/2}$ dipole transitions independently, so that a much larger contribution in $\Delta\omega_m$ due to the pure $E2$ coupling remains the same while that due to the interference term changes sign for the magnetic sublevels of the ground state. Fortson [13] showed that this is achieved when a single ion is placed ($x = z = 0$) simultaneously at the antinode and the node of two standing-wave lasers represented, respectively, as

$$E' = \hat{x}E'_0 \cos kz \text{ and } E'' = i\hat{z}E''_0 \sin kx. \tag{6}$$

These lasers produce $\Delta m = \pm 1$ dipole and quadrupole transitions, respectively. In the presence of these two lasers, the Larmor frequency between the ground magnetic sublevels is given by

$$\omega'_L = \omega_L \sim 2{\rm Re} \sum_{m'} \left(\Omega^{\rm PNC*}_{m'm}\Omega^Q_{m'm}\right)/\Omega^Q_m, \tag{7}$$

where ω_L is the Larmor frequency between the same sublevels in the absence of the lasers. Thus the PNC shift can be extracted from the measurement of the ground-state Larmor frequency in the absence and in the presence of the laser fields. Fortson calculated the shift to be 0.2 Hz for Ba^+ in the presence of a strong laser field $E'_0 = 2 \times 10^6$ V/m [13]. However, it is still a challenge to measure such a small change by applying the usual rf spectroscopic technique. It demands a magnetic field of stability one part in 10^8 for a few hundred kilohertz magnetic splitting in order to achieve an accuracy of 1%.

By employing the generalized Ramsey interferometric technique to the maximally correlated atomic state, it is possible to determine the PNC light shift with the required precision. Under free precision, a maximally entangled atomic state, similar to one of Bell's states, $\psi(0) = \frac{1}{\sqrt{2}}(|u_1\rangle\,|u_2\rangle + |v_1\rangle\,|v_2\rangle)$, evolves into $\psi(\tau) = \frac{1}{\sqrt{2}}(|u_1\rangle\,|u_2\rangle + \exp^{i\Delta\lambda\tau}|v_1\rangle\,|v_2\rangle)$ after a time τ. The phase evolution rate $\Delta\lambda = [(E_{u_1} + E_{u_2}) - (E_{v_1} + E_{v_2})]/\hbar$ corresponds to the energy difference between the atomic states u_k and v_k. The real part of the phase factor $\exp^{i\Delta\lambda t}$ can be measured by projecting the ions on the states $|\pm\rangle = \frac{1}{\sqrt{2}}(|u\rangle_k \pm |v\rangle_k)$ and measuring the relative phase. For states in the DFS, the free precision time τ can be made very long and hence the phase can be measured accurately [25,26]. By a careful choice of the state, it is possible to measure the PNC shift in DFS, thereby avoiding the possible systematic effects in coupling to the environment.

Instead of a single ion, in the following we consider a string of two Ba^+ ions (even isotope, $I = 0$) confined in a linear Paul trap. The relevant electronic levels are shown in Fig. 1. The ions were cooled into their ground motional state of the first two normal modes of motion [27] using laser Doppler cooling, by applying 493- and 650-nm lasers followed by sideband cooling with a 1.76-μm laser (Fig. 1).

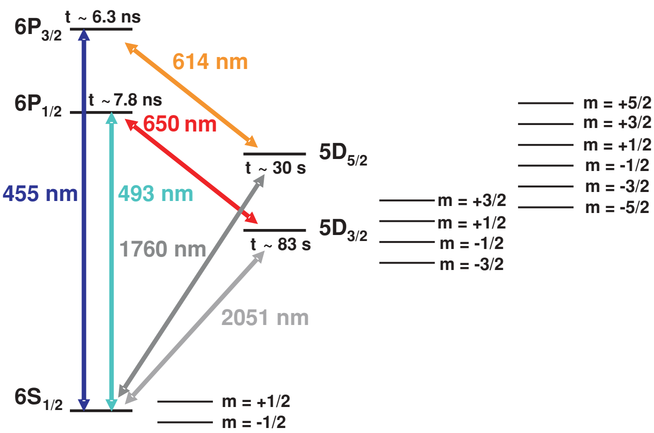

FIG. 1. (Color online) Relevant atomic levels of Ba^+. The Zeeman sublevels are also shown for clarity.

Both of the ions are prepared in a Zeeman sublevel of the ground electronic state (e.g., $6S_{1/2}, m = 1/2$). The ions are then individually treated with a sequence of laser pulses. A $\pi/2$ pulse at the blue sideband on the first ion prepares it in a superposition of the electronic ground and metastable (e.g., $D_{5/2}, m = 1/2$) states and the motional ground and first excited states. A π pulse at the carrier on the second ion brings it to the electronic excited state ($D_{5/2}, m = 1/2$), keeping the motional state unchanged. Another blue sideband π pulse on the second ion transfers the excited-state population back to ground electronic and motional states. One more π pulse at the carrier on each ion coherently transfers the quadrupole excited-state population into the other Zeeman sublevel in the ground state, thus preparing the state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1|0\rangle_2 + |0\rangle_1|1\rangle_2)|0\rangle, \tag{8}$$

where $|1\rangle_i$ and $|0\rangle_i$ stand for $m = 1/2$ and $-1/2$ of the $6S_{1/2}$ state of the ith ion, and $|0\rangle$ describes the ground motional state of center-of-mass (c.m.) mode. The presence of the two-ion state makes it decoherence free as compared to the superposition state of a single ion. The Zeeman shifts of the two parts of the entangled state cancel out in the absence of the magnetic-field gradient along the trap axis. This state is immune to any decoherence effects arising from the magnetic-field fluctuation common to both ions, spontaneous decay, etc., and therefore the state, in principal, possesses an infinitely long coherence.

After preparing such an entangled state in DFS, two laser fields E' and E'' in a standing-wave configuration are applied for a time interval τ on one ion (e.g., ion 1) as shown in Fig. 2. The magnetic splitting, quadrupole light shift, and PNC light shift of the ground-state magnetic sublevels for the two ions are shown schematically in Fig. 3, which depicts that the ground-state Larmor frequency of one ion shifts only due to PNC interaction, while that of the other ion remains unchanged. Thus, a small perturbation is introduced within the entangled state $|\Psi\rangle$ [Eq. (8)] and it evolves as

$$|\Psi(\tau)\rangle = \frac{1}{\sqrt{2}}[|1\rangle_1|0\rangle_2 + \exp(i\Delta\lambda\tau)|0\rangle_1|1\rangle_2]|0\rangle, \tag{9}$$

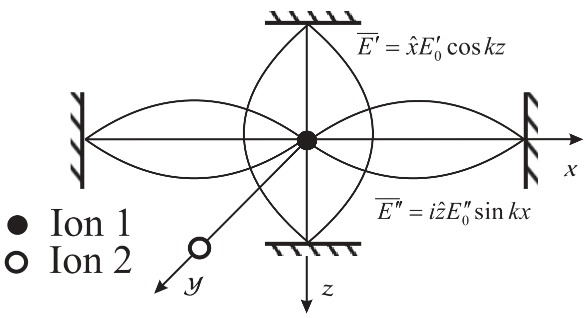

FIG. 2. Schematic of the experiment with two ions placed in a linear ion trap and interrogated by two standing-wave lasers. The amplitude E_0' should be orders of magnitude larger as compared to E_0'' for an improved systematic.

where the phase evolution rate $\Delta\lambda$ corresponds to the energy difference between the two parts of the entangled state [i.e., the PNC light shift given by Eq. (7)].

The state $|\Psi(\tau)\rangle$ should be projected on $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle_i \pm |1\rangle_i)$ in order to observe the time evolution of the expectation value $\langle\sigma_x^{(1)} \otimes \sigma_x^{(2)}\rangle$, where $\sigma_x^{(i)}$ denotes the Pauli spin matrix for the ith ion. It oscillates with a frequency $2\pi/\Delta\lambda$ [25]. Thus, the PNC light shift can be extracted directly from the measurement of the oscillation frequency.

The uncertainty in the PNC light shift measurement will be determined by the decoherence time of the maximally entangled state, which is practically infinite in the absence of external perturbation but limited by the natural lifetime (τ) of the $5D_{3/2}$ state in our case. The uncertainty in the frequency measurement on N maximally correlated atomic systems is inversely proportional to NT instead of $\sqrt{NT}$ for uncorrelated systems [17]. Here, T is the time of a single measurement that can be made as large as τ is. Therefore the statistical signal-to-noise ratio for n number of measurements can be approximated as

$$\frac{\varepsilon^{\rm PNC}}{\delta\varepsilon^{\rm PNC}} \approx \frac{\varepsilon^{\rm PNC} E_0'}{\hbar} f\sqrt{n}N\tau, \qquad (10)$$

where f signifies an experimental efficiency factor. It is determined by how well the entangled state is formed and detected. In our case, it can be close to 1 since it has been shown that such a state can be prepared with a fidelity of nearly 95%. N in this case is 2, since a two-ion maximally correlated state is used for the measurement. By considering the same f as in the single-ion experiment [13], the figure of merit will be twice as high in the present experiment. In other words, it would be possible to achieve the same precision by performing one-quarter as many experiments as compared to that on a single ion. It can, in principle, be further improved by considering the correlated state of more than two ions.

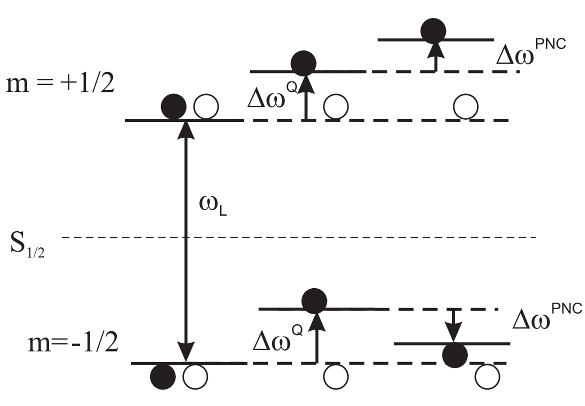

FIG. 3. Energy shifts of ground-state magnetic sublevels of two ions in the presence of the magnetic field and lasers E' and E''. $\Delta\omega^Q$ and $\Delta\omega^{\rm PNC}$ denote quadrupole and PNC light shifts, respectively.

The size of the PNC light shift could, in principle, be increased by increasing the amplitude E_0'. However, as the amplitude is increased, the off-resonant couplings become more and more important, effectively deteriorating the coherence of the entangled state. The induced loss rate is [28]

$$\Gamma_{\gamma jm}^{\rm loss} = \frac{e^2}{4\hbar^2} \sum_{\gamma',m',\pm\omega} \frac{|\langle\gamma' j' m'|\mathbf{E}\cdot\mathbf{r}|\gamma jm\rangle|^2}{(\omega_{\gamma'} - \omega_\gamma \pm \omega)^2} \frac{\omega^3}{(\omega_{\gamma'} - \omega_\gamma)^3} \Gamma_{\gamma' j'}, \qquad (11)$$

where $\Gamma_{\gamma' j'}$ is the spontaneous transition rate out of $|\gamma' j'\rangle$. By considering the off-resonant coupling from $6P$ levels, $\Gamma_{\gamma jm}^{\rm loss}$ values for the states $|6S_{1/2}, m = 1/2\rangle$, $|5D_{3/2}, m = 1/2\rangle$, and $|5D_{3/2}, m = 3/2\rangle$ of Ba^+ have been estimated for an electric field $E_0' = 1.6 \times 10^6$ V/m, and they turn out to be 0.0044, 0.007, and 0.0008 Hz, respectively. The total induced loss rate is comparable to the natural decay rate of $5D_{3/2}$ (0.012 Hz), and hence the electric-field amplitude mentioned earlier is the maximum for this experiment. The off-resonant light shift for such a larger electric field is also significant, but for a linear polarization the ground-state Zeeman sublevels suffer a scalar shift and it does not change the Larmor frequency. However, in the presence of a small circular polarization in the E' laser, the sublevels experience a vector shift [29] that can mimic the PNC measurement. This systematic can be measured by performing the same experiment described earlier but in the absence of the E'' laser so that there is no interference. Alternatively, the ions could be placed at the antinodes of the E' laser, while one of them could be placed at the node of the E'' laser. A small magnetic-field gradient along the trap axis is a major source of systematic effects, but it can also be eliminated by repeating the experiment and exchanging the role of the two ions or of the same ion in the absence of the laser fields. In order to finally extract the PNC-induced $E1$ amplitude, it is necessary to know the electric field at the ion position $E_0'(0)$, $E_0''(0)$ and the quadrupole light shift. The electric fields could be measured by off-resonant excitations, since the related matrix elements for Ba^+ are well known [30]. The quadrupole light shift can be measured as well by using the technique of the generalized Ramsey interference experiment [31,32]. Using two ions instead of one ion in a linear ion trap may lead to an unwanted stray electric field, which is a major concern for parity mixing. Since the ions are sideband cooled to the ground state of their c.m. mode, the field at the ion equilibrium position must be zero. The ions in a linear string of Coulomb crystals have a wave-packet span that is negligible as compared to the wavelength of the standing wave. Therefore, they can be considered to be at rest. The first-order effects due to stray fields as well as the trapping potential are not only displaced from the PNC transition by multiples of trap frequency but are also negligibly small due to sideband cooling.

In case of nonzero nuclear spin isotopes the physics of PNC is even richer because of the presence of a tiny nuclear-spin-dependent (NSD) contribution. The measurement of the nuclear-spin-dependent part, and hence the nuclear anapole moment in $E1_{\rm PNC}$, appears to be difficult by driving the rf spin-flip transition on a single ion, but it is feasible with the technique described here. For example, in spin $I = 3/2$ isotopes, there is only one $M1$ allowed transition (between $m_F = 1,0$ of $F = 2, S_{1/2}$ in the presence of laser fields connecting $F = 2, S_{1/2}$ and $F' = 3, D_{3/2}$) in which the quadrupole-transition-induced light shift does not change the Larmor frequency. This is essential for measuring the total PNC light shift. In order to extract the NSD part in $E1_{\rm PNC}$, other transitions of the same isotope need to be considered to measure the light shift due to the total PNC amplitude. The Larmor frequency between those two levels contains not only the differential PNC shift but also the differential quadrupole light shift, which is a serious systematic effect. However, two entangled states [Eq. (8)] with $|1\rangle_i = |m_F = 2, F = 2, 6S_{1/2}\rangle_i$, $|0\rangle_i = |m_F = -2, F = 2, 6S_{1/2}\rangle_i$ and $|1\rangle_i = |m_F = 1, F = 2, 6S_{1/2}\rangle_i$, $|0\rangle_i = |m_F = -1, F = 2, 6S_{1/2}\rangle_i$ can be formed to measure the NSD contribution. The measured light shifts with these two states contain both nuclear-spin-dependent and -independent parts, which are the same in the two transitions but multiplied by associated Clebsch-Gordan coefficients. It is therefore convenient to separate out both contributions with high precision.

We have shown that a two-ion entangled state is a better tool for the measurement of parity-violating light shift as compared to the single-ion experiment. Various systematics present in a single-ion experiment are absent in this case, and some of them can even be measured in this case. The statistical signal-to-noise ratio can be improved with this maximally correlated state. The measurement of the nuclear-spin-dependent contribution and the nuclear anapole moment is feasible using correlated atomic states, as shown here. The experimental techniques involved here are regularly in use by the quantum computation community. Therefore, it is feasible with today's technology.

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