Vacuum-induced Berry phases in single-mode Jaynes-Cummings models

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Motivated by work [Phys. Rev. Lett. **89**, 220404 (2002)] for detecting the vacuum-induced Berry phases with two-mode Jaynes-Cummings models (JCMs), we show here that, for a parameter-dependent single-mode JCM, certain atom-field states also acquired photon-number-dependent Berry phases after the parameter slowly changed and eventually returned to its initial value. This geometric effect related to the field quantization still exists, even if the field is kept in its vacuum state. Specifically, a feasible Ramsey interference experiment with a cavity quantum electrodynamics system is designed to detect the vacuum-induced Berry phase.

PACS number(s): 42.50.Ct, 03.65.Vf

In 1984, Berry showed that the state of a quantum system can acquire a purely geometric phase (now called the Berry phase), in addition to the usual dynamical phase, after slowly changing and eventually returning to its initial form [1]. Basically, the Berry phase does not depend on the dynamical properties of the system but only on the topological features of the parameter space of the evolved system. Up to now, the Berry phase has been found in various systems, such as spins, polarized light, atoms, and so on [2]. Also, recent studies have shown that the geometric phases can be utilized to implement quantum logic gates for realizing quantum computation [3–5].

Quantized optical fields, as well as their interactions with atoms, are the main objects in quantum optics. Originally, the famous Jaynes-Cummings model (JCM) [6] was introduced to describe the interaction between an undamped two-level atom and a nondecaying single-mode quantized field, under the rotating-wave approximation. This model has been widely generalized to treat various interactions between atoms and photons. These include, for example, the interaction of multilevel atoms with multimode quantized fields and various multiphoton processes in quantum optics [7]. One of the basic phenomena in quantum optics is that the vacuum of a quantized field can behave as a physical reality with certain observable effects. For example, in terms of vacuum fluctuations of the quantized electromagnetic field [8], certain important quantum effects, such as Lamb shifts and spontaneous emissions, can be well explained. Recent works [9,10] indicate that the vacuum of a quantized optical field could also induce the observable Berry phases. In order to observe these vacuum-induced geometric effects, two field modes were introduced [9,10] to interact with a two-level atom. As a consequence, the experimental tests are relatively complicated.

In this work, we show that only one field mode interacting with a two-level atom could be utilized to detect the vacuum-induced Berry phase. Beginning with a generic model, that is, the m-quantum JCM, we show how the desirable Berry phase can be acquired by an evolved quantum state in a parameter-dependent single-mode JCM. Furthermore, we design a Ramsey interference device involved with only one

field mode to detect such a geometric effect related to the field quantization.

The Hamiltonian of an m-quantum JCM [11], that is, a two-level system coupled to a quantized mode via an m-photon process, can be expressed (under the rotating-wave approximation) as

$$H = \nu a^\dagger a + \frac{\omega}{2}\sigma_z + \lambda_m(\sigma_+ a^m + \sigma_- a^{\dagger m}). \quad (1)$$

Here, $a^\dagger$ and a are the creation and annihilation operators of the cavity field with frequency ν, and σ_+, σ_-, and σ_z are the Pauli operators of the atom. The symbol ω is the transition frequency of the atom between the excited state $|2\rangle$ and ground state $|1\rangle$, and λ_m is the coupling coefficient between the atom and cavity mode. Under the time-dependent unitary transformation $\hat{S}(t) = \exp(i\Delta_m\sigma_z t/2)$, this Hamiltonian can be rewritten as

$$H_m = \frac{\Delta_m}{2}\sigma_z + \lambda_m(\sigma_+ a^m + \sigma_- a^{\dagger m}), \quad (2)$$

where $\Delta_m = \omega - m\nu$ is the detuning. The eigenstates of such a Hamiltonian read

$$|\Psi_n^-\rangle = \cos\frac{\theta_{nm}}{2}|1, n+m\rangle - \sin\frac{\theta_{nm}}{2}|2, n\rangle \quad (3)$$

and

$$|\Psi_n^+\rangle = \cos\frac{\theta_{nm}}{2}|2, n\rangle + \sin\frac{\theta_{nm}}{2}|1, n+m\rangle, \quad (4)$$

respectively. Here, $\{|n\rangle, n = 0, 1, 2, \ldots\}$ are the number states of the quantized bosonic field, and $\theta_{nm} = \arccos[\Delta_m/\sqrt{\Delta_m^2 + 4\lambda_m^2(n+m)!/n!}]$.

Following Fuentes-Guridi *et al.* [9], we introduce a phase-shift operation $U[\phi(t)] = \exp[-i\phi(t)a^\dagger a]$ to change the Hamiltonian (2) to the following parameter-dependent form:

$$H_m(\phi) = \frac{\Delta_m}{2}\sigma_z + \lambda_m(\sigma_+ a^m e^{im\phi} + \sigma_- a^{\dagger m}e^{-im\phi}). \quad (5)$$

Obviously, such a ϕ-dependent Hamiltonian describes a two-level atom interacting (via an m-photon process) with a quantized field mode. Here, the phase parameter $\phi(t)$ changes with the time t and can be changed slowly from 0 to 2π, generating a cyclic path in the parameter space during the evolution. As a consequence, if the system begins with one of its eigenstates, $|\Psi_n^+\rangle$ or $|\Psi_n^-\rangle$, then it returns to this state
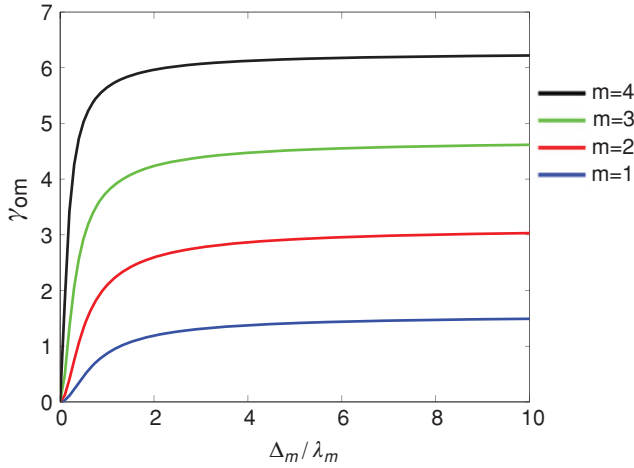
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FIG. 1. (Color online) The curves, going from bottom to top, and indicated in color by blue, red, green, and black, represent the vacuum-induced Berry phase of $m = 1$ ($m = 2$, $m = 3$, $m = 4$), respectively.

but acquires a geometric phase (besides the dynamical one not shown here):

$$\gamma_+ = i \int_c d\phi \langle \Psi_n^+ | U^\dagger(\phi) \frac{d}{d\phi} U(\phi) | \Psi_n^+ \rangle$$
$$= m\pi(1 - \cos\theta_{nm}) + 2\pi n \qquad (6)$$

or

$$\gamma_- = i \int_c d\phi \langle \Psi_n^- | U^\dagger(\phi) \frac{d}{d\phi} U(\phi) | \Psi_n^- \rangle$$
$$= -m\pi(1 - \cos\theta_{nm}) + 2\pi(n + m). \qquad (7)$$

The geometric phase acquired here depends on the photon number n. Physically, this nontrivial quantum effect can be measured by using an interference procedure between the eigenstate $|\Psi_0^+\rangle$ (or $|\Psi_0^+\rangle$) and the ground state $|1,0\rangle$, for which no geometric phase is acquired. Typically, if the system begins with the state $|2,0\rangle = \cos\theta_{0m}/2|\Psi_0^+\rangle - \sin\theta_{0m}/2|\Psi_0^-\rangle$, that is, the field is in a vacuum state, this adiabatic operation performed on the degrees of freedom of the field still yields a Berry phase

$$\gamma_{0m} = m\frac{\pi}{2}(1 - \cos 2\theta_{0m}) = m\frac{\Omega_m}{4}, \qquad (8)$$

with the solid angle $\Omega_m = 2\pi[1 - \cos(2\theta_{0m})]$. Figure 1 shows how the vacuum-induced Berry phase varies with the parameter Δ_m/λ_m. Through adiabatic evolution, the initial state $|2\rangle$ coupling to vacuum mode in cavity acquires a geometric phase. In this case, the atom-field entanglement in the eigenstates (3), (4) cannot be neglected [9]. Note that the expression (8) cannot be interpreted only as a geometric phase of the two-level system, as the origin of the geometric phase is related to the vacuum fluctuation of the field. Clearly, for a common $\Delta_m/\lambda_m \equiv \Delta/\lambda$, the more quantum m corresponds to the greater vacuum-induced Berry phase. Basically, the photon-dependent Berry phase shown in Eqs. (6) and (7) is due to the performance of the field quantization. Thus, even the photon number of the field is 0, and the geometric phase is still nontrivially induced. Any classical correspondence of such a phenomenon does not exist.
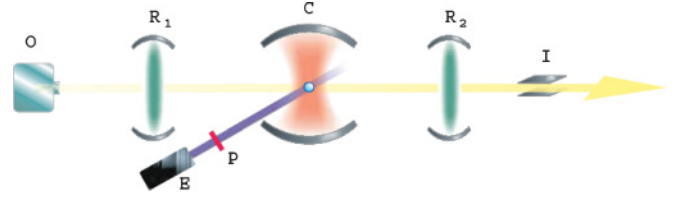

FIG. 2. (Color online) An experimental Ramsey interference setup [10] for observing the Berry phase generated in the one-mode JCM. Here, an atom is emitted from the source O; flies sequentially across the first Ramsey zone R_1, high-Q quantized cavity C (wherein the cavity is kept in the vacuum state and the desirable vacuum-induced Berry phase is generated by a classical driving from E), and the second Ramsey zone R_2; and then is finally detected in I. The information of the vacuum-induced Berry phase is extracted by the measured atomic probability.

Berry phases related to field quantization could be measured with the usual one-photon JCMs, which has been experimentally demonstrated in the well-known cavity QED systems. Indeed, various quantum properties [12] of the radiation field interacting with atoms have been successfully demonstrated with these systems. Typically, a cavity QED experiment [10] involved with *two* quantized bosonic modes has been proposed to test the geometric phases generated in a two-mode JCM. In what follows, we show that a cavity QED system involving only one quantized bosonic mode could also be utilized to test this vacuum-induced Berry phase. Our proposed setup for such a test is shown in Fig. 2, wherein an atom is emitted from the source O; flies sequentially across R_1, C, and R_2; and is finally detected in I.

Initially, the atom is assumed to be prepared in the upper level $|2\rangle$ in the source O and then emitted. After the first Ramsey zone R_1, the state of the atom reads

$$|\Psi_1\rangle = \cos\left(\frac{\Omega_{R1}\tau_1}{2}\right)|2\rangle + i\sin\left(\frac{\Omega_{R1}\tau_1}{2}\right)|1\rangle, \qquad (9)$$

with τ_1 being the time spent by the atom inside the zone R_1.

During the atom flies across the high-Q quantized cavity C, the parameter-dependent Hamiltonian (5) can be obtained. For example, a Raman configuration shown in Fig. 3 is utilized to achieve the ϕ-dependent one-photon JCM. Here, an auxiliary external classical laser beam $E(t)$ is applied to drive the transition $|2\rangle \leftrightarrow |3\rangle$ with the Rabi frequency
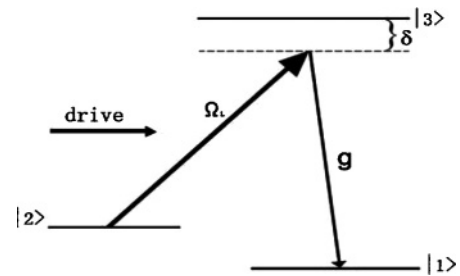

FIG. 3. Schematic diagram of a three-level atom interacting with a quantized field in C cavity (see Fig. 2), which induces the transitions $|3\rangle \leftrightarrow |1\rangle$ with Rabi frequency g. In addition, a classical laser field driving the transition $|3\rangle \leftrightarrow |2\rangle$ (with Rabi frequency $\Omega_L = \Omega_0 e^{i\phi}$) is applied to produce the desirable ϕ-dependent single-mode JCM.

$\Omega_L = \Omega_0 \exp(i\phi)$, while the quantized cavity mode $(a, a^\dagger)$ couples to the transition $|1\rangle \leftrightarrow |3\rangle$ with the strength g. The Hamiltonian describing such a configuration in the interaction picture reads ($\hbar \equiv 1$) (see, e.g., [13])

$$H_{\text{int}} = \Omega_L \sigma_{32} e^{-i\delta t} + g\sigma_{31} a e^{-i\delta t} + \text{H.c.}, \quad (10)$$

with δ being the detuning. Generally, the corresponding time-evolution operator can be formally expressed as

$$U_I(t) = 1 - i \int_0^t dt' H_I(t')$$
$$- \int_0^t dt' H_I(t') \int_0^t dt'' H_I(t'') + \cdots. \quad (11)$$

Under the so-called large-detuning limit, that is, $\delta \gg g, \Omega_0$, the second-order contribution to $U_I(t)$ is significantly more important than the first-order one. This is because the former involves terms linear in time, whereas the latter involves terms that are just oscillatory or constant in time. Therefore, we can retain only the second-order terms and rewrite the time-evolution operator (14) as

$$U_I(t) \approx 1 - i \left\{ \frac{\Omega_L^2}{\delta} \sigma_{22} + \frac{g^2}{\delta} a a^\dagger \sigma_{11} \right. $$
$$\left. + \frac{\Omega_0 g}{\delta} [\sigma_{21} a e^{i\phi} + \sigma_{12} a^\dagger e^{-i\phi}] \right\} t = 1 - i H_{\text{eff}} t, \quad (12)$$

with an effective Hamiltonian

$$H_{\text{eff}} = \frac{\Omega_0^2}{\delta} \sigma_{22} + \frac{g^2}{\delta} a a^\dagger \sigma_{11}$$
$$+ \lambda_1 [\sigma_{21} a e^{i\phi} + \sigma_{12} a^\dagger e^{-i\phi}], \quad \lambda_1 = \frac{\Omega_0 g}{\delta}. \quad (13)$$

Obviously, this effective Hamiltonian is equivalent (apart from the unimportant Stark shifts) to the ϕ-dependent Hamiltonian (5) with $m = 1$. Therefore, after passing the cavity vacuum wherein the driving parameter ϕ changes from 0 to 2π, the atom undergoes the following evolution:

$$|\Psi_1\rangle \longrightarrow |\Psi_2(\tau_1)\rangle = e^{i\gamma_{01} + i\xi} \cos\left(\frac{\Omega_{R1}\tau_1}{2}\right) |2\rangle$$
$$+ i \sin\left(\frac{\Omega_{R1}\tau_1}{2}\right) e^{-i\xi} |1\rangle. \quad (14)$$

Here, $\gamma_{01} = \Omega_1/4$ is the vacuum-induced Berry phase acquired in the pass of the cavity, and $\xi = \lambda_1 \tau$, with τ being the time during which the atom stayed in the cavity.

Furthermore, after passing the second Ramsey zone R_2, the atom evolves to the state

$$|\Psi_2\rangle \longrightarrow |\Psi_3(\tau_1, \tau_2)\rangle = c_1(\tau_1, \tau_2)|1\rangle + c_2(\tau_1, \tau_2)|2\rangle, \quad (15)$$

with

$$c_1(\tau_1, \tau_2) = e^{-i\xi} i \sin\left(\frac{\Omega_{R1}\tau_1}{2}\right) \cos\left(\frac{\Omega_{R2}\tau_2}{2}\right)$$
$$+ e^{i\gamma_{01} + i\xi} i \cos\left(\frac{\Omega_{R1}\tau_1}{2}\right) \sin\left(\frac{\Omega_{R2}\tau_2}{2}\right),$$
$$c_2(\tau_1, \tau_2) = e^{i\gamma_{01} + i\xi} \cos\left(\frac{\Omega_{R2}\tau_2}{2}\right) \cos\left(\frac{\Omega_{R1}\tau_1}{2}\right)$$
$$- e^{-i\xi} \sin\left(\frac{\Omega_{R2}\tau_2}{2}\right) \sin\left(\frac{\Omega_{R1}\tau_1}{2}\right).$$

If the two Ramsey zones are properly set such that the condition $\Omega_{R1}\tau_1 = \Omega_{R2}\tau_2 = \pi/2$ is exactly satisfied, then the probability of detecting the atom in its upper level $|2\rangle$ in I is

$$P_2 = |c_2(\tau_1, \tau_2)|^2 = \frac{1 - \cos(\gamma_{01} + 2\xi)}{2}. \quad (16)$$

If $\xi = n\pi$ is set inside the cavity, this probability can be further simplified to

$$\tilde{P}_2 = \frac{1 - \cos(\gamma_{01})}{2}, \quad (17)$$

which is directly related to the Berry phase acquired by the atom flying across the high-Q quantized cavity C. Therefore, the Berry phase generated in the one-photon JCM could be observed by the Ramsey interference method.

Experimentally, the one-photon Rabi frequency is set as $g/2\pi \simeq 50$ kHz [14,15]. This implies that if the solid angle is required to be $\Omega_1 = \pi$, then the parameter θ_{01} should be set to satisfy the condition $\cos 2\theta_{01} = 1/2$. Since the parameter θ_{01} is determined by $\theta_{01} = \arccos[\Delta_1/\sqrt{\Delta_1^2 + 4\lambda_1^2}]$, with $\Delta_1 = (\Omega_0^2 - g^2)/\delta$, $\lambda_1 = \Omega_0 g/\delta$, the Rabi frequency of the applied classical driving should be designed as $\Omega_0/2\pi \simeq 173$ kHz. On the other hand, in order to satisfy the large-detuning condition required, that is, $\delta \gg g, \Omega_0$, we may typically set $\delta = 3\Omega_0$ yielding $\lambda_1/2\pi \simeq 15$ kHz. This means that the atom-field interaction cam perform ten complete Rabi cycles during an effective atom-cavity interaction time of 0.6 ms [8]. This interaction time is manifestly shorter than the decay time (1 ms) of the cavity (see, e.g., [14]).

We now discuss how the dissipation of the cavity influences the observable effect of the vacuum-induced geometric phase. Following [16], the effective Hamiltonian of the atom-cavity system becomes $\tilde{H}_{\text{eff}} = H - i\Gamma\hat{n}/2$. With the same procedure, we can prove that through a cyclic and adiabatic evolution the
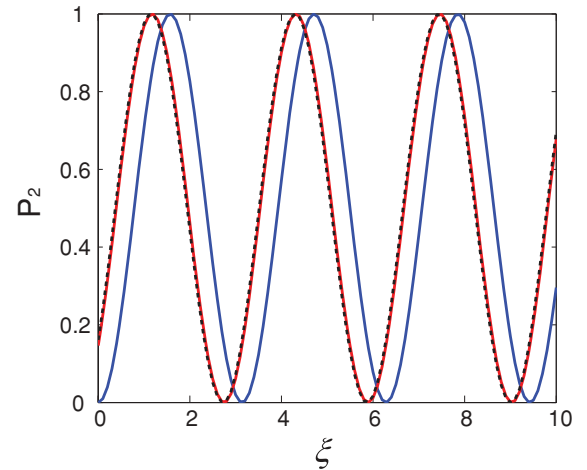

FIG. 4. (Color online) Experimental predictions to observe the vacuum-induced Berry phase by measuring P_2, the probability of the atom being detected in the state $|2\rangle$. This probability is a function of the controllable parameter ξ related to the atom interacting with the cavity. Here, the second solid (blue) curve corresponds to the Ramsey interferometry without Berry phase, while the first solid (red) curve shows the situation in which a geometric phase shift ($\pi/4$) is induced. Additionally, the dashed (black) curve shows that in the presence of the cavity decay $\Gamma = 1$ kHz, the P_2 is little influenced.

acquired geometric phase reads

$$\gamma_{01}^d = \frac{\pi}{2}\left(1 - \text{Re}\frac{(\Delta_1 - i\Gamma/2)^2 - 4\lambda_1^2}{(\Delta_1 - i\Gamma/2)^2 + 4\lambda_1^2}\right). \tag{18}$$

Since Γ/R should be a perturbation quantity, we can expand this geometric to the second order in Γ/R, $R = \sqrt{\Delta_{01}^2 + 4\lambda_1^2}$, and then obtain

$$\gamma_{01}^d \approx \gamma_{01} + \frac{\pi}{4}\frac{\cos^2\theta_{01}}{8\sin^2\theta_{01} + 16\sin^4\theta_{01} + \cos^4\theta_{01}}\left(\frac{\Gamma}{R}\right)^2. \tag{19}$$

Consequently, the probability of the atom being detected in the state $|2\rangle$ is changed to be $\tilde{P}_2^d = [1 - \cos(\gamma_{01}^d)]/2$. Notice that in the case of low decoherence, the lowest order correction of the expected geometric phase is only quadratic in Γ/R, suggesting that the field decoherence may not play such an important role in the proposed experiment. This can be numerically verified from the comparison in the Fig. 4: after considering the presence of a typical cavity dissipation $\Gamma = 1$ kHz [17], the probability of the atom being detected in the state $|2\rangle$ (the black line) is almost unchanged. This means that the experimental detection of the vacuum-induced Berry phase in JCM with the Ramsey interference is feasible, even in presence of the cavity losses.

In summary, we have calculated the Berry phase of the m-quantum JCM and proposed an experimental setup to observe and measure such a geometric phase induced by the vacuum field in a one-photon single-mode JCM. Basically, geometric phases acquired by the atom-field system are dependent on the number of photons in the field. This is different from those attained in the semiclassical counterpart. Our results show also that, for a common $\Delta_m/\lambda_m \equiv \Delta/\lambda$, the more quantum m corresponds to the larger vacuum-induced Berry phase. A Ramsey interference experiment with cavity QED is designed to detect the vacuum-induced Berry phase.

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