Making beam splitters with dark soliton collisions

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We show with numerical simulations that for certain simple choices of parameters, the waveguides induced by colliding dark solitons in a Kerr medium yield a complete family of beam splitters for trapped linear waves, ranging from total transmission to total deflection. The way energy is transferred from one waveguide to another is similar to that of a directional coupler, but no special fabrication is required. Dark soliton beam splitters offer potential advantages over their bright soliton counterparts: Their transfer characteristics do not depend on the relative phase or speed of the colliding solitons; dark solitons are generally more robust than bright solitons; and the probe peaks at nulls of the pump, enhancing the signal-to-noise ratio for probe detection. The last factor is especially important for possible application to quantum information processing.

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I. INTRODUCTION

Solitons, both bright and dark, and both temporal and spatial, imprint waveguides through their induced refractive-index profile. These induced waveguides can be used to guide linear waves, and this has been demonstrated theoretically and experimentally. The literature at this point is extensive (for bright solitons, see, e.g., [1–9]; for dark solitons, see the review by Kivshar [10] and [11,12]).

An important application of the idea of guiding linear waves with solitons is controlling light with light. The switching of light waves using induced directional couplers has been demonstrated experimentally for both bright [8] and dark [13] solitons. The directional couplers used in these experiments consist of a length of parallel waveguides; the length and separation of the waveguides determine the extent of the energy transfer. Dark solitons can also undergo Y splitting, which produces a bifurcation of the induced waveguide, and beam splitting has similarly been demonstrated with these Y junctions [11,13].

An attractive alternative to a strip of parallel waveguides for directional coupling is the X junction formed by a soliton collision. Akhmediev and Ankiewicz [14] analyze the use of two colliding Kerr solitons, both bright and dark, for constructing induced X junctions and couplers, and Miller and Akhmediev [15] give a comprehensive analysis of the effect of an N-way bright soliton collision on the scattering of guided waves, also in a Kerr medium. The corresponding analysis for dark solitons is provided by Miller [16]. While useful energy transfer results for bright solitons, the conclusion of [14,16] is that the induced X junctions for dark solitons have zero cross talk, the equivalent of a wire crossing, and no coupling of energy can be exploited. This work assumes, however, that the propagation of both the solitons and the guided waves is governed by the same group velocity dispersion (GVD) and nonlinear parameters.

This assumption—that the soliton pump and the signal probe propagation equations have the same GVD and nonlinear coupling parameters—does not ordinarily hold in practice, especially when we aim at detecting the probe in the presence of the much more intense pump and therefore use different wavelengths and polarizations for the pump and probe. Consider, for example, solitons in a birefringent fiber. The solitons see the self-phase modulation index, while the probe sees the cross-phase modulation index, and these depend on the polarization choices of the implementation. Furthermore, both the GVD and coupling constants are in general functions of wavelength [17].

Recently, Rand and the author discussed the use of bright soliton collisions to build beam splitters, including mode-separating beam splitters [18], and phase shifters [19]. This suggests the possibility of using dark solitons in the same way. Dark solitons have simpler collision dynamics than their bright counterparts [10], are generally more stable in the presence of perturbations [11], and may offer some important advantages in this application. This hope might seem dashed by the result mentioned previously [14,16]: Linear waves guided by colliding dark solitons are not scattered at junctions in the induced waveguides, assuming that the propagation of pump and probe are governed by the same GVD and nonlinear parameters. In this article, we relax this unnecessarily restrictive assumption and show through simulations that certain simple choices of GVD and nonlinear coupling constant in the probe equation yield a complete, one-parameter family of beam splitters.

II. THE MODEL AND BOUND STATE SOLUTIONS

We adopt the normalized self-defocusing cubic nonlinear Schrödinger equation for the normal dispersion region to model the pump,

$$i\frac{\partial P}{\partial x}+\frac{\partial^2 P}{\partial t^2}-2|P|^2P=0, \tag{1}$$

and use Blow and Doran's analytical solution to this equation for two dark solitons [20] in what follows. Here, as usual, t is time relative to the frame moving with the soliton, and x is distance in the direction of propagation. The propagation of the probe signal is described by the linear equation

$$i\frac{\partial u}{\partial x}+\kappa_1\frac{\partial^2 u}{\partial t^2}-2\kappa_2|P|^2u=0, \tag{2}$$

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which is exactly the linear Schrödinger wave equation, with potential $|P|^2$ determined by the pump. The intensity of the probe is assumed to be very much less than that of the pump so that the probe signal has no effect on the pump. We also assume that the interaction interval is short enough so that walk-off between the pump and probe can be neglected. The parameters κ_1 and κ_2 reflect the facts that in general the pump and probe can differ in wavelength and that the nonlinear term in the pump equation is due to self-phase modulation while the corresponding term in the probe equation is due to cross-phase modulation. A similar two-parameter probe equation is derived in some detail by de la Fuente and Barthelemy [3].

We consider first the special case when the wavelengths and relative polarizations of the pump and probe can be adjusted so that the two free parameters in the probe equation are equal: $\kappa_1 = \kappa_2 = \kappa$. When the pump signal is just a single dark (actually, black) soliton, $|P|^2 = \tanh^2(t)$. To reduce the corresponding probe equation to a familiar eigenvalue problem, let

$$u(x,t) = w(t)e^{-i\kappa(E+2)x}, \tag{3}$$

which results in

$$w'' + [E + 2\text{sech}^2(t)]w = 0. \tag{4}$$

This is a form of the associated Legendre equation [21] with degree ℓ and order m, where $\ell(\ell+1) = 2$ and $m^2 = -E$. Taking $\ell = 1$ and $m^2 = -E = 1$ then shows that $w = \text{sech}(t)$ is a fundamental trapped mode of the waveguide induced by the dark soliton. Finally, the probe signal in this case is

$$u(x,t) = \text{sech}(t)e^{-i\kappa x}. \tag{5}$$

We conclude that the guided wave when $\kappa \neq 1$ differs from the special case when $\kappa = 1$ only by a phase factor $e^{-i(\kappa-1)x}$. This corresponds to what can be called an additional dynamical phase if the probe equation is interpreted as a Schrödinger wave equation. We will next see that this difference completely changes the probe's scattering behavior at the junctions produced by dark soliton collisions.

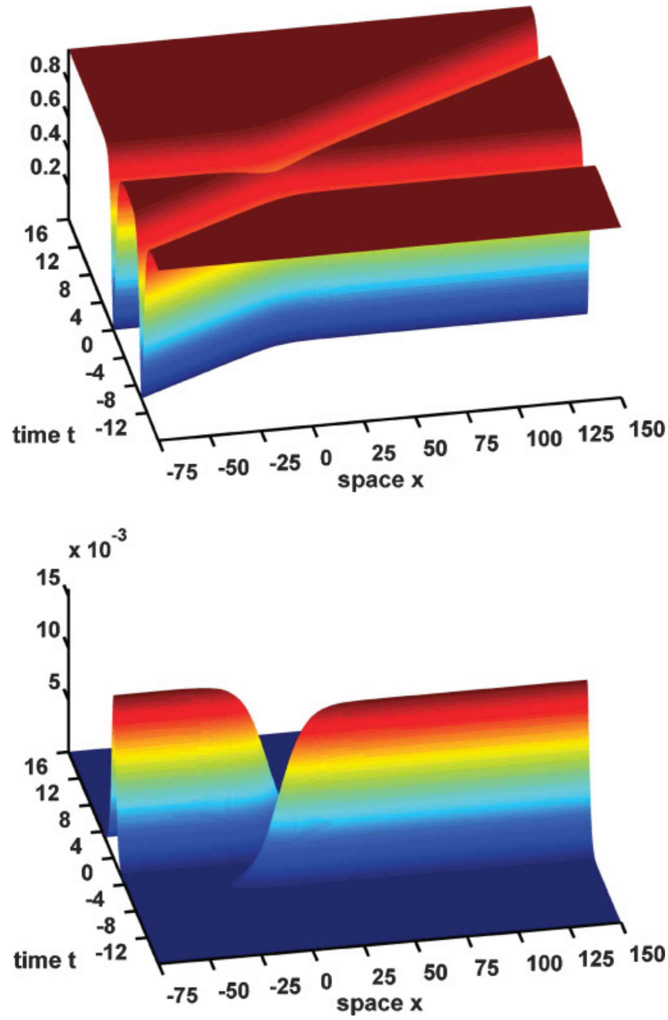

FIG. 1. (Color online) (top) The pump signal, a dark soliton collision, using the analytical solution of Eq. (1) from Blow and Doran [20]. The velocity parameters are $\lambda_1 = 0.0$ and $\lambda_2 = 0.04$. (bottom) The corresponding probe signal for the case when the pump and probe equations have the same constants ($\kappa = 1$), illustrating zero cross talk [14,16].

III. SCATTERING AT JUNCTIONS

The behavior of the guided wave at the collision-induced junction depends on κ in a very regular way. First, Fig. 1 shows the pump and probe for the case when $\kappa = 1$. The probe emerges unchanged (except for a displacement) after passing the junction, as predicted in [14,16]. Figure 2 then shows the intensity of the propagating probe signal in the same induced waveguide for the more general cases $\kappa = 1.5, 2$, and 2.5. As κ increases from 1, the effect of scattering at the junction changes from transparent ($\kappa = 1$) to a 50:50 beam splitter ($\kappa = 1.5$) to total deflection ($\kappa = 2$) and back to a 50:50 beam splitter ($\kappa = 2.5$). This behavior continues periodically as κ continues to increase. For example, Fig. 3 shows the intensity of the probe (as seen from above) for the case $\kappa = 12$, which corresponds to a total transfer of energy to the deflected beam. The captured wave bounces back and forth between the two arms of the induced waveguide $\kappa/2$ times, as in a directional coupler [22].

Thus the net effect of the induced waveguide is the same as two linear polarizers with their axes rotated at relative angle $(\kappa - 1)\pi/2$. Figure 4 shows the transmitted and deflected energy as a function of κ for $-1 \leqslant \kappa \leqslant 6$, measured from numerical simulations. The behavior is remarkably close to the square of a sinusoid, and in fact, in the range $1 \leqslant \kappa \leqslant 2$, the measured energy ratios differ from $\sin^2(\kappa\pi/2)$ and $\cos^2(\kappa\pi/2)$ by less than 1%. Note, however, that the point $\kappa = 0$ is singular because there the probe propagates independently of the pump; points for very small $|\kappa|$ are omitted from the plot.

The way the soliton-guided beam splitter works is similar to the way a directional coupler works, but they differ in important ways. In the usual kind of directional coupler, two identical waveguides are brought together to run in parallel, and the amount of energy transfer is determined by the length of the run and their distance apart. In the same way, the two waveguide arms induced by a dark soliton collision, which is always repulsive, approach each other, and energy in one of the arms can then couple to the second. By analogy, one might suppose that the relative speed of the colliding solitons, which determines the interaction time between the trapped waves,

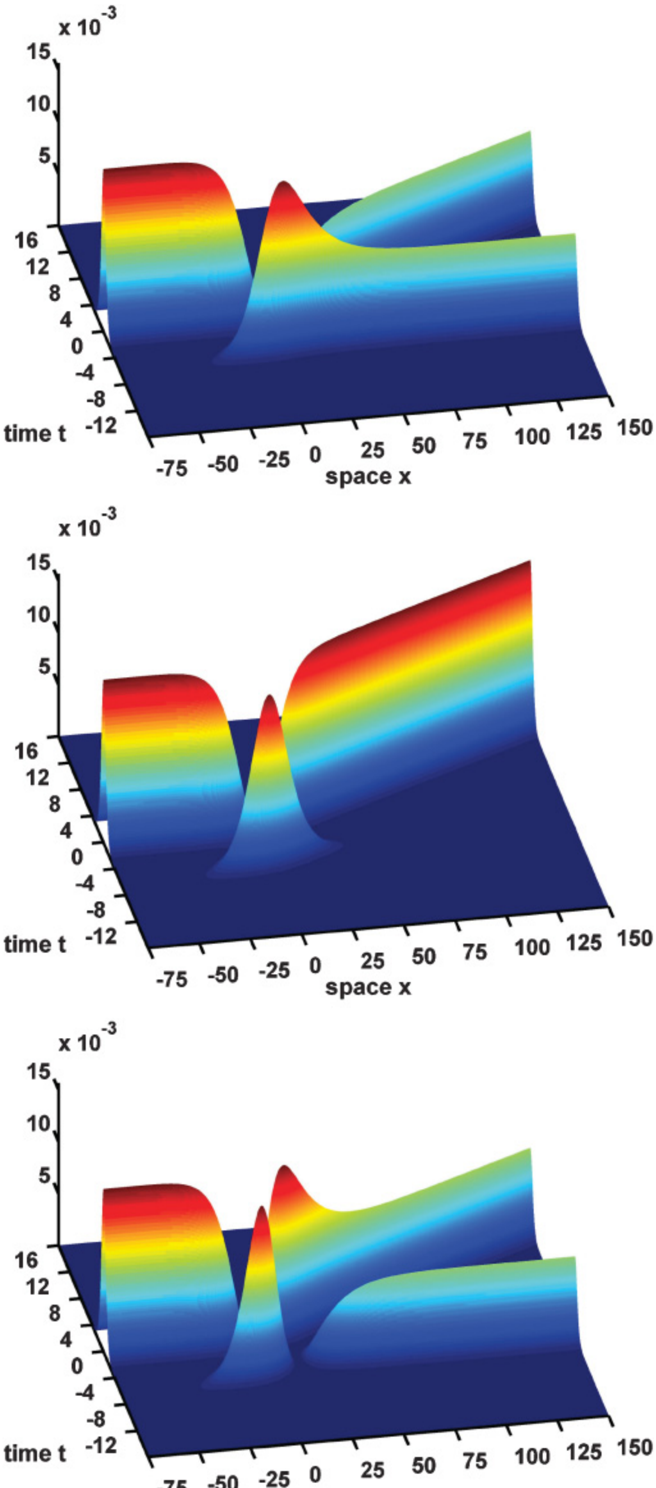

FIG. 2. (Color online) (top to bottom) The probe signal in the same induced waveguide for the cases $\kappa = 1.5, 2$, and 2.5.

also determines the amount of energy transfer. Surprisingly, this is not the case—it is only the parameter κ that determines the nature of the induced coupler. For example, Fig. 5 shows the probe wave for the case $\kappa = 2$ (total transfer) and relative speeds 0.015 and 0.12; the behavior is qualitatively the same as for relative speed 0.04, which is illustrated in Fig. 2.

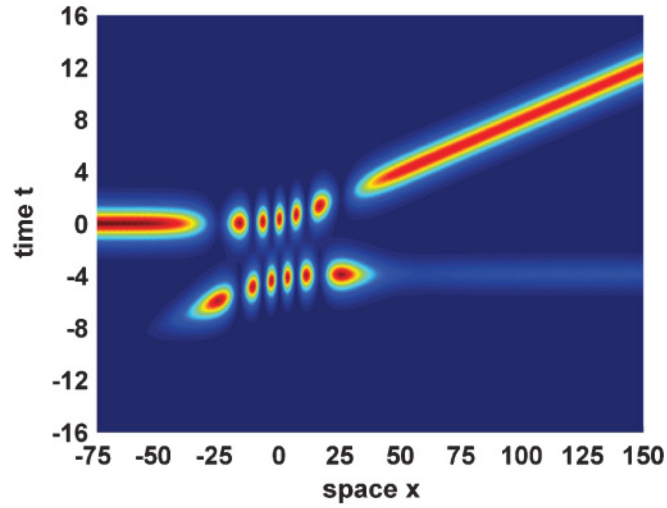

FIG. 3. (Color online) The intensity of the probe signal (as seen from above) for $\kappa = 12$.

A hint of this phenomenon is provided by the theoretical result mentioned earlier [14,16]: When $\kappa = 1$, captured waves do not couple at all, regardless of the relative collision velocity.

IV. HIGHER ORDER MODES AND MODE SEPARATION

Relaxing the requirement that $\kappa_1 = \kappa_2$ in Eq. (2) allows higher order modal solutions, as in the case of bright solitons [18]. If we then write the probe equation as the associated Legendre equation in this more general case, we get

$$w'' + (\kappa_2/\kappa_1)[E + 2\text{sech}^2(t)]w = 0. \quad (6)$$

The eigenfunctions, which are associated Legendre functions of degree ℓ and order m, now correspond to the choices $\ell(\ell+1) = 2\kappa_2/\kappa_1$ and $m^2 = -(\kappa_2/\kappa_1)E$, and

$$u(x,t) = w(t)e^{-i\kappa_2(E+2)x}. \quad (7)$$

From this we see that the first-degree case ($\ell = 1$) requires that $\kappa_2 = \kappa_1$, and the second-degree case ($\ell = 2$) requires that

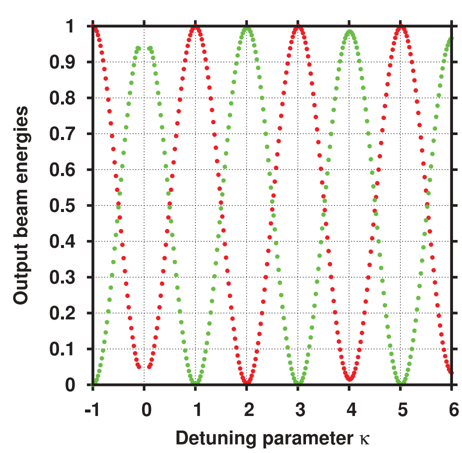

FIG. 4. (Color online) The relative energy transmission of the dark-soliton beam splitter as a function of κ.

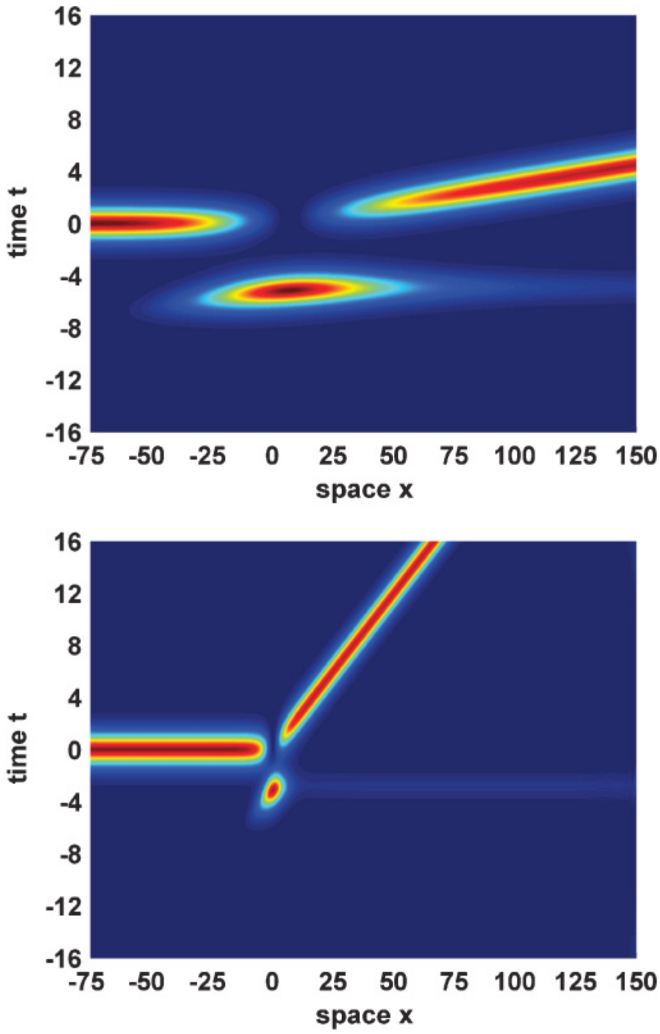

FIG. 5. (Color online) Full transfer of a captured wave, corresponding to the case $\kappa = 2$, for waveguides resulting from soliton collisions at different speeds. (top) Relative speed ($\lambda_2 - \lambda_1$ in [20]) is 0.015, instead of 0.04; (bottom) relative speed is 0.12.

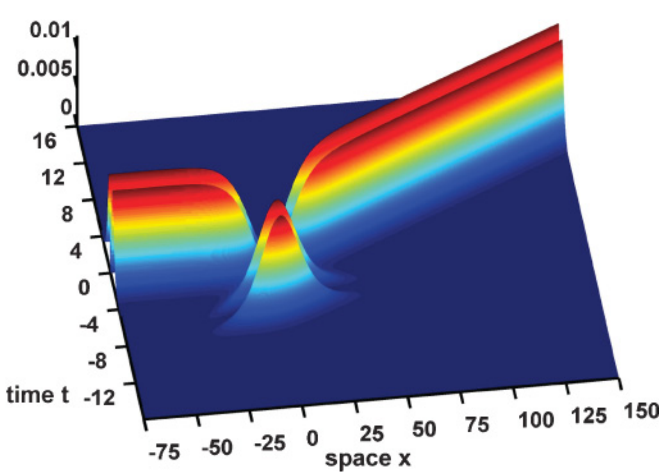

FIG. 6. (Color online) The effect of a junction on the second-degree modal function u_{21}. In this example, $\kappa_1 = 2/3$, $\kappa_2 = 2$, and the modal function u_{21} is totally deflected.

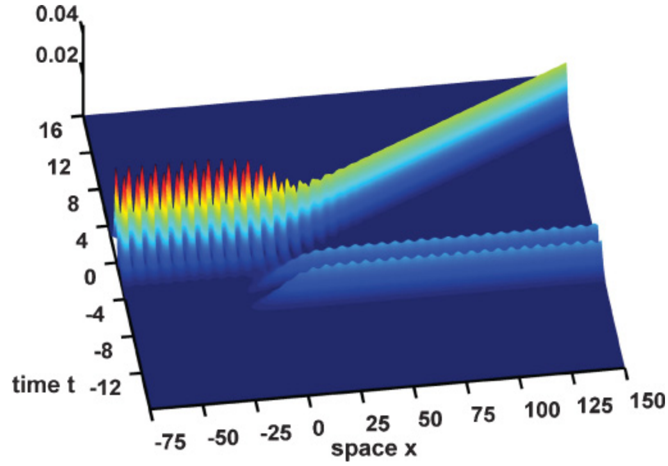

FIG. 7. (Color online) The effect of a junction on an equal superposition of the second-degree modal functions u_{21} and u_{22}. In this example, $\kappa_1 = 1/3$, $\kappa_2 = 1$, and the modes are separated, with only u_{22} being deflected.

$\kappa_2 = 3\kappa_1$. These values for κ_2 will be used in the following examples. As in [18], we denote the orthogonal modal functions by $u_{\ell m}$. The behavior of mode u_{21} at a waveguide junction exhibits the same behavior as does u_{11}, but with one-third the period. For example, $\kappa_1 = 1/3$ results in transparency, $\kappa_1 = 2/3$ in total transfer, $\kappa_1 = 1$ in transparency, and so on. Figure 6 shows an example for the case when $\kappa_1 = 2/3$: The probe wave is transferred totally from the soliton with speed 0 to the soliton with speed 0.04.

In contrast, the modal function u_{22} is transferred almost perfectly for a very broad range of κ_1 around 1. Thus, when $\kappa_1 = 1/3$, u_{21} is not affected by the junction, while u_{22} is totally deflected. This means that if the probe signal is originally a linear superposition of the two, the junction, acting as a beam splitter, will separate the two modes, as in the bright soliton case [18]. Figure 7 illustrates this. The strong beat in the linear superposition is caused by the fact that the modal functions u_{21} and u_{22} propagate with dynamical phases determined by two different eigenvalues, as observed in the corresponding bright case [18,23].

V. DISCUSSION AND IMPLICATIONS

Dark soliton collisions offer some real advantages over bright soliton collisions and directional couplers in controlling light waves: First, dark solitons are known to be more stable in the presence of noise and are generally more robust than bright solitons [11,17]. Second, the probe, which is of much lower intensity, peaks at the dip in the intensity of its host soliton, thus increasing the signal-to-noise ratio and making it easier, in principle, to detect. Third, the characteristics of the dark soliton beam splitter do not depend on the relative phase or relative speed of the colliding solitons, whereas bright solitons need to have their phases and speeds carefully controlled to produce a given result [18,19]. Finally, no special fabrication is required, as might be the case for directional couplers.

One might ask how dependent the behavior of this scheme is on the accuracy of the coefficients κ_1 and κ_2 in Eq. (2), which, in a practical implementation, would depend on proper tuning of the wavelength and polarization. Some simple numerical experiments were conducted in the $\kappa_1 = \kappa_2$ case by perturbing κ_2 and keeping κ_1 fixed at 1, a transparent case. The results show that the effect of this kind of perturbation is to deflect some energy to the alternate channel rather than causing radiation or some other, irregular loss of energy. Also, the magnitude of the diversion is moderate. For example, if κ_2 is perturbed by $+5\%$, the transmitted energy decreases from essentially 100% to 98.6%, and the deflected energy, captured by the second arm of the induced waveguide, increases from essentially zero to 1.4%. These experiments suggest that a practical implementation would not be overly sensitive to the realization of physical parameters.

While everything we have said so far is classical, in the limit of low linear wave energy, the probe becomes quantized and can be thought of as a photon wave packet—as pointed out in [18]. There is now experimental verification that coherent photons can be transmitted and detected in the presence of normal, classical traffic in fibers using wavelength multiplexing (e.g., see [24]). Furthermore, pulse shaping of individual photons in single-mode fibers has recently been reported [25,26]. The trapping of the probe in the traveling potential of a dark (or bright) soliton is a kind of wave shaping, and this possibility suggests a variety of experiments and possible application to quantum information processing. Dark solitons seem especially attractive for this application because the photon detection takes place when the pump is near zero in intensity.

VI. CONCLUSIONS

We have shown that the waveguides induced by dark solitons can be used to control weak probes in the same way that bright solitons can, in contrast with the zero–cross talk case reported in [16], provided that the GVD and nonlinear coupling parameter for the probe are chosen appropriately. These degrees of freedom are readily available if we use different wavelengths and polarizations for the pump and probe. For the probes corresponding to the degree 1–associated Legendre modal functions, the dark soliton junctions behave in a way that is very closely analogous to a beam splitter made of crossed polarizers, with a single parameter playing the role of angle between polarizing filters. For the probes corresponding to the degree 2–associated Legendre functions, the junction can act as a mode-separating beam splitter.

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[1] J. T. Manassah, Opt. Lett. **14**, 396 (1989).
[2] J. T. Manassah, Opt. Lett. **16**, 587 (1991).
[3] R. de la Fuente and A. Barthelemy, IEEE J. Quantum Electron. **28**, 547 (1992).
[4] A. W. Snyder and A. P. Sheppard, Opt. Lett. **18**, 482 (1993).
[5] M. Morin, G. Duree, G. Salamo, and M. Segev, Opt. Lett. **20**, 2066 (1995).
[6] M.-F. Shih, M. Segev, and G. J. Salamo, Opt. Lett. **21**, 931 (1996).
[7] M.-F. Shih, Z. Chen, M. Mitchell, M. Segev, H. Lee, R. S. Feigelson, and J. P. Wilde, J. Opt. Soc. Am. B **14**, 3091 (1997).
[8] S. Lan, E. DelRe, Z. Chen, M.-F. Shih, and M. Segev, Opt. Lett. **24**, 475 (1999).
[9] E. A. Ostrovskaya, Y. S. Kivshar, D. Mihalache, and L.-C. Crasovan, IEEE J. Sel. Top. Quantum Electron. **8**, 591 (2002).
[10] Y. S. Kivshar, IEEE J. Quantum Electron. **29**, 250 (1993).
[11] B. Luther-Davies and Y. Xiaoping, Opt. Lett. **17**, 496 (1992).
[12] G. Zhang, S. Liu, J. Xu, G. Zhang, and Q. Sun, Chin. Phys. Lett. **13**, 101 (1996).
[13] Z. Chen, M. Segev, D. N. Christodoulides, and R. S. Feigelson, Opt. Lett. **24**, 1160 (1999).
[14] N. Akhmediev and A. Ankiewicz, Opt. Commun. **100**, 186 (1993).
[15] P. D. Miller and N. N. Akhmediev, Phys. Rev. E **53**, 4098 (1996).
[16] P. D. Miller, Phys. Rev. E **53**, 4137 (1996).
[17] G. P. Agrawal, *Nonlinear Fiber Optics*, 4th ed. (Academic, London, 2006).
[18] K. Steiglitz and D. Rand, Phys. Rev. A **79**, 021802(R) (2009).
[19] K. Steiglitz, Phys. Rev. A **81**, 033835 (2010).
[20] K. J. Blow and N. J. Doran, Phys. Lett. A **107**, 55 (1985).
[21] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-relativistic Theory*, 3rd ed. (Pergamon, Oxford, 1977).
[22] A. Yariv and P. Yeh, *Optical Waves in Crystals* (Wiley, New York, 1983).
[23] J. T. Manassah, Opt. Lett. **15**, 670 (1990).
[24] N. I. Nweke *et al.*, Appl. Phys. Lett. **87**, 174103 (2005).
[25] P. Kolchin, C. Belthangady, S. Du, G. Y. Yin, and S. E. Harris, Phys. Rev. Lett. **101**, 103601 (2008).
[26] H. Specht, J. Bochmann, M. Mücke, B. Weber, E. Figueroa, D. Moehring, and G. Rempe, Nat. Photon. **3**, 469 (2009).