Taking apart the enhanced backscattering cone: Interference fringes from reciprocal paths in multiple light scattering

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We report the decomposition of the enhanced backscattering cone into its constitutive interference fringes. These fringes are due to the constructive interference between reciprocal paths of any multiply scattered wave after ensemble averaging. An optical setup combining a two-point continuous-wave illumination and matching detection allows the observation of the fringes and, therefore, the quantitative characterization of the Green's function for light propagation between the two points in a multiple-scattering media.

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I. INTRODUCTION

The study of multiple wave scattering has given rise to many new physical insights in the last two decades. Without interference, or after ensemble averaging over the configuration of the scatterers, multiple scattering can be well described as a diffusion process [1]. In the diffusion regime, the wave is transported across the diffusive material as a random walk, with the average step being the transport mean free path ℓ. Perhaps the most well-known manifestation of interference in a multiply scattering medium is the enhanced backscattering (EBS) cone [2], which has been observed in optic [2] as well as acoustic [3] and seismic [4] waves. The constructive interference between reciprocal paths leads to a scattered intensity in the exact backscattering direction that is twice as high as that which would be expected from treating the wave propagation as an incoherent diffusive process. This interference is theoretically explained as being the sum of unseen interference fringes [5]. Until now, all experimental evidence for interference effects in the propagation of waves in multiple-scattering media has involved a spatial average over paths that connect points on an extensive area of the sample. In the present article, we describe an experimental optical technique based on a two-point illumination and matched two-point observation scheme. This setup allows us to see, using a steady state technique, the interference fringes of waves traveling between two specific well-localized points on the surface of a random scattering sample, effectively taking apart the EBS cone. The ability to vary the optical phase between the two illumination beams allows us to unambiguously identify the fringes as originating from reciprocal paths. The dependence of the fringe visibility on the distance between the two illumination points is in quantitative agreement with the sustaining theory. This scheme permits us to characterize the Green's function for light propagation between the two localized points.

II. THEORY

The standard explanation for the EBS effect focuses on the fact that, for every arbitrary scattering path between two given points on the surface of a random scattering media illuminated by a plane wave, the reciprocal path, traveled in the reverse direction, is also possible. Provided all the scattering events are elastic, the optical phase acquired by light traveling in either direction is identical. This leads to interference fringes in the far field with a period that is inversely proportional to the distance between the two points. Summing up the result for all possible pairs of illuminated points on the surface, the interference fringes with different periods average out to the diffuse background, except along the exact backscattering direction where all the fringe patterns add up in phase. The net result is an enhanced backscattering cone with a peak twice as high as the diffuse background and a width that is inversely proportional to the transport mean free path in the medium.

To explore this interference effect in more detail, consider two coherent point sources for diffusive light, **A** and **B**, separated by a vector **d**, at the surface of a material with transport mean free path ℓ where reciprocity holds [6]. A variable optical phase delay between the two sources introduces a phase factor $\exp(-i\varphi)$ on the source at point **B**. Let the field scattered at **A** created by light incident on the sample at **B**, which has traveled along a specific path m, be designated by $E_m(\mathbf{A},\mathbf{B})$. This field is scattered one last time before exiting the material, through the scatterer's T matrix **T**. The simplest case is that of point scatterers, which give rise to a T matrix proportional to the identity matrix [7]. The field propagation from this last scattering event to a point **R** in the far field is described by the free-space Green's function $g(\mathbf{r}) = -\exp(ikr)/(4\pi r)$. To describe the average intensity in the far field of the light backscattered only from points **A** and **B**, it is necessary to take the ensemble average of the square norm of the sum of fields for all possible paths entering at **A** or **B** and exiting at **A** or **B**:

$$I = \left\langle \left\| T \sum_m g(\mathbf{R}-\mathbf{A})[E_m(\mathbf{A},\mathbf{A}) + e^{-i\varphi} E_m(\mathbf{A},\mathbf{B})] + g(\mathbf{R}-\mathbf{B})[E_m(\mathbf{B},\mathbf{A}) + e^{-i\varphi} E_m(\mathbf{B},\mathbf{B})] \right\|^2 \right\rangle_{\text{ens}}. \quad (1)$$

In calculating the norm squared, several types of terms are present. The ensemble average over any pairs of different paths is, by the independent scattering approximation, equal to zero. The only nonzero terms are those which involve propagation along the same physical path, either in the same

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direction such as $E_m(\mathbf{A},\mathbf{B})E_m^*(\mathbf{A},\mathbf{B})$ or in reciprocal directions such as $E_m(\mathbf{A},\mathbf{B})E_m^*(\mathbf{B},\mathbf{A})$. In the far field (i.e., $R \gg d$), the two Green's functions $g(\mathbf{R}-\mathbf{A})$ and $g(\mathbf{R}-\mathbf{B})$ can be approximated as having the same amplitude, differing only by a relative phase factor $\exp(i\mathbf{k}\cdot\mathbf{d})$. In a reciprocity-conserving material, the field is transported equally along one path and its reciprocal, so that $E_m(\mathbf{A},\mathbf{B})E_m^*(\mathbf{B},\mathbf{A}) = ||E_m(\mathbf{A},\mathbf{B})||^2 = ||E_m(\mathbf{B},\mathbf{A})||^2$. Recognizing that the ensemble average of the intensity of all the light that travels from **A** to **B**, regardless of the path taken, is just the diffusion intensity Green's function $G(\mathbf{B},\mathbf{A})$ multiplied by the source intensity (set as S_0, equal at **A** and **B**), one can write

$$\begin{aligned} I &= S_0\left(\frac{T}{4\pi R}\right)^2\{G(\mathbf{A},\mathbf{A})+G(\mathbf{B},\mathbf{B})+G(\mathbf{B},\mathbf{A})\\ &\quad+G(\mathbf{A},\mathbf{B})+[e^{i(\mathbf{k}\cdot\mathbf{d}-\varphi)}+e^{-i(\mathbf{k}\cdot\mathbf{d}-\varphi)}]G(\mathbf{A},\mathbf{B})\}\\ &= 2S_0\left(\frac{T}{4\pi R}\right)^2\{G(0)+[1+\cos(\mathbf{k}\cdot\mathbf{d}-\varphi)]G(d)\}. \end{aligned} \tag{2}$$

The backscattered intensity displays a constant background and a set of interference fringes, with period inversely proportional to the distance d between the two points **A** and **B**, and a magnitude proportional to the diffusion intensity Green's function $G(d)$. Note that the same setup, with two pointlike sources, where the diffusive sample is replaced with a mirror, also displays fringes:

$$\begin{aligned} I_m &= S_0||g(\mathbf{R}-\mathbf{A})+g(\mathbf{R}-\mathbf{B})e^{-i\varphi}||^2\\ &= [2S_0/(4\pi R)^2]\cos(\mathbf{k}\cdot\mathbf{d}+\varphi). \end{aligned} \tag{3}$$

The two sets of fringes, from the mirror [Eq. (3)] and diffusive sample [Eq. (2)], have the same period, but they shift in opposite directions when the relative incident optical delay φ is changed. This phase behavior provides a clear means of distinguishing between "mirror-like" fringes and fringes originating from constructive interference between light traveling along reciprocal paths within the diffusive medium. This distinctive behavior can be intuitively understood by considering the following argument: When a plane wave impinges on a diffusive sample at an angle θ from normal incidence, the specular reflection is at $-\theta$, whereas the EBS direction is at θ. When shifting the incident plane wave away from normal incidence, for example, the specular reflection and the center of the EBS cone shift in opposite directions.

The infinite-medium diffusion Green function [1] is $G_\infty(r) = 1/(4\pi r)$, where r is the distance between input and output points. To theoretically treat the case of a semi-infinite diffusive medium [1], the infinite-medium Green function is mirrored in order to prevent diffusive paths from crossing the interface, such that $G(\mathbf{r}',\mathbf{r}) = G_\infty(\mathbf{r}',\mathbf{r}) - G_\infty(\mathbf{r}',\mathbf{r}^*)$. Here, $\mathbf{r}^*$ is the mirror image of $\mathbf{r}$ through a plane at a distance $z_e = \tau_e\ell$ away from the interface of the diffusive material. The so-called extrapolation length z_e and extrapolation ratio τ_e can be specified quantitatively at various levels of approximations [8]. For a reflectionless interface, the simple value $\tau_e = 2/3$ is appropriate. Both the incoming and outgoing diffuse light scatters, on average, at a distance of one mean free path inside the material; thus both the source and exit point of interest of the diffusion Green's function are at a depth $z = \ell$. The Green's function for a semi-infinite diffusive material for two points "on the surface" can finally be written as

$$G(\mathbf{r}) = (4\pi r)^{-1} - [4\pi\sqrt{r^2+4\ell^2(1+\tau_e)^2}]^{-1}. \tag{4}$$

According to Eq. (2), this diffusion Green's function characterizes the amplitude of the interference fringes. The fringe amplitude decreases rapidly with the distance between the two points, over a typical length scale equal to the transport mean free path ℓ.

In this theoretical model, both the input and output for the diffuse light were assumed to be pointlike, and the constant background backscattered by **A** and **B**, proportional to $G(0)$, is infinite. Of course, for an actual experimental setup, it is necessary to convolute this expression over the spatial profile of the incident beams and the observation region, removing the singularity. Furthermore, there are several other experimental artifacts which reduce the observed visibility. For the actual experimental setup, shown in Fig. 1, the incident beams have a roughly Gaussian spatial profile, while the observation region has a square format. There exist light paths that begin outside the observation area, but end up inside. The reciprocal of these paths is not observed. Also, due to the Gaussian spatial profile, the incident intensity is not constant as assumed above. The scattered intensity from paths starting closer to the beam center and ending up farther away will be higher than light traveling in the reciprocal direction [9], also reducing the visibility. Both of these effects effectively increase the weight of the constant background. One can take them into account by replacing $G(0)$ in Eq. (2) by a constant C, independent of the distance d. The magnitude of the fringes is still approximated, in a realistic setup, by $[1+\cos(i\mathbf{k}\cdot\mathbf{d})]G(d)$, provided that d is greater than or equal to the typical size of the matched source and observation. In the optical setup described here, the equal size of the source and observation is the resolution, and the visibility is measured for distances larger than the resolution. The fringe visibility, following these approximations of C and $G(d)$, is

$$\text{Visibility} \equiv \frac{\max - \min}{\max + \min} = \frac{G(d)}{C+G(d)}. \tag{5}$$

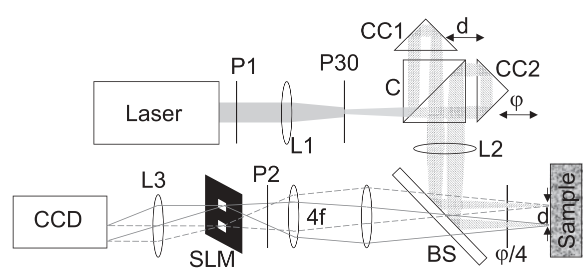

FIG. 1. Schematic of the double-beam EBS setup to measure interference fringes from reciprocal paths in diffusive media. Two coherent 30-μm spots with variable distance d and phase delay φ are impinging on the surface of the sample. The spatial light modulator (SLM) only transmits light that is reflected from the sample from the same position as the illumination spots. The angular distribution of the backscattered light, filtered by the SLM, is measured through a lens L3 on a CCD camera. Such a scheme of double-spot illumination and matching observation is necessary to measure the EBS fringes.

III. DOUBLE-BEAM ENHANCED BACKSCATTERING SETUP

In Fig. 1, we show the experimental setup used to observe the individual interference fringes from reciprocal paths in a diffusive material and to quantify their dependence on the transport mean free path of the material. The interference fringes are present after ensemble averaging, accomplished by rotating the sample about an axis close, but not parallel, to the incident beam direction. The output of a single-mode continuous-wave laser (Coherent Verdi, 532-nm wavelength) is focused through lens L1 on a 30-μm pinhole P30. This light is then split by a beam splitter cube C, reflected by corner cubes CC1 and CC2, and subsequently imaged (with unity magnification) by lens L2 onto the sample after being partially reflected by the beam splitter BS. Two incident spots are produced on the sample, with an approximately Gaussian spatial profile of 30-μm full width at half maximum. Moving CC1 along an axis perpendicular to the incoming beam direction shifts the first spot on the sample by a distance d compared to the second one. Moving CC2 along an axis parallel to the incoming beam direction changes the phase delay φ of the second spot on the sample compared to the first one. The fringe visibility, like the enhancement factor of the EBS cone [9], is maximum when illumination and observation areas are matched. Therefore, only light backscattered from these two illumination spots should be observed. A good approximation is made by spatial filtering: The surface of the sample is imaged, through a $4f$ pair of lenses, onto a spatial light modulator (SLM), which can be programed to have two pixels open and the rest black. The two open pixels of the SLM are the images through the $4f$ lenses of the illumination spots on the sample. The SLM pixels are 32.2 μm wide, which sets the width of the illumination spots for best contrast. The angular distribution of the light filtered by the SLM is focused through lens L3 onto the surface of a charge-coupled device (CCD) camera. The polarizer P1 and analyzer P2 along with the quarter-wave plate $\lambda/4$ between BS and the sample are adjusted to select the circular preserving polarization channel. In this polarization channel, the maximum enhancement of the EBS cone is observed [9] and, likewise, the largest visibility of interference fringes. Great care must be taken to filter out the specular reflection and single scattering, as the diffuse light transmitted through the two small apertures of the SLM is weak. Note that the double aperture made by the SLM is in itself able to produce a Young's fringe pattern, provided it is illuminated by coherent light. In this setup, however, due to the $4f$ imaging system, the SLM by itself is not responsible for any additional fringes beyond those that originate from light backscattered by a diffusive sample illuminated with two spots. A simple test is to block either one of the illumination spots, which makes the fringes disappear. The function of the SLM is to filter out most of the constant background which originates from nonreciprocal paths, namely those which exit outside the illumination spots. Essentially the same interference fringes would be observed with or without the spatial filtering, but with an extremely small visibility in the latter case.

IV. EXPERIMENTAL RESULTS

The double-beam EBS setup, as described in Fig. 1, is used to observe interference fringes from a series of diffusive samples with different transport mean free paths, in the range 12–215 μm. These diffusive samples are of two origins: commercially available white products (80 g/m^2 paper and Teflon) and homemade suspensions of polystyrene spheres within a sol-gel matrix [10]. Figure 2 visually presents the fringes, measured from both a mirror and a diffusive sample, as a function of the incident optical delay φ. Given the experimental limitations discussed above, the visibility of the observed EBS fringes is at best 10%. To carry out an analysis, each fringe profile is first normalized to its average in order to reduce the effect of the incident power fluctuations. Then the difference between each profile at φ and the corresponding one at $\varphi + \pi$ is taken in order to remove the constant background and increase the visibility. After this normalization and π differentiation, the fringe pattern of the diffusive sample is well recovered, as shown for a typical case in Fig. 2(b). The variation of the fringe shift with the optical delay confirms that the observed fringes originate from reciprocal paths; that is, they are the constituent fringes of the EBS cone.

For a more quantitative approach, the treated fringe profiles are Fourier transformed. The amplitude and phase of the Fourier component at the fringe frequency describe the fringe visibility and its relative position, respectively. In the two insets of Fig. 2, the phase of the fringe Fourier component is plotted vs. the optical delay for each case: mirror and diffusive sample. The observed phase behavior agrees well with that predicted by Eqs. (2) and (3). The reflective and EBS fringes shift with the optical delay φ, with a respective slope of -1 and $+1$.

Having confirmed that the fringes measured from the diffusive sample are due to constructive interference of

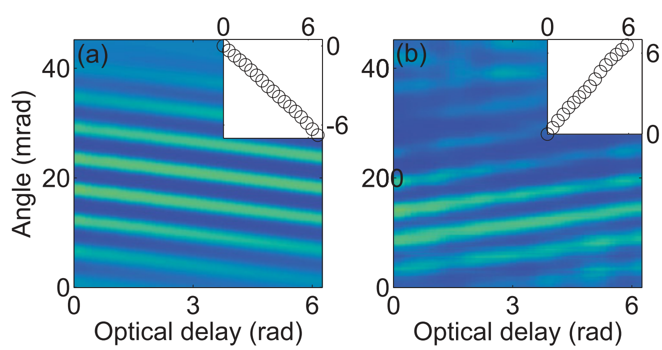

FIG. 2. (Color online) Fringe behavior as a function of incident optical delay φ. The fringe angular profiles as a function of optical delay are presented for a fixed distance between the incident beams, $d = 96$ μm. Panel (a) corresponds to a mirror while panel (b) is from a sol-gel-based diffusive sample with mean free path $\ell = 57$ μm. In each case, the inset shows the phase (rad) of the fringe Fourier component vs. φ (rad). The fringe pattern is well recovered in the diffusive case. The upward shift of the fringes for the diffusive sample, compared to the downward shift for the mirror, for increasing optical delay, confirms that the fringes in (b) are due to interference between light traveling along reciprocal paths. The corresponding amplitude of the fringe Fourier component measured on the diffusive sample therefore correctly represents the EBS visibility.

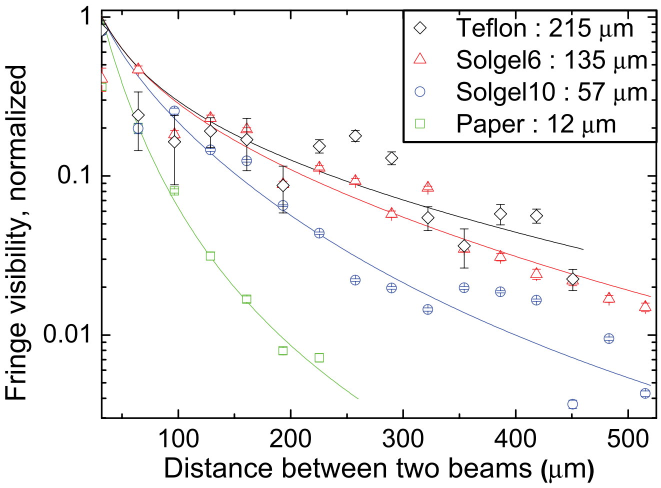

FIG. 3. (Color online) EBS visibility vs. distance d and mean free path ℓ. Equation (5) is fit to each set of visibility, for a given diffusive sample, and with a fixed ℓ. The nature and ℓ values of each sample are summarized in the legend. The visibility for each sample is normalized to the first point (at 32.2 μm) of the fitted curve. The theoretical expectation, from Eq. (5), is in good agreement with the visibility measurements. Each diffusive fringe is therefore a constitutive part of the EBS cone.

reciprocal paths in the multiple-scattering material, it is sound to compare the amplitude of the fringe Fourier component to the expected visibility of Eq. (5). In Fig. 3, the average amplitude of the fringe Fourier component is plotted as a function of distance between incident spots and for several diffusive samples of known ℓ. For each sample, the transport mean free path is determined by the width of a standard EBS cone measurement [10]. The theoretical expectation, from Eq. (5), is fit to the measured visibility for each sample with a known ℓ, with C being the only fitting parameter. The constant C in Eq. (5) mainly sets the overall magnitude of the visibility, whereas the visibility dependence with distance is set by the fixed mean free path, except at distances smaller than the resolution (pixel size) when the theory does not hold. The Green's function can also be directly determined from the visibility measurements, according to Eq. (2). Good agreement between the theoretical expectation and the measured fringe visibility is found for several diffusive samples with a wide range of transport mean free paths, from 12 to 215 μm. Both the identification of the diffusive fringe as originating from reciprocal-path interference and the expected dependence of the visibility on distance indicate that these diffusive fringes are indeed the constitutive parts of the EBS cone.

V. CONCLUSION

In conclusion, an optical setup was presented, along with sustaining theory, which allows the observation of interference fringes stemming from reciprocal paths in a multiple scattering material, and we observe cosine-like interference fringes in reflection of a diffusive material. Effectively, the conjugation of a dual spot illumination and observation scheme allows one to "take apart" the EBS cone by unraveling its constitutive fringes and, at the same time, evaluate the two-point Green's function for light propagation in the multiple-scattering medium between the pair of illuminated points. The spatial shift of the fringes with the incident optical delay allows one to distinguish the EBS fringes from nondiffusive reflective fringes. The EBS visibility decreases with distance between the two coherent incident beams, in very good agreement with theory.

A robust experimental procedure was devised to reveal reciprocal paths in a multiple-scattering medium, where the distance between the first and last scatterers was fixed. Such a procedure allows one to probe multiple scattering of light with path-length dependence in a continuous-wave setting, in contrast to other typical methods necessitating time-resolved measurements [11]. Given the method's ability to characterize the two-point spatial Green's function for light transport, one can envision the possibility of using it to probe nonuniform scattering media. In particular, the method may be developed into an effective means of spatially resolving the disorder in systems where the spatial inhomogeneity actually plays a dominant role (a recent example is the observation of localized resonators in some random laser materials [12], associated with disorder which cannot be quantified by the EBS technique, using plane wave illumination). Another very interesting possibility is related to the recently realized Lévy Glass [13], in which energy transport does not obey the standard diffusion theory. Noting that Eq. (2) does not rely on the diffusion approximation, contrary to Eq. (4), the fringe visibility is directly related to the Green's function. Our method therefore provides a convenient way to measure the Green's function for light transport in these innovative materials.

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