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Spin optodynamics analog of cavity optomechanics

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The dynamics of a large quantum spin coupled parametrically to an optical resonator is treated in analogy with the motion of a cantilever in cavity optomechanics. Distinct spin optodynamic phenomena are predicted, such as cavity-spin bistability, optodynamic spin-precession frequency shifts, coherent amplification and damping of spin, and the spin optodynamic squeezing of light.

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Cavity optomechanical systems are currently being explored with the goal of measuring and controlling mechanical objects at the quantum limit, using interactions with light [1]. In such systems, the position of a mechanical oscillator is coupled parametrically to the frequency of cavity photons. A wealth of phenomena result, including quantum-limited measurements [2], mechanical response to photon shot noise [3], cavity cooling [4], and ponderomotive optical squeezing [5].

Concurrently, spins and psuedospins coupled to electromagnetic cavities are being researched in atomic [6], ionic [7], and nanofabricated systems [8,9], with applications including magnetometry [10], atomic clocks [11], and quantum information processing [6,9,12]. In contrast to mechanical objects, spin systems are more easily disconnected from their environment and prepared in quantum states, including squeezed states [11].

In this Rapid Communication, we seek to link these two fields by exploiting the similarities between large-spin systems and harmonic oscillators [13] to construct a cavity spin-optodynamics system in analogy to cavity optomechanics. Optomechanical phenomena map directly to our proposed system, resulting in spin cooling and amplification [14,15], nonlinear spin sensitivity and spin-cavity bistability [16,17], and spin optodynamic squeezing of light [14,18]. Such a system may find application as a quantum-limited spin amplifier or as a latching spin detector. We detail these phenomena using currently accessible parameters, and we propose realizations either using cold atoms and visible light or using cryogenic solid-state systems and microwaves.

An ideal cavity optomechanics system, consisting of a harmonic oscillator coupled linearly to a single-mode cavity field, obeys the Hamiltonian

$$\mathcal{H} = \hbar\omega_c\hat{n} + \hbar\omega_z\hat{a}^\dagger\hat{a} - f z_{\mathrm{HO}}(\hat{a}^\dagger + \hat{a})\hat{n} + \mathcal{H}_{\mathrm{in,out}}. \quad (1)$$

Here $\hat{a}$ is the oscillator's phonon annihilation operator, $\hat{n}$ is the photon number operator, ω_z is the natural frequency of the oscillator in the dark, and ω_c is the bare cavity resonance frequency. f is the radiation-pressure force applied by a single photon, while $z_{\mathrm{HO}} = \sqrt{\hbar/2m\omega_z}$ is the harmonic oscillator length for oscillator mass m. $\mathcal{H}_{\mathrm{in,out}}$ describes the coupling of the cavity field to external light modes. Under this Hamiltonian, the cantilever position $\hat{z}$ and momentum $\hat{p}$ evolve as $d\hat{z}/dt = \hat{p}/m$ and $d\hat{p}/dt = -m\omega_z^2\hat{z} + f\hat{n}$.

To construct a spin analog of this system, we consider a non-birefringent Fabry-Perot cavity with its axis along $\mathbf{k}$ (Fig. 1). For the collective spin, we first consider a gas of N hydrogenlike atoms in a single hyperfine manifold of their electronic ground state, each with dimensionless spin s and gyromagnetic ratio γ. The atoms are optically confined at an antinode of the cavity field. An external magnetic field $\mathbf{B} = B\mathbf{b}$ is applied to the atoms. The detuning Δ_{ca} between the cavity and atomic resonance frequencies is chosen to be large compared to both the natural linewidth and the hyperfine splitting of the atoms' excited state. In this limit, spontaneous emission may be ignored and the single-atom cavity-field interaction energy, $\mathcal{H}_{\mathrm{Stark}} = (\hbar g_0^2/\Delta_{\mathrm{ca}})\hat{n}(1 \pm \upsilon\mathbf{k}\cdot\hat{\mathbf{s}})$, comprises the scalar and vector ac Stark shifts [19], where g_0 quantifies the atom-cavity coupling and υ the vector shift.

Summing over all atoms q, we obtain the system Hamiltonian,

$$\mathcal{H} = \hbar\omega_c(\hat{n}_+ + \hat{n}_-) + \mathcal{H}_{\mathrm{in,out}} + \sum_q\Bigg(-\hbar\gamma\mathbf{B}\cdot\hat{\mathbf{s}}_q + \frac{\hbar g_0^2}{\Delta_{\mathrm{ca}}}[(\hat{n}_+ + \hat{n}_-) + \upsilon(\hat{n}_+ - \hat{n}_-)\mathbf{k}\cdot\hat{\mathbf{s}}_q]\Bigg), \quad (2)$$

with number operators $\hat{n}_\pm$ for the $\sigma^\pm$ polarized optical modes.

The preceding Hamiltonian can be rewritten as the interaction of the collective spin operator $\hat{\mathbf{S}} \equiv \sum_q \hat{\mathbf{s}}_q$ with an effective total magnetic field $\mathbf{B}_{\mathrm{eff}} \equiv \mathbf{\Omega}_{\mathrm{eff}}/\gamma$, giving [20]

$$\mathbf{\Omega}_{\mathrm{eff}} = \Omega_L\mathbf{b} + \Omega_c(\hat{n}_+ - \hat{n}_-)\mathbf{k}. \quad (3)$$

Here $\Omega_L = \gamma B$ and $\Omega_c = -\upsilon g_0^2/\Delta_{\mathrm{ca}}$. Altogether, the cavity-spin optodynamical Hamiltonian is

$$\mathcal{H} = \hbar\left(\omega_c + \frac{N g_0^2}{\Delta_{\mathrm{ca}}}\right)(\hat{n}_+ + \hat{n}_-) + \mathcal{H}_{\mathrm{in,out}} - \hbar\mathbf{\Omega}_{\mathrm{eff}}\cdot\hat{\mathbf{S}}. \quad (4)$$

Now consider the external magnetic field to be static and oriented along $\mathbf{i}$, orthogonal to the cavity axis. In the limit $\langle\hat{S}\rangle \simeq S\mathbf{i}$, the spin dynamics become

$$\frac{d\hat{S}_j}{dt} = \Omega_L\hat{S}_k - \Omega_c S(\hat{n}_+ - \hat{n}_-), \quad \frac{d\hat{S}_k}{dt} = -\Omega_L\hat{S}_j. \quad (5)$$

The analogy between cavity optomechanics and spin optodynamics is established by assigning $\hat{z} \to -z_{\mathrm{HO}}\hat{S}_k/\Delta S_{SQL}$ and $\hat{p} \to p_{\mathrm{HO}}\hat{S}_j/\Delta S_{SQL}$, where z_{HO} and $p_{\mathrm{HO}} = \hbar/(2z_{\mathrm{HO}})$ are defined with $\omega_z \to \Omega_L$ [13] and $\Delta S_{SQL} = \sqrt{S/2}$ is the standard quantum limit for transverse spin fluctuations.

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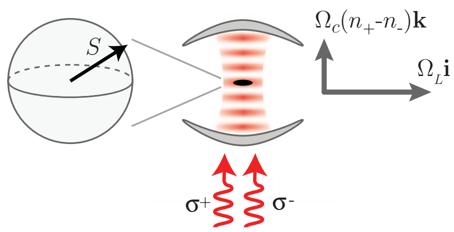

FIG. 1. (Color) An ensemble of atoms trapped within a driven optical resonator experiences an externally imposed magnetic field along **i** and a light-induced effective magnetic field along the cavity axis **k**. The evolution of the collective spin $\hat{\mathbf{S}}$ resembles that of a cantilever in cavity optomechanics.

Equations (5) now match the optomechanical equations of motion with the optomechanical coupling defined through $f z_{\rm HO}\hat{n} \to -\hbar\Omega_c\Delta S_{SQL}(\hat{n}_+ - \hat{n}_-)$. The main result of this work, that various cavity optomechanical phenomena are manifest also in cavity-spin optodynamical systems, is immediately established.

Let us now elaborate on these phenomena. To obtain general results, we will proceed without assuming $\hat{S} \simeq S\mathbf{i}$, except in certain cases, noted in the text, where some physical insight is gained. We begin with effects for which both the light field and the ensemble spin may be treated classically, that is, by letting $\mathbf{S} = \langle\hat{\mathbf{S}}\rangle$ and $\bar{n}_\pm = \langle\hat{n}_\pm\rangle$.

Cavity-spin bistability. We begin our examination by finding the fixed points of the system. The collective spin vector is static when $\mathbf{S}$ is parallel to $\mathbf{\Omega}_{\rm eff}$. Writing $\mathbf{S} = S(\mathbf{i}\sin\theta_0 + \mathbf{k}\cos\theta_0)$, this condition requires $\bar{n}_+ - \bar{n}_- = (\Omega_L/\Omega_c)\cot\theta_0$. The intracavity photon numbers are determined also by the standard expression for a driven cavity of half linewidth κ; that is, $\bar{n}_\pm = \bar{n}_{\max,\pm}[1 + (\Delta_{p,\pm} \pm \Omega_c S\cos\theta_0)^2/\kappa^2]^{-1}$ with $\omega_\pm = (\omega_c + Ng_0^2/\Delta_{\rm ca}) + \Delta_{p,\pm}$ being the frequency of laser light of polarization $\sigma^\pm$ driving the cavity and $\bar{n}_{\max,\pm}$ characterizing its power. These two expressions for $\bar{n}_+ - \bar{n}_-$ may admit several solutions (Fig. 2).

As typical in instances of cavity bistability [22], several of the static solutions for the intracavity intensities may be unstable. To identify such instabilities, we consider the torque on the collective spin when it is displaced slightly toward $+\mathbf{k}$ from its static orientation. Stable dynamics result when such displacement yields a torque $\mathbf{N}\cdot\mathbf{j}$ with the sign $\alpha = \mathrm{sgn}(\sin\theta_0)$. Geometrically, this stability requires that the spin vector be displaced further in the $+\mathbf{k}$ direction than the vector $\alpha\mathbf{\Omega}_{\rm eff}$. Quantifying the linear response of the intracavity effective magnetic field to variations of the collective spin via $\lambda = \Omega_c d(\bar{n}_+ - \bar{n}_-)/dS_k$, the static spin orientations are found to be unstable when $\alpha\lambda > \Omega_L|\csc^3\theta_0|/S$.

Optodynamical Larmor frequency shift. The dynamics of the spin precessing about one of the stable configurations can be parametrized by the spin precession frequency, which is shifted from Ω_L by two effects. First, there is an upward frequency shift from the static modification of the effective magnetic field, leading to precession at the frequency $\Omega_L{}' = \Omega_L|\csc\theta_0|$ when $\lambda = 0$. A second shift occurs when the spin dynamics are slow compared to the response time of the cavity field ($\Omega_L{}' \ll \kappa$). Here the precessing spin modulates the cavity field, which, in turn, acts back upon the spin to modify its precession frequency. When the precession amplitude is small, a solution of the spin equations of motion derived from the Hamiltonian in Eq. (4) yields an overall precession frequency $\Omega_L{}''$, where

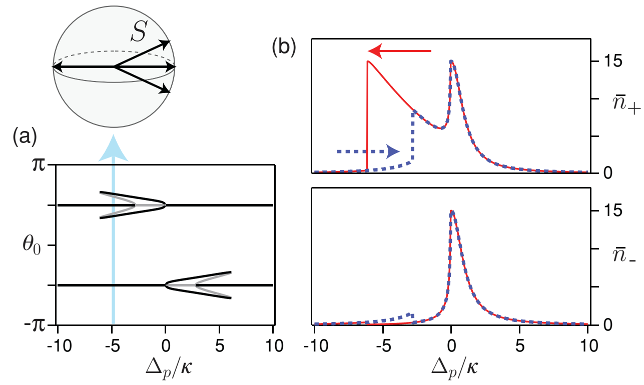

FIG. 2. (Color) Cavity-spin bistability in a cavity driven with linearly polarized light. We consider $N = 5000$ spin-2 ^{87}Rb atoms, $\Omega_c/\kappa = 1.25\times10^{-3}$, $\Omega_L/\kappa = 3.3\times10^{-2}$, and $\bar{n}_{\max,\pm} = 15$ (similar to Ref. [21]). (a) As Δ_p is varied, several stable (black) and unstable (gray) static spin configurations are found. Configurations for $\Delta_p/\kappa = -4.8$ are depicted. (b) The cavity exhibits hysteresis as the probe is swept with positive (dashed blue lines) or negative (solid red lines) frequency chirps, with the spin initially along **i**. Rapid transitions as Δ_p/κ is swept upward from -7 or downward from 0 involve symmetry breaking as the cavity becomes birefringent; we display $\bar{n}_+$ and $\bar{n}_-$ assuming the stable branch closer to $\theta_0 = 0$ is selected. Here $\Delta_{\rm ca}/2\pi = 20$ GHz from the D_2 transition, $g_0/2\pi = 15$ MHz, $\kappa/2\pi = 1.5$ MHz.

$$\Omega_L{}''^2 = \Omega_L{}'^2 - \lambda\Omega_L S\sin\theta_0. \tag{6}$$

The quantity $k_S \equiv -\lambda\Omega_L S\sin\theta_0$ serves as the analog of the optical spring constant [23] and leads to shifts of the Larmor precession frequency with a sign and magnitude that depend on the spin orientation λ and the frequency, intensity, and polarization of the cavity probe fields. When the precession amplitude is large, the dynamics become essentially nonlinear. In this case the dynamics can be described by numerical simulation (Fig. 3).

Coherent amplification and damping of spin. Now we consider the effects of the finite cavity response time κ^{-1} on the spin dynamics. To develop an intuitive picture, we consider the unresolved sideband regime $\Omega_L < \kappa$, in a frame (indicated by the index "r") corotating with the collective spin, with $\mathbf{i}_r$ aligned to the fixed point. We assume the spin to be precessing at a near constant rate and the cavity field response to this precession to be simply delayed by κ^{-1}. Employing the rotating-wave approximation, the delay causes the effective field $\mathbf{\Omega}_{\rm eff,r}$ to point out of the $\mathbf{i}_r$-$\mathbf{k}_r$ plane, with $\mathbf{\Omega}_{\rm eff,r}\cdot\mathbf{j}_r = -(\alpha\lambda S_{k,r}\sin^2\theta_0\sin\phi)/2$, where $\phi = \Omega_L{}''/\kappa$. The collective spin now experiences a torque in the $\mathbf{k}_r$ direction, giving

$$\frac{dS_{k,r}}{dt} = \frac{-\alpha\lambda\sin^2\theta_0\sin\phi S_{i,r}}{2}S_{k,r}. \tag{7}$$

For positive (negative) values of $\alpha\lambda$, the Larmor precession frequency is shifted down (up) and the spin is damped toward (amplified away from) its stable point. Similar relations apply

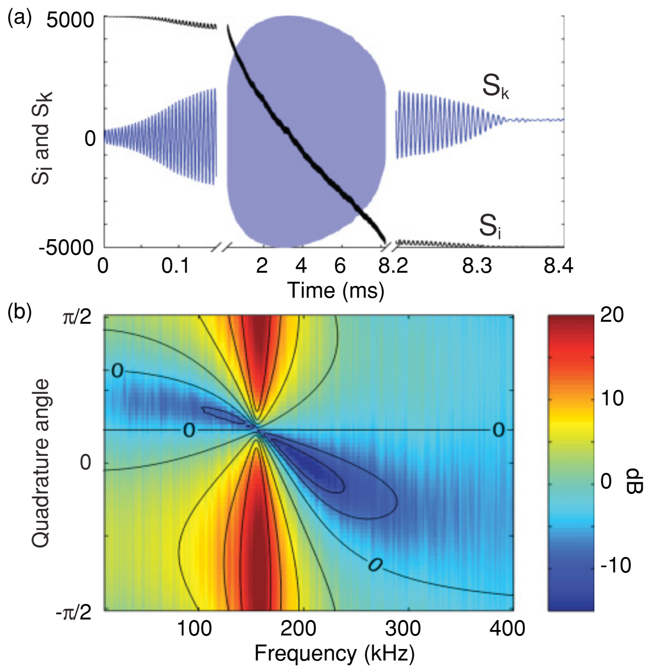

FIG. 3. (Color) Simulations of spin dynamics for $S = 5000$, $\Omega_L/2\pi = 200$ kHz, $\Omega_c/2\pi = -2.3$ kHz, $\kappa/2\pi = 1.8$ MHz, $\hat{n}_+ = 10$, and $\Delta_{p,+} = 0.37\kappa$. (a) Time evolution of S_i (black) and S_k (blue), following spin preparation along **i**, shows amplification, reorientation, and damping toward the high-energy stable orientation along $-$**i**. Note the different scales on the horizontal axis. (b) Logarithmic optical spectral noise power relative to that of coherent light, plotted vs quadrature angle ϕ (amplitude quadrature at $\phi = 0$), shows inhomogeneous optical squeezing. Simulation results shown in color, and linearized theory [Eq. (10)] as contour lines every 5 dB.

to cavity optomechanics [24]. The deflection of the spin toward or away from the stable points persists for large precession amplitudes (Fig. 3).

This cavity-induced spin amplification or damping differs from conventional optical pumping in two important respects. First, while the spin polarization generated by optical pumping relies on the polarization of the pump light, the target state for cavity-induced spin damping is selected energetically. Similar to cavity optomechanical cooling [4], cavity enhancement of Raman scattered light drives spins to the high- or low-energy spin state according to the detuning of probe light from the cavity resonance, independent of the polarization. Second, this amplification or damping of the intracavity spin is coherent, preserving the phase of Larmor precession, at least within the limits of a quantum amplifier.

Spin optodynamical squeezing of light. We now consider quantum optical effects of cavity-spin optodynamics. One such effect is the disturbance of the collective spin due to quantum optical fluctuations of the cavity fields. In cavity optomechanics, intracavity photon number fluctuations disturb the motion of a cantilever, providing the necessary backaction of a quantum measurement of position [25]. The analogous disturbance of optically probed atomic spins (or pseudospins) has been studied both in free-space [26] and intracavity [11,27] configurations. In an optomechanics-like configuration, for example, with $\mathbf{B} \propto \mathbf{i}$, backaction heating of the atomic spin enforces quantum limits to measurement of the precessing ensemble and also set limits on optodynamical cooling. In contrast with optomechanical systems, optically probed spin ensembles readily present the opportunity to perform quantum-nondemolition (QND) measurements; with $\mathbf{B} \parallel \mathbf{k}$, the detected spin component S_k is a QND variable representing the energy of the spin system.

The noise-perturbed spin acts back upon the cavity optical field, mediating a self-interaction of the light field that can result in optical squeezing. To exhibit this effect, we consider a cavity illumined with σ^+ circular polarized probe light with detuning Δ_p. The dynamics of the cavity field are given by

$$\frac{d\hat{c}_+}{dt} = (i\Delta_p - \kappa + i\Omega_c \hat{S}_k)\hat{c}_+ + \kappa(\eta + \hat{\xi}_+). \tag{8}$$

Here, η gives the coherent-state amplitude of the drive field and the noise operator $\hat{\xi}_+$ represents its fluctuations. When evaluating the dynamics numerically, we consider a semiclassical Langevin equation, converting $\hat{\xi}_+$ into a Gaussian stochastic variable with statistics related to those of the noise operator, and replacing the operators $\hat{c}_+$ and $\hat{\mathbf{S}}$ with c numbers. This substitution is appropriate for moderately large values of $\bar{n}$ and S.

Figure 3 portrays the simulated evolution of a spin prepared initially in a low-energy spin orientation (close to **i**), driven by a blue-detuned cavity probe. Coherent spin amplification directs the spin toward the stable high-energy configuration (near $-$**i**), yielding a dynamical steady state characterized by a negative temperature.

To obtain analytical expressions for the evolution dynamics, we follow the example of cavity optomechanics [5] by linearizing the Langevin equations for spin and optical fluctuations about their steady-state value. The spin projection S_k responds to amplitude-quadrature fluctuations of the cavity field $\xi_A(\omega)$ with susceptibility

$$\chi(\omega) \equiv \frac{S_k(\omega)}{\xi_A(\omega)} = \frac{-\Omega_L{}'\Omega_c\sqrt{\bar{n}_+}}{\Omega_L{}'^2 + k_S R(\omega) - \omega^2 + i\omega\Gamma_o(\omega)}, \tag{9}$$

where $\Gamma_o(\omega) = 2\kappa \frac{\Omega_L{}'^2 - \omega^2}{\kappa^2 + \Delta_p^2 - \omega^2}$ is the cavity optodynamic spin damping and $R(\omega) = \frac{\kappa^2 + \Delta_p^2}{\kappa^2 + \Delta_p^2 - \omega^2}$. The susceptibility is largest for $\omega \simeq \Omega_L{}''$. The driven spin feeds the fluctuations back onto the cavity field, yielding the intracavity field fluctuation spectrum

$$c_+(\omega) = \frac{\Omega_L{}'^2 + i\frac{\kappa+\omega}{\Delta_p} k_S R(\omega) - \omega^2 + i\omega\Gamma_o(\omega)}{\Omega_L{}'^2 + k_S R(\omega) - \omega^2 + i\omega\Gamma_o(\omega)}\xi_A + i\xi_P, \tag{10}$$

where $\xi_P(\omega)$ is the input spectrum of phase fluctuations. This fluctuation spectrum exhibits inhomogeneous optical squeezing [Fig. 3(b)].

Applications. The analogy of cavity optodynamics widens the range of phenomena accessed through the manipulation and detection of quantum spins within optical cavities, enabling several applications. For example, bistability in cavity-coupled single-spin systems serves to increase the readout fidelity of cavity-coupled qubits [28]. Similarly here, cavity-spin bistability could be used as a Schmitt trigger for the collective spin: If the probe power is turned on diabatically in the bistable regime, the cavity transmission will latch into either a bright or dark state, depending on whether the initial spin state is below or above a parametrically chosen threshold value. The cavity-spin system may also be used as a phase-preserving amplifier for spin dynamics occurring near the shifted precession frequency Ω_L'', with amplification noise given in Eq. (10). Both applications may aid measurements of ac magnetic fields, amplifying weak signals above technical sensitivity limits.

Conversely, cavity-spin optodynamics may be applied as a powerful simulator of cavity optomechanics, with the spin system allowing for new means of control. For example, precession frequencies may be tuned rapidly by varying the applied magnetic field, simulating optomechanics with a dynamically variable mechanical spring constant. Alternately, spatial control of inhomogeneous magnetic fields may be used to divide a spin ensemble into several independent subensembles, simulating optomechanics with several mechanical modes.

In addition to the dilute gas implementation discussed so far, a similar system could be constructed using solid-state spin ensembles and microwave resonators. For example, using ensembles of nitrogen-vacancy defects in diamond coupled to the circular polarized evanescent radiation of a crossed microwave resonator [29], the Hamiltonian of Eq. (4) is obtained by using the ground $m_s = \pm 1$ states for the pseudospin and replacing the ac Stark shift with an ac Zeeman shift from microwave radiation near the 2.8-GHz crystal-field-split Zeeman transition.

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