Bell-like inequality for the spin-orbit separability of a laser beam

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In analogy with Bell's inequality for two-qubit quantum states, we propose an inequality criterion for the nonseparability of the spin-orbit degrees of freedom of a laser beam. A definition of separable and nonseparable spin-orbit modes is used in consonance with the one presented in Phys. Rev. Lett. **99**, 160401 (2007). As the usual Bell's inequality can be violated for entangled two-qubit quantum states, we show both theoretically and experimentally that the proposed spin-orbit inequality criterion can be violated for nonseparable modes. The inequality is discussed in both the classical and quantum domains.

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I. INTRODUCTION

Experiments to show violation of Bell-like inequalities have attracted much attention in the last years due to the possibility of ruling out classical hidden-variable theories which jeopardize the need of a quantum-mechanical model to describe nature. The majority of proposed experiments rely on a *pair* of entangled quantum particles for genuine nonlocality tests [1] or not necessarily entangled in the case of noncontextuality tests [2]. Entanglement in *single*-particle degrees of freedom has already been investigated in Ref. [3], where a Bell-like inequality was violated by entangling the spin and the beam path of single neutrons in an interferometer. The same kind of single-particle scheme has been proposed for photonic setups using the polarization and transverse (spin-orbit) modes [4]. We have proposed a similar setup to investigate the spin-orbit separability of a laser beam in [5]. Simulations of Bell inequalities in classical optics have also been discussed in waveguides [6] and imaging systems [7]. The combination of the polarization and spatial photonic degrees of freedom open interesting possibilities in the quantum optics and quantum-information domains [8–10]. The ability to produce and transform beams carrying orbital angular momentum has allowed the development of important techniques with potential applications to quantum information [11,12].

In this work we present our experimental results together with the theoretical background developed for the analogy between the usual quantum-mechanical context of Bell inequality and our classical spin-orbit counterpart. Moreover, we discuss the quantum-mechanical description of the experiment and investigate its predictions for different initial quantum states, namely, a coherent, a single-photon, and a mixed state. Although no genuine entanglement is present in the classical optics implementation, entanglement will be present if single photons are sent through the same apparatus.

II. SPIN-ORBIT SEPARABILITY OF THE CLASSICAL AMPLITUDE

Following Ref. [13], we define as separable those spin-orbit modes that can be written in the form $\vec{E}_S(\vec{r}) = \psi(\vec{r})\hat{\mathbf{e}}$, where $\psi(\vec{r})$ is a normalized c-number function of the transverse spatial coordinates (transverse mode) and $\hat{\mathbf{e}}$ is a normalized polarization vector. However, there are modes that cannot be written in this form, which we shall refer to as nonseparable. For example, consider the following normalized mode:

$$\vec{E}_{\mathrm{MNS}}(\vec{r}) = \frac{1}{\sqrt{2}}[\psi_V(\vec{r})\hat{\mathbf{e}}_V + \psi_H(\vec{r})\hat{\mathbf{e}}_H], \quad (1)$$

where $\psi_H(\vec{r})$ and $\psi_V(\vec{r})$ are the first-order Hermite-Gaussian transverse modes with horizontal (H) and vertical (V) orientations [14], and $\hat{\mathbf{e}}_H$ and $\hat{\mathbf{e}}_V$ are the horizontal (H) and vertical (V) linear polarization unit vectors. This mode cannot be written as a product of a spatial part times a polarization vector. In the space of spin-orbit modes of a classical beam, it plays a role analogous to a maximally entangled two-qubit state, and we shall refer to it as a maximally nonseparable mode (MNS). While the separable spin-orbit modes have a single polarization state over the beam wave front, the nonseparable modes exhibit a polarization gradient leading to a polarization-vortex behavior [15].

Let us take the arbitrary spin-orbit mode

$$\begin{aligned}\vec{E}(\vec{r}) = {} & A_1\psi_V(\vec{r})\hat{\mathbf{e}}_V + A_2\psi_V(\vec{r})\hat{\mathbf{e}}_H \\ & + A_3\psi_H(\vec{r})\hat{\mathbf{e}}_V + A_4\psi_H(\vec{r})\hat{\mathbf{e}}_H \end{aligned} \quad (2)$$

and discuss its separability with the aid of a concurrence-like quantity [13,16] $C = 2|A_2A_3 - A_1A_4|$, where A_i ($i = 1,\ldots,4$) are complex numbers satisfying $\sum_{i=1}^4 |A_i|^2 = 1$. It turns out that $0 < C \leqslant 1$ for nonseparable modes. In particular we say that $C = 1$ corresponds to a maximally nonseparable mode. It can be easily verified that any separable mode $\vec{E}_S(\vec{r})$ has $C = 0$.

To develop the spin-orbit inequality, it will be useful to define the following rotated basis of polarization and transverse modes:

$$\begin{aligned}\hat{\mathbf{e}}_{\alpha+} &= (\cos\alpha)\hat{\mathbf{e}}_V + (\sin\alpha)\hat{\mathbf{e}}_H,\\ \hat{\mathbf{e}}_{\alpha-} &= (\sin\alpha)\hat{\mathbf{e}}_V - (\cos\alpha)\hat{\mathbf{e}}_H,\\ \psi_{\beta+}(\vec{r}) &= (\cos\beta)\psi_V(\vec{r}) + (\sin\beta)\psi_H(\vec{r}),\\ \psi_{\beta-}(\vec{r}) &= (\sin\beta)\psi_V(\vec{r}) - (\cos\beta)\psi_H(\vec{r}).\end{aligned} \quad (3)$$

Rewriting the maximally nonseparable mode given by Eq. (1) in the rotated basis, we get

$$\begin{aligned}\vec{E}_{\mathrm{MNS}}(\vec{r}) = {} & \frac{1}{\sqrt{2}}\{\cos(\beta-\alpha)[\psi_{\beta+}(\vec{r})\hat{\mathbf{e}}_{\alpha+} + \psi_{\beta-}(\vec{r})\hat{\mathbf{e}}_{\alpha-}] \\ & + \sin(\beta-\alpha)[\psi_{\beta-}(\vec{r})\hat{\mathbf{e}}_{\alpha+} - \psi_{\beta+}(\vec{r})\hat{\mathbf{e}}_{\alpha-}]\}. \end{aligned} \quad (4)$$

Let $I_{(\pm)(\pm)}(\alpha,\beta)$ be the squared amplitude of the $\psi_{\beta(\pm)}(\vec{r})\hat{\mathbf{e}}_{\alpha(\pm)}$ component in the expansion of $\vec{E}_{\rm MNS}(\vec{r})$ in the rotated basis. They play the same role as the detection probabilities in the quantum-mechanical context. Due to the orthonormality of $\{\psi_{\beta+},\psi_{\beta-}\}$ and $\{\hat{\mathbf{e}}_{\alpha+},\hat{\mathbf{e}}_{\alpha-}\}$, it can be easily shown that

$$I_{++}(\alpha,\beta) + I_{+-}(\alpha,\beta) + I_{-+}(\alpha,\beta) + I_{--}(\alpha,\beta) = 1. \quad (5)$$

Following the analogy with the usual quantum-mechanical Clauser-Horne-Shimony-Holt (CHSH) [17] inequality for spin 1/2 particles, we can define

$$M(\alpha,\beta) = I_{++}(\alpha,\beta) + I_{--}(\alpha,\beta) - I_{+-}(\alpha,\beta) - I_{-+}(\alpha,\beta) = \cos\,[2(\beta-\alpha)], \quad (6)$$

and derive a Bell-type inequality for the quantity

$$S = M(\alpha_1,\beta_1) + M(\alpha_1,\beta_2) - M(\alpha_2,\beta_1) + M(\alpha_2,\beta_2). \quad (7)$$

For any separable mode, $-2 \leqslant S \leqslant 2$, however, this condition can be violated for nonseparable modes. A maximal violation of the previous inequality, corresponding to $S = 2\sqrt{2}$, can be obtained for the set $\alpha_1 = \pi/8$, $\alpha_2 = 3\pi/8$, $\beta_1 = 0$, $\beta_2 = \pi/4$.

III. THE EXPERIMENT

The experimental setup to observe maximal violation of the nonseparability inequality is shown in Fig. 1 and is composed of two stages: preparation of the maximally nonseparable mode and measurement of the intensities $I_{(\pm)(\pm)}(\alpha,\beta)$. The preparation stage consists of a Mach-Zender (MZ) interferometer with a half-wave plate (HWP) oriented at 45° with respect to the horizontal plane in one arm and a Dove prism (DP) also oriented at 45° with respect to the horizontal plane in the other arm. Before the MZ interferometer, the horizontally polarized TEM_{00} beam from a high-stability laser (Lightwave 142H-532-400SF) at 532 nm passes through a holographic mask [18–20] and produces mode $\vec{E}(\vec{r}) = \psi_V(\vec{r})\hat{\mathbf{e}}_H$ at the first diffraction order. In the MZ interferometer, the half-wave plate converts $\hat{\mathbf{e}}_H$ into $\hat{\mathbf{e}}_V$, and the Dove prism changes $\psi_V(\vec{r})$ into $\psi_H(\vec{r})$ so the resulting mode at the output beam splitter (BS2) is

$$\vec{E}(\vec{r}) = \frac{1}{\sqrt{2}}[\psi_H(\vec{r})\hat{\mathbf{e}}_H + e^{i\phi}\psi_V(\vec{r})\hat{\mathbf{e}}_V], \quad (8)$$

where ϕ is the phase difference between the two arms of the MZ interferometer. Mirror M1 is mounted on a piezoelectric transducer (PZT) to allow fine control of the phase difference ϕ. The other output of BS2 is used to check the alignment between the two components of the mode prepared.

The measurement stage is composed by a Dove prism oriented at a variable angle $\beta/2$, a half-wave plate oriented at a variable angle $\alpha/2$, a Mach-Zender interferometer with an additional mirror (MZIM) [21], and one polarizing beam splitter (PBS) after each of the MZIM outputs. Four photocurrent detectors (D1–D4) are used to measure the intensities at the PBS outputs. The HWP at $\alpha/2$ combined with DP at $\beta/2$ define in which basis we are going to measure our initial mode.

We want MZIM to work as a parity selector delivering odd modes $\psi_V(\vec{r})\hat{\mathbf{e}}_H$ and $\psi_H(\vec{r})\hat{\mathbf{e}}_V$ in one port and even modes $\psi_V(\vec{r})\hat{\mathbf{e}}_V$ and $\psi_H(\vec{r})\hat{\mathbf{e}}_H$ in the other port. Parity is evaluated according to the eigenvalue of the respective mode under reflection over the horizontal plane. Let χ be the optical phase difference between the two arms of the MZIM. Note that proper functioning of the MZIM as a parity selector occurs only when $\chi = 2n\pi$ $(n = 0,1,2,\ldots)$. For $\chi = (2n+1)\pi$, the even and odd outputs are interchanged.

After propagating through DP oriented at $\beta/2$ and through HWP oriented at $\alpha/2$, the maximally nonseparable mode given by Eq. (1) transforms to

$$\vec{E}'(\vec{r}) = A_e^+(\phi)\psi_V(\vec{r})\hat{\mathbf{e}}_V + A_o^-(\phi)\psi_V(\vec{r})\hat{\mathbf{e}}_H + A_o^+(\phi)\psi_H(\vec{r})\hat{\mathbf{e}}_V + A_e^-(\phi)\psi_H(\vec{r})\hat{\mathbf{e}}_H, \quad (9)$$

where

$$A_e^{\pm}(\phi) = e^{i\phi/2}\{\cos(\phi/2)\cos(\beta-\alpha) \pm i\sin(\phi/2)\cos(\beta+\alpha)\},$$
$$A_o^{\pm}(\phi) = e^{i\phi/2}\{\pm\cos(\phi/2)\sin(\beta-\alpha) + i\sin(\phi/2)\sin(\beta+\alpha)\}. \quad (10)$$

If MZIM phase $\chi = 0$, then the four amplitudes above would be the ones measured by the detectors, since the MZIM interferometer together with PBS1 and PBS2 would separate the modes $\psi_V(\vec{r})\hat{\mathbf{e}}_V$, $\psi_V(\vec{r})\hat{\mathbf{e}}_H$, $\psi_H(\vec{r})\hat{\mathbf{e}}_V$, and $\psi_H(\vec{r})\hat{\mathbf{e}}_H$. But we will still consider the case in which χ may differ from zero, and then the corresponding intensities normalized to the total intensity are given by

$$I_1 = I_2 = \cos^2\left(\frac{\chi}{2}\right)|A_e^{\pm}(\phi)|^2 + \sin^2\left(\frac{\chi}{2}\right)|A_o^{\pm}(\phi)|^2,$$
$$I_3 = I_4 = \sin^2\left(\frac{\chi}{2}\right)|A_e^{\pm}(\phi)|^2 + \cos^2\left(\frac{\chi}{2}\right)|A_o^{\pm}(\phi)|^2. \quad (11)$$

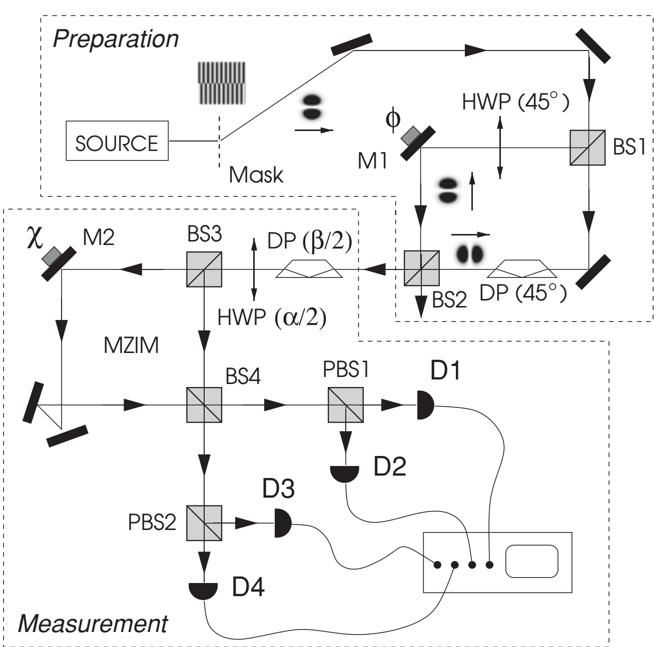

FIG. 1. Experimental setup for the Bell-type inequality violation using a nonseparable classical beam. HWP, half-wave plate; DP, Dove prism; (P)BS, (polarizing) beam splitter; D1–D4, photocurrent detectors.

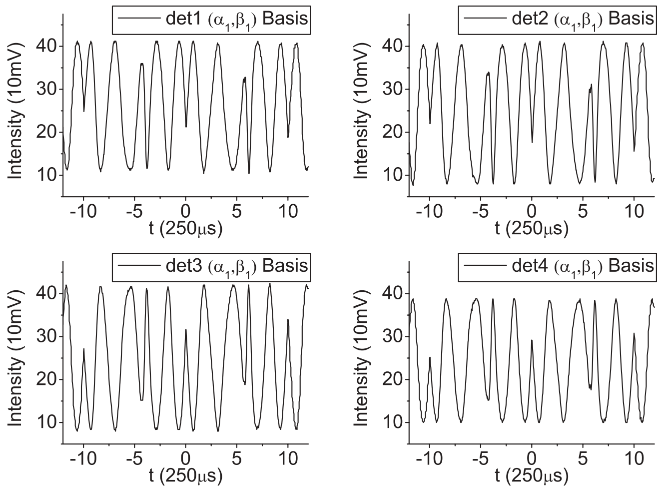

FIG. 2. Experimental results for the maximally nonseparable initial mode, measured in the (α_1,β_1) basis. Time parametrizes the MZIM phase χ.

We can test the violation of the nonseparability inequality by making measurements in the bases (α_1,β_1), (α_1,β_2), (α_2,β_1), and (α_2,β_2) and obtaining the values of $M(\alpha,\beta)$ and subsequently S. The value of S for arbitrary ϕ and χ is given by

$$S(\chi,\phi) = 2\sqrt{2}\cos\chi\cos^2(\phi/2). \tag{12}$$

Thus maximal violation of the nonseparability inequality is accomplished for $\phi = \chi = 0$. This is a key result, because it shows that experimental errors in the phases will only diminish the violation, not increase it.

In our experiment, the MZIM phase χ is continuously varied by applying a voltage ramp to the PZT on M2 while intensities I_1 through I_4 are monitored at the oscilloscope. An example of our experimental results is presented in Fig. 2, showing the oscillations caused by the variation of χ. We know from the intensities dependence on ϕ and χ that $\chi = 0$ corresponds to the peaks in the graphics, and $\phi = 0$ corresponds to a maximal visibility of these oscillations. Since we have repetitions of these peaks, we obtain an ensemble of intensities which allows us to calculate the averages and standard deviations of $M(\alpha,\beta)$ and S. For this end, we take 30 points distributed over the ten peaks measured. This procedure is repeated for all four bases, and the results are shown in Table I. The value of S obtained for the MNS mode is 2.10 ± 0.03, which violates the inequality by 3.3 standard deviations. In this table, we also show our experimental results for a separable mode $\psi_V(\vec{r})\hat{\mathbf{e}}_V$, which is easily obtained by blocking the Dove prism arm of the preparation MZ interferometer.

TABLE I. Mean values for M and S for maximally nonseparable and separable modes.

	Maximally nonseparable	Separable
$\overline{M}(\alpha_1,\beta_1)$	0.609 ± 0.006	0.490 ± 0.008
$\overline{M}(\alpha_1,\beta_2)$	0.486 ± 0.009	0.000 ± 0.005
$\overline{M}(\alpha_2,\beta_1)$	-0.522 ± 0.004	-0.56 ± 0.01
$\overline{M}(\alpha_2,\beta_2)$	0.482 ± 0.009	0.000 ± 0.004
$\overline{S}$	2.10 ± 0.03	1.05 ± 0.03

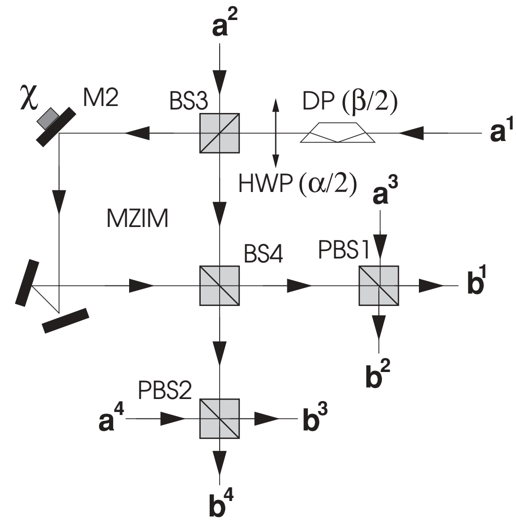

FIG. 3. Input and output modes of the measurement apparatus.

IV. QUANTUM-MECHANICAL FORMULATION

To describe our experiment in a complete quantum-mechanical frame, we must take into account all possible inputs and describe the evolution of the field operators inside the apparatus. In our notation, the mode amplitudes that enter the interferometer are $a^i_{\sigma\lambda}$, where the index i represents the input, σ stands for polarization, and λ for transverse mode. In Fig. 3, the input and output modes are indicated in the sketch of the measurement setup. The output mode amplitude $b^j_{\mu\nu}$ is a combination of the input amplitudes with coefficients $u^{i\sigma\lambda}_{j\mu\nu}$. Therefore, the field amplitude $\mathbf{E}_j$ at output j is

$$\mathbf{E}_j = \sum_{\mu,\nu} b^j_{\mu\nu}\hat{e}_\mu\psi_\nu e^{i\vec{k}_j\cdot\vec{r}}, \tag{13}$$

where

$$b^j_{\mu\nu} = \sum_{i,\sigma,\lambda} u^{i\sigma\lambda}_{j\mu\nu} a^i_{\sigma\lambda}. \tag{14}$$

Since the intensities are expectation values calculated in normal ordering, all input modes in the vacuum state (a^1_{HV}, a^1_{VH}, $a^2_{\sigma\lambda}$, $a^3_{\sigma\lambda}$, and $a^4_{\sigma\lambda}$) will not contribute. Assuming that MZIM is set for proper functioning ($\chi = 0$), the only occupied output modes are

$$\begin{aligned} b^1_{HH} &= \cos\alpha\cos\beta a^1_{HH} + \sin\alpha\sin\beta a^1_{VV}, \\ b^2_{VV} &= \sin\alpha\sin\beta a^1_{HH} + \cos\alpha\cos\beta a^1_{VV}, \\ b^3_{VH} &= -\sin\alpha\cos\beta a^1_{HH} + \cos\alpha\sin\beta a^1_{VV}, \\ b^4_{HV} &= -\cos\alpha\sin\beta a^1_{HH} + \sin\alpha\cos\beta a^1_{VV}. \end{aligned} \tag{15}$$

The only nonzero contributions to the normalized detected intensities $I_j = \langle\mathbf{E}^\dagger_j\cdot\mathbf{E}_j\rangle/I_0$ are

$$\begin{aligned} I_1 &= \langle b^{1\dagger}_{HH} b^1_{HH}\rangle / I_0, \\ I_2 &= \langle b^{2\dagger}_{VV} b^2_{VV}\rangle / I_0, \\ I_3 &= \langle b^{3\dagger}_{VH} b^3_{VH}\rangle / I_0, \\ I_4 &= \langle b^{4\dagger}_{HV} b^4_{HV}\rangle / I_0, \end{aligned} \tag{16}$$

where $I_0 = \sum_{j=1}^4\langle\mathbf{E}^\dagger_j\cdot\mathbf{E}_j\rangle$.

To evaluate the inequality, according to the CHSH prescription, we calculate $M(\alpha,\beta) = I_1 + I_2 - I_3 - I_4$ for the four bases and S as given by Eq. (7). With the description of the inequality above, we can calculate the theoretical value of S for different input quantum states.

A. Coherent state

For measurements made within its coherence length, the laser source can be described by a coherent state. The preparation part of our experiment (taking $\phi = 0$) produces the two-mode coherent state $|\nu,\nu\rangle$, where the first slot of the ket refers to mode HH and the second one to VV. We can use this state to calculate the mean values of Eq. (16), knowing that the complex amplitude $\nu = \sqrt{I_0/2}$, where I_0 is the total initial intensity of the laser. The results are in the equations below:

$$\begin{aligned} I_1 = I_2 = \tfrac{1}{2}\cos^2(\beta - \alpha), \\ I_3 = I_4 = \tfrac{1}{2}\sin^2(\beta - \alpha). \end{aligned} \tag{17}$$

They give $S = 2\sqrt{2}$ for the same choice of bases used in the experiment, showing maximal violation for the two-mode coherent state. Since this state is a tensor product, this violation cannot be attributed to entanglement. In fact, we will see that this violation is closely related to optical coherence when we investigate the statistical mixture case.

In order to investigate the quantum-mechanical relation between the nonseparable mode and the HH and VV modes, let us first write the two-mode coherent state in the Fock basis:

$$\begin{aligned} |\nu,\nu\rangle &= e^{-|\nu|^2} \sum_{n,m=0}^{\infty} \frac{\nu^{n+m}}{\sqrt{n!m!}} |n,m\rangle \\ &= e^{-|\nu|^2} \sum_{N=0}^{\infty} \sum_{m=0}^{N} \frac{\nu^N}{\sqrt{(N-m)!m!}} |N-m,m\rangle. \end{aligned} \tag{18}$$

In the second equality, terms corresponding to the same total photon number ($N = n + m$) were grouped together. Note that the single-photon component ($N = 1$) of the two-mode coherent state is $\nu(|1,0\rangle + |0,1\rangle)$ which is clearly entangled. Let us define the operator $a^\dagger_{\rm MNS}$ that creates this single-photon component when acting on the vacuum state $|0\rangle$:

$$a^\dagger_{\rm MNS} = \frac{(a^\dagger_{VV} + a^\dagger_{HH})}{\sqrt{2}}. \tag{19}$$

Now one can easily put the two-mode coherent state given by Eq. (18) in the form

$$\begin{aligned} |\nu,\nu\rangle &= e^{-|\nu|^2} \sum_{n=0}^{\infty} \frac{(\sqrt{2}\nu)^n (a^\dagger_{\rm MNS})^n}{n!} |0\rangle \\ &= |\sqrt{2}\nu\rangle_{\rm MNS}, \end{aligned} \tag{20}$$

where $|\sqrt{2}\nu\rangle_{\rm MNS}$ is a coherent state in the maximally nonseparable mode of Eq. (1). Of course, there are three other nonseparable modes orthogonal to this one, in which photons are created by the operators $(a^\dagger_{VV} - a^\dagger_{HH})/\sqrt{2}$, $(a^\dagger_{HV} + a^\dagger_{VH})/\sqrt{2}$, and $(a^\dagger_{HV} - a^\dagger_{VH})/\sqrt{2}$. Since these modes are in the vacuum state, they were omitted in the above equation.

B. Single-photon Fock state

Let us now assume the input mode prepared in a single-photon state $a^\dagger_{\rm MNS}|0\rangle = \frac{1}{\sqrt{2}}(|1,0\rangle + |0,1\rangle)$. As we already mentioned, this state is clearly entangled. In this regime, the intensity measurements translate to photocounts associated with the detection probabilities in each output port of the measurement device. These probabilities violate the CHSH inequality, and one is left in the traditional framework of Bell's experiments, in this case for the spin-orbit degrees of freedom of single photons [4]. The detection probabilities are proportional to the mean values of the intensity operator in each output port, and they are easily calculated to be the same as the normalized intensity values shown in Eq. (17), obviously giving the same value $S = 2\sqrt{2}$.

C. Statistical mixture

It is interesting to investigate the spin-orbit separability for a statistical mixture of two coherent states such as

$$\rho = \tfrac{1}{2}(|\nu,0\rangle\langle\nu,0| + |0,\nu\rangle\langle 0,\nu|), \tag{21}$$

where, as before, the first and second slots in the kets correspond to the HH and VV mode, respectively. Such a statistical mixture can model two independent lasers (random relative phase) prepared in modes HH and VV, and combined in a beam splitter. The results for the intensity calculations of Eq. (16) are shown below:

$$\begin{aligned} I_1 = I_2 = \tfrac{1}{2}(\cos^2\alpha\cos^2\beta + \sin^2\alpha\sin^2\beta), \\ I_3 = I_4 = \tfrac{1}{2}(\sin^2\alpha\cos^2\beta + \cos^2\alpha\sin^2\beta). \end{aligned} \tag{22}$$

For the bases used in the experiment, they give $S = \sqrt{2}$ which does not violate the separability inequality. This shows that optical coherence plays an important role, so no violation would be obtained if the modes were incoherently combined.

V. CONCLUSION

We have proposed an inequality criterion, as a sufficient condition, for the spin-orbit nonseparability of a classical laser beam. The notion of separable and nonseparable spin-orbit modes in classical optics builds a useful analogy with entangled quantum states, allowing the study of some of their important mathematical properties. This analogy has already been successfully exploited to investigate the topological nature of the phase evolution of an entangled state under local unitary operations [13]. Many quantum computing tasks require entanglement but do not need nonlocality, so using different degrees of freedom of single particles can be useful. This is the type of entanglement whose properties can be studied in the classical optical regime allowing one to replace time-consuming measurements based on photocount by the much more efficient measurement of photocurrents.

Although helpful, the notion of mode nonseparability must not be confused with genuine quantum entanglement. In order to avoid this confusion, we have included a detailed quantum optical description of the experiment in which different quantum states were considered. In the intense-beam regime, the comparison between the results obtained for the tensor product of two coherent states and those obtained for a statistical mixture reveals the important role played by optical coherence. In the single-photon regime, we recover the traditional scenario of the CHSH inequality in which violation implies entanglement.

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[1] D. N. Matsukevich, P. Maunz, D. L. Moehring, S. Olmschenk, and C. Monroe, Phys. Rev. Lett. **100**, 150404 (2008), and references therein.
[2] G. Kirchmair *et al.*, Nature (London) **460**, 494 (2009), and references therein.
[3] Y. Hasegawa *et al.*, Nature (London) **425**, 45 (2003).
[4] L. Chen and W. She, J. Opt. Soc. Am. B **27**, A7 (2010).
[5] C. V. S. Borges, M. Hor-Meyll, J. A. O. Huguenin, and A. Z. Khoury, in Proceedings of the Quantum Optics IV International Conference, 2008 (unpublished) [http://www.fisica.ufmg.br/qo4]; in Proceedings of the II Quantum Information School and Workshop, 2009 (unpublished) [http://www.paratyquantum.info].
[6] J. Fu, Z. Si, S. Tang, and J. Deng, Phys. Rev. A **70**, 042313 (2004).
[7] M. A. Goldin, D. Francisco, and S. Ledesma, J. Opt. Soc. Am. B **27**, 779 (2010).
[8] M. F. Santos, P. Milman, A. Z. Khoury, and P. H. S. Ribeiro, Phys. Rev. A **64**, 023804 (2001).
[9] D. P. Caetano, P. H. S. Ribeiro, J. T. C. Pardal, and A. Z. Khoury, Phys. Rev. A **68**, 023805 (2003).
[10] M. Hor-Meyll, A. Auyuanet, C. V. S. Borges, A. Aragão, J. A. O. Huguenin, A. Z. Khoury, and L. Davidovich, Phys. Rev. A **80**, 042327 (2009).
[11] C. E. R. Souza, C. V. S. Borges, A. Z. Khoury, J. A. O. Huguenin, L. Aolita, and S. P. Walborn, Phys. Rev. A **77**, 032345 (2008).
[12] C. E. R. Souza and A. Z. Khoury, Opt. Express **18**, 9207 (2010).
[13] C. E. R. Souza, J. A. O. Huguenin, P. Milman, and A. Z. Khoury, Phys. Rev. Lett. **99**, 160401 (2007).
[14] A. Yariv, *Quantum Electronics* (Wiley, New York, 1989).
[15] A. A. Ishaaya, L. T. Vuong, T. D. Grow, and A. L. Gaeta, Opt. Lett. **33**, 13 (2008).
[16] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
[17] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969).
[18] N. R. Heckenberg, R. McDuff, C. P. Smith, and A. G. White, Opt. Lett. **17**, 221 (1992).
[19] J. A. O. Huguenin, B. Coutinho dos Santos, P. A. M. dos Santos, and A. Z. Khoury, J. Opt. Soc. Am. A **20**, 1883 (2003).
[20] N. K. Langford, R. B. Dalton, M. D. Harvey, J. L. OBrien, G. J. Pryde, A. Gilchrist, S. D. Bartlett, and A. G. White, Phys. Rev. Lett. **93**, 053601 (2004).
[21] H. Sasada and M. Okamoto, Phys. Rev. A **68**, 012323 (2003).