Total reflection, frequency, and velocity tuning in optical pulse collision in nonlinear dispersive media

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The total internal reflection of a weak signal pulse from a powerful reference pulse at another frequency in a dispersive nonlinear medium is demonstrated. As a result of the two-wave collision, the signal pulse changes its frequency and group velocity, and time delay occurs. We develop both a wave and a pulse-tracing theory of the total reflection from the moving inhomogeneity induced by the optical pulse. An analytical expression for the critical value of group-velocity mismatch as a function of pump intensity and the group-velocity dispersion are obtained. The possibility of signal pulse reflection from a bright soliton in a cubic medium is considered.

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Control of light by light in a nonlinear media is of particular interest in nonlinear optics and photonics [1–3]. In this paper we investigate the effect of optical pulse time delay and frequency tuning as a result of a two-frequency interaction in a cubic medium. This phenomenon is the temporal analog to nonlinear total reflection of a wave beam intercrossing the reference beam at small angle less than the critical value [4–9].

The phenomenon of pulse reflection is as follows: A signal pulse propagating in a nonlinear crystal meets with a high-power pump pulse at a different frequency. Due to the nonlinear cross-phase modulation the pump pulse creates an effective inhomogeneity of the refractive index and the signal pulse can be totally reflected from such induced inhomogeneity and can propagate with another velocity. The elaborated theory predicts that such an effect of total internal reflection of the signal pulse from the moving reference pulse occurs if the group-velocity dispersion (GVD) is anomalous at the signal frequency and the nonlinearity is defocusing (or normal dispersion and focusing nonlinearity) and if the initial group-velocity mismatch is less than the critical value defined by the pump peak intensity.

As a result the signal frequency changes and group-velocity mismatch reverses its sign. Due to the frequency tuning the signal velocity decreases: It cannot overtake the main pulse and is delayed compared with the linear medium. Interestingly, beam reflection takes place in a medium with a defocusing nonlinearity only. The described effect of the signal frequency tuning is very similar to the well-known phenomenon of the frequency change as a result of the reflection from a moving inhomogeneity (e.g., rapidly moving electron clouds and ionized and plasma regions [10–15]) or inhomogeneities, induced in the quick-response nonlinear media either by itself [16] or by a co- or counterpropagating high-power pump pulse [17,18].

The remarkable fact is that the total reflection of pulses considered here occurs not in two- or three-dimensional space but in (1+1)-dimensional space-time domain. Note that the effective repulsion can be realized if the group-velocity mismatch (GVM) is small enough. However, in the opposite case of a large GVM when the pulse reflection does not occur a weak interaction also allows time delay control, which was shown previously [19–21]. As suitable media for the observation of such an effect of the total reflection, nonlinear optical fibers or photonic crystals can be considered.

Consider the dynamics of a nonlinear interaction of a high-power pump pulse at a frequency ω_1 with a weak signal pulse at a frequency ω_2, which overtakes the reference pulse. The pulse electric field is represented as $E_j(z,t) = 1/2\{A_j(z,t)\exp[i(\omega_j t - k_j z)] + \text{c.c.}\}$. For analysis the following set of equations for slowly varying envelopes is used:

$$\begin{gathered}\frac{\partial A_1}{\partial z} + iD_1\frac{\partial^2 A_1}{\partial \tau^2} = F(A_1),\\ \frac{\partial A_2}{\partial z} + \nu_2\frac{\partial A_2}{\partial \tau} + iD_2\frac{\partial^2 A_2}{\partial \tau^2} = ik_2 n_{\mathrm{nl}}(A_1)A_2.\end{gathered} \tag{1}$$

Here $\nu_2 = 1/u_2 - 1/u_1$ is the group-velocity mismatch, $u_j = (\partial k_j/\partial\omega)^{-1}$ is the group velocity ($j = 1,2$), $\tau = t - z/u_1$ is the retarded time, $D_j = -\frac{1}{2}\frac{\partial^2 k_j}{\partial\omega^2}$ is the group-velocity dispersion coefficient, $F(A_1)$ is a nonlinear term describing self-action of the reference pulse, $k_j = n_{j0}\omega_j/c$ is the wave number, n_{20} is the linear refractive index at the frequency ω_2, and $n_{\mathrm{nl}} = -\Delta n_2/n_{20}$ is the inhomogeneity of the refractive index induced by the reference pulse at the signal frequency.

If the signal intensity is smaller than the pump intensity, the nonlinear terms in Eq. (1) describing the signal self-action and cross-action of the signal pulse on the reference pulse can be neglected. In a cubic medium one has $F(A_1) = -ik_{10}n_2|A_1|^2A_1$, where n_2 is the nonlinearity coefficient (positive for a focusing nonlinearity and negative for the defocusing case), and the nonlinear term is $n_{\mathrm{nl}}(A_1) = -2n_2|A_1|^2/n_{20}$.

Let the high-power reference pulse propagates along the z axis, $A_1(z=0) = E_1\exp(-\tau^2/T_1^2)$, and let the weak signal pulse with amplitude $E_2 \ll E_1$ has a time delay θ at the entrance of the medium, $A_2(z=0) = E_2\exp[-(\tau-\theta)^2/T_2^2]$. In order to observe pulses interaction it is necessary that $u_2 > u_1$, i.e., $\nu_2 < 0$, and $-\theta/\nu_2 < L$, where L is the medium length. One more condition can be imposed: The reference pulse should not spread significantly at the distance necessary for the signal pulse to overtake the reference one, i.e., $\theta/|\nu_2| < T_1^2/2|D_1|$. By using a temporal analog of the ray optics approximation the signal pulse envelope can be represented as $A_2 = B_2\exp(iS_2)$ and substituting this expression into the second equation of the set (1) we obtain the eikonal equation

$$\frac{\partial S_2}{\partial z} + \nu_2\frac{\partial S_2}{\partial \tau} - D_2\left(\frac{\partial S_2}{\partial \tau}\right)^2 = k_2 n_{\mathrm{nl}}(\tau). \tag{2}$$

The solution of Eq. (2) can be written as

$$S_2 = qz + S_0(\tau). \tag{3}$$

Then substituting (3) into (2) we obtain the following algebraic equation for the variable $\partial S_0/\partial\tau$:

$$D_2\left(\frac{\partial S_0}{\partial\tau}\right)^2 - \nu_2\frac{\partial S_0}{\partial\tau} + [k_2 n_{\mathrm{nl}}(\tau) - q] = 0,$$

which has the solution

$$\frac{\partial S_0}{\partial\tau} = \frac{\nu_2}{2D_2} \pm \sqrt{\left(\frac{\nu_2}{2D_2}\right)^2 - \frac{[k_2 n_{\mathrm{nl}}(\tau) - q]}{D_2}}. \qquad (4)$$

Using (4) in (3), we can find the general solution of the eikonal Eq. (2) in the form

$$S_2 = qz - \frac{\nu_2}{2D_2}\tau \pm \int\sqrt{\left(\frac{\nu_2}{2D_2}\right)^2 - \frac{[k_2 n_{\mathrm{nl}}(\tau) - q]}{D_2}}d\tau. \qquad (5)$$

Differentiating expression (5) by the parameter q, we obtain the equation for the signal pulse trajectory in the space-time domain:

$$z = \mp\frac{1}{2D_2}\int\frac{d\tau_p}{\sqrt{(\nu_2/2D_2)^2 - [k_2 n_{\mathrm{nl}}(\tau_p) - q]/D_2}},$$

or in differential form:

$$\frac{d\tau_p}{dz} = \pm\sqrt{\left(\nu_2^2 + 4qD_2\right) - 4D_2 k_2 n_{\mathrm{nl}}(\tau_p)}.$$

The subscript p in variable τ_p is used to discriminate between the retarded time τ and the trajectory coordinate τ_p. The parameter q can be defined from the boundary condition

$$\left.\frac{\partial S_2}{\partial\tau}\right|_{z=0} = 0.$$

Then we get $D_2(\frac{\partial S_2}{\partial\tau})^2|_{z=0} - \nu_2(\frac{\partial S_2}{\partial\tau})|_{z=0} + k_2 n_{\mathrm{nl}}(\theta) = q$ and, consequently, $q = k_2 n_{\mathrm{nl}}(\theta)$. So the final form of the trajectory equation is the following:

$$\frac{d\tau_p}{dz} = \pm\sqrt{\nu_2^2 - 4D_2 k_2[n_{\mathrm{nl}}(\tau_p) - n_{\mathrm{nl}}(\theta)]}. \qquad (6)$$

The last expression can be interpreted as an analog of Snell's law for pulses. It is obvious from the analysis of the radicand in the Eq. (6) that total reflection occurs if the following conditions are met:

$$\begin{gathered} 4D_2 k_2[n_{\mathrm{nl}}(\tau_p) - n_{\mathrm{nl}}(\theta)] > 0, \\ \nu_2^2 \leqslant \max\{4D_2 k_2[n_{\mathrm{nl}}(\tau_p) - n_{\mathrm{nl}}(\theta)]\}. \end{gathered} \qquad (7)$$

The first of the conditions (7) gives the correspondence between the signs of dispersion and nonlinearity necessary for the realization of the pulse reflection. For a cubic media one gets $-D_2 n_2(|A_1|^2 - |A_1(\theta)|^2) > 0$; thus, if the initial time separation of the pulses is so large that $A_1(\theta) \approx 0$, it can be simplified and can be written as $n_2 D_2 < 0$. So, if $D_2 > 0$ (i.e., the GVD is anomalous at the frequency ω_2), the nonlinearity must be defocusing; if $D_2 < 0$, nonlinearity must be focusing. Note that beam reflection takes place in the case of defocusing nonlinearity only.

As in the case of optical beam parametric reflection [4,7] one can use a cascaded quadratic nonlinearity, the sign of which can be controlled by the sign of the respective wave vector mismatch.

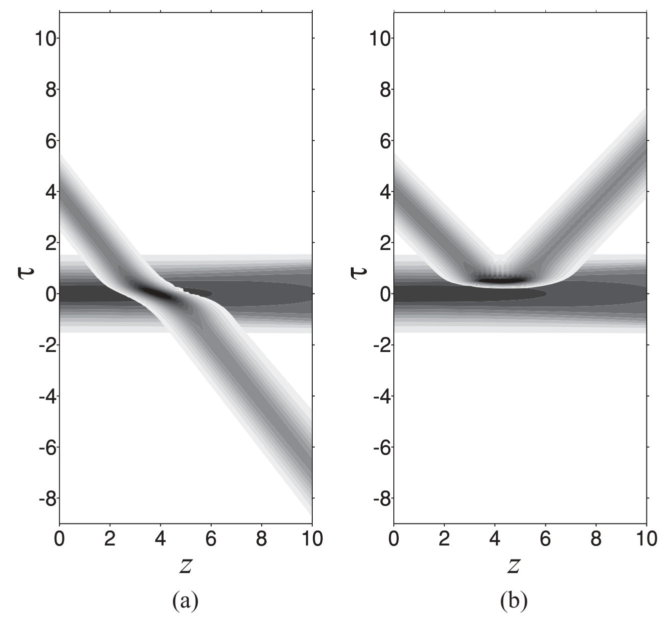

FIG. 1. Intensity distribution on the plane (τ, z) for the two-pulse interaction (numerical simulation of wave equations): (a) signal pulse delay as a result of the collision with the reference pump when the GVM exceeds the critical value, $|\nu_2| > \nu_{\mathrm{cr}}$; (b) the total reflection of the signal pulse from the reference pulse when GVM is small, $|\nu_2| < \nu_{\mathrm{cr}}$. All quantities are plotted in arbitrary dimensionless units.

The second condition of (7) is imposed on the value of group-velocity mismatch, $|\nu_2| < \nu_{\mathrm{cr}}$. The critical value of the mismatch plays the same role as the critical angle of total internal reflection of the beams: If the group-velocity mismatch exceeds the critical value, the signal pulse overtakes the reference pulse without reflection. Total reflection takes place at values of the GVM less than the critical one, which depends on the reference pulse peak intensity, the nonlinearity coefficient, and the GVD value:

$$|\nu_{\mathrm{cr}}| = (8k_2|A_{1\,\max}|^2|n_2 D_2|/n_{20})^{1/2}. \qquad (8)$$

The theoretical results presented here are confirmed by the data from the numerical simulation of Eq. (1) for slowly varying envelopes (see Fig. 1). We emphasize that in the case of a large group-velocity mismatch total reflection does not occur but the pulse trajectory is also modified in the interaction region, and that results in the appearance of a time delay of the signal pulse. A similar approach was used for group velocity control in a quadratic medium [19–21].

As a result of the reflection (nonlinear interaction of pulses) the signal pulse instantaneous frequency $\Omega(z) = \frac{\partial S_0}{\partial\tau}$ changes according to Eq. (4) as

$$\Omega(z) = \frac{\nu_2}{2D_2} \pm \sqrt{\left(\frac{\nu_2}{2D_2}\right)^2 + \frac{k_2[n_{\mathrm{nl}}(\tau_p) - n_{\mathrm{nl}}(\theta)]}{D_2}}. \qquad (9)$$

If the initial time separation of the two pulses is rather large and $n_{\mathrm{nl}}(\theta) \approx 0$ one can obtain a simple expression for the frequency shift from the expression (9):

$$\Delta\Omega = \nu_2/D_2. \qquad (10)$$

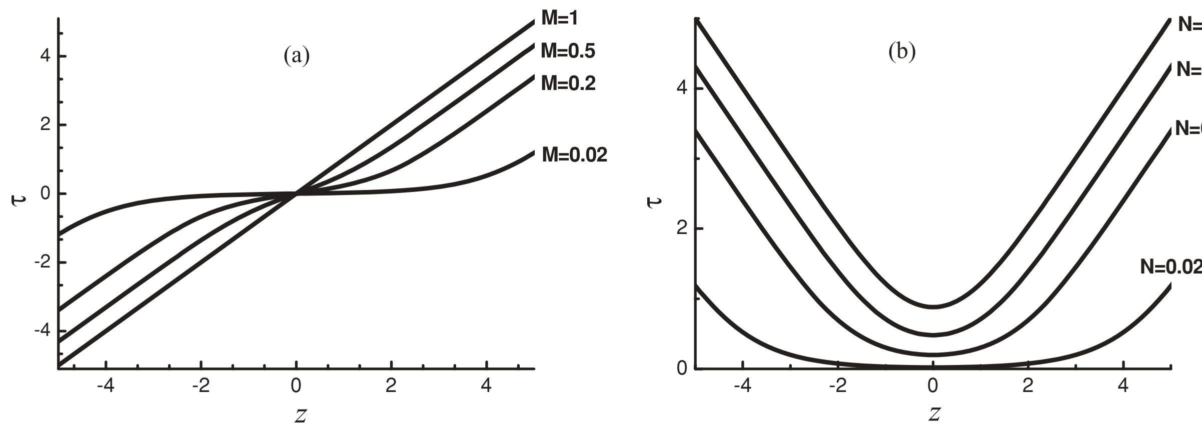

FIG. 2. The trajectories of the signal pulse colliding with the bright soliton in the Kerr medium for (a) large GVM, $|\nu_2| > \nu_{cr}$, and (b) small GVM, $|\nu_2| < \nu_{cr}$. All quantities are plotted in arbitrary dimensionless units.

Using Eqs. (8) and (10) one can estimate the maximum value of the frequency shift at a given pump intensity:

$$|\Omega_{\max}| = |\nu_{cr}|/D_2 = (8k_2|A_{1\max}|^2|n_2/n_{20}D_2|)^{1/2}. \quad (11)$$

According to Eq. (11), the maximal frequency shift value is proportional to the pump peak amplitude or proportional to the square root of the induced inhomogeneity value, $|\Omega_{\max}| \sim \sqrt{-\Delta n_{2\max}}$. Similar results can be obtained also for a cascaded quadratic nonlinearity [22].

Based on the obtained conditions (7) one can use several approaches for observing the pulse reflection effect. In the first method a reference frequency must be chosen in such a way that the GVD and consequently the dispersive spreading of the pulse are minimal. The signal frequency is selected according to the nonlinearity sign as follows from the first condition (7). The second approach is based on pulse reflection from a bright temporal soliton that retains its amplitude profile in a dispersive medium. Use of the soliton pulse as a reference wave deserves separate consideration.

Let nonlinearity be defocusing and let interacting pulses have sufficient frequency separation that the reference pulse spectrum corresponds to the region of normal dispersion and the signal pulse spectrum to the anomalous dispersion region, correspondingly. Thus one can generate a bright soliton at the reference frequency and realize pulse reflection from the soliton. Another variant is possible when nonlinearity is focusing, the reference GVD is anomalous, and the signal GVD is normal.

Pulse reflection occurs if the soliton amplitude is higher than the critical value defined by the value of the group-velocity mismatch:

$$A_{sol} > |\nu|\sqrt{n_{20}/8k_2\,|n_2 D_2|}. \quad (12)$$

Also, the signal pulse amplitude should be significantly smaller than the soliton amplitude.

Note that in the case of a pulse collision with an optical soliton in a Kerr medium when $A_1 = A_{10}\,\text{sech}(\tau/T)$ and $n_{nl} = n_{nl0}\,\text{sech}^2(\tau/T)$ Eq. (6) for the signal pulse trajectory has an exact analytical solution. One gets $\sinh(\tau/T) = M\sinh(\nu_2 z/T)$, $M = \sqrt{1-\nu_{cr}^2/\nu_2^2}$ for $|\nu_2| > \nu_{cr}$ and $\sinh(\tau/T) = N\cosh(\nu_2 z/T)$, $N = \sqrt{\nu_{cr}^2/\nu_2^2 - 1}$ for $|\nu_2| < \nu_{cr}$. In linear media with $n_{nl} = 0$, $\nu_{cr} = 0$, and $M = 1$, the pulse trajectory is the straight line: $\tau = \nu_2 z$. The signal pulse trajectories for different parameters are shown in Fig. 2.

In summary, the effect of total reflection of a weak signal pulse from a high-power reference pulse with another frequency is demonstrated in a dispersive Kerr medium. It is shown that as a result of such a binary collision, a signal pulse frequency shift occurs, the propagation velocity changes, and a time delay takes place. The conditions for signal pulse reflection from a moving inhomogeneity induced by a pump pulse in a nonlinear medium are found. The analytical expression for the reflected wave frequency shift is obtained. The possibility of pulse reflection from bright solitons is considered. We believe that the described phenomenon allows realizing ultrafast all-optical switching and efficient control of light pulse frequency and velocity.

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