Polychromatic quadripartite entanglement from concurrent four-wave mixing in a three-level atomic system

Hua-tang Tan and Gao-xiang Li[*]
Department of Physics, Huazhong Normal University, 430079 Wuhan, China
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In this paper, we investigate the generation of polychromatic quadripartite entanglement of continuous variables from a three-level Λ-type atomic system inside an optical quadruply resonant cavity. The atoms are driven by external lasers and simultaneously coupled to four cavity modes by means of multiply concurrent four-wave mixing interactions. The general master equation of the cavity field is derived explicitly. By solving the Gaussian-type master equation and using the negative-partial-transpose criterion for bipartite entanglement, we show that the genuine quadripartite entanglement of the field can be generated over a wide range of parameters. The entanglement properties of the four-mode field are discussed in detail. We find that the optimal quadripartite entanglement can be obtained when the cavity modes are tuned to be resonant with the Rabi sidebands of the driven atoms.

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I. INTRODUCTION

Entanglement, one of the most striking features of quantum mechanics, has become an essential resource for quantum information processing. Multipartite optical entanglement for continuous variables (CVs), which embodies quantum correlations in phase and amplitude quadratures of light fields, has been proven to be a key ingredient for constructing multiparty quantum communication networks [1–4]. For instance, CV multipartite entanglement has recently been produced and used for realizing CV quantum error correction [5], one-way quantum computations [6], and universal phase gating [7]. Also, it has recently been shown that CV quadripartite entanglement of the optical field is critical for building quantum teamwork [8] and quantum dense coding communication networks [9]. By injecting individual squeezed optical beams into combined beam splitters, experiments have realized multiple entangled beams with the same frequency [2,10]. Besides, a lot of work has also been concentrated on the preparation of multifrequency entangled beams, by utilizing nondegenerate parametrical downconversion (NPDC) or cascaded second-order optical nonlinearities inside an optical cavity [11–16]. This kind of multicolor entanglement is useful in the area of quantum communication where physical systems, such as trapped ions or atomic ensembles, with different resonances at the nodes of a quantum network need to be connected by entangled photons of different frequencies [17,18]. Experimentally, the three-color pump-signal-idler entanglement from a single intracavity NPDC has been realized recently [19]. In addition, the generation of CV quadripartite entanglement of optical fields with different frequencies via concurrent NPDC processes has been investigated very recently [20].

In contrast, the preparation of CV entangled light via coherent atomic systems has also been attracted a lot of attention. Entangled photons from atomic systems have potential applications in quantum memory [21] and long-distance quantum communication [22], since the low frequency and narrow linewidth of the light can ensure efficient couplings between photons and atomic memories in a quantum network [23,24]. Experimentally, low-frequency two-color entangled light from nondegenerate four-wave mixing (NFWM) in a three-level Λ atomic system has been generated [24–29]. Theoretically, a number of proposals for producing CV bichromatic entanglement via NFWM in atomic systems have also been put forward in Refs. [30–36]. For example, it shows that two-color highly entangled light beams can be generated via NFWM from N intracavity three-level Λ atomic system close to electromagnetically induced transparency [37].

In this paper, in view of the usefulness of polychromatic multipartite entanglement and based on the experimental achievements in producing entangled light from coherently driven atomic systems [24–29], we investigate the generation of CV quadripartite entanglement of optical fields with different frequencies in a three-level Λ atomic system inside a cavity. Atoms are driven by external coherent lasers and simultaneously coupled to the four cavity modes. The general master equation of the cavity field is obtained explicitly. With the help of the negative-partial-transpose criterion for bipartite entanglement, we show that, through multiply concurrent NFWM interactions between the cavity modes, genuine quadripartite entanglement can be achieved inside and outside the cavity over a wide range of parameters. It is found that the genuine quadripartite entanglement becomes maximum when the cavity fields are tuned to be resonant with the Rabi sidebands of the laser-driven atoms. We also find that quadripartite entanglement is absent for laser-atom detuning in the vicinity of the Rabi frequencies of the lasers. This paper is arranged as follows: In Sec. II, the model is introduced and the general master equation of the cavity field is derived explicitly. In the following section, III, correlation matrices for states of the intracavity and output fields are obtained. In Sec. V, the properties of the quadripartite entanglement are discussed in detail. Finally, in Sec. VI we give our summary.

II. MODEL AND MASTER EQUATION

We consider an ensemble of independent three-level Λ atoms inside a four-mode resonant optical cavity. As shown

[*]gaox@phy.ccnu.edu.cn

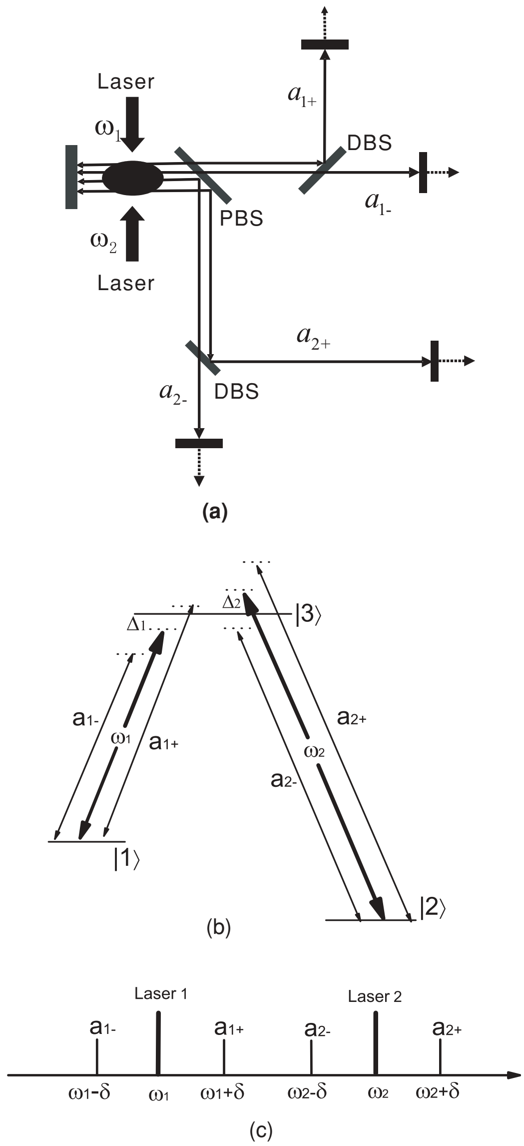

FIG. 1. (a) Schematic diagram of the quadruply resonant cavity in which the atomic ensemble is trapped. PBS, polarization beam splitter; DBS, dichroic beam splitter. (b) Level configuration of the three-level Λ atoms and the atom-field couplings. (c) Frequencies of the pumps and cavity fields.

in Fig. 1, the two lower levels, $|1\rangle$ and $|2\rangle$, of the atoms are coupled to the excited level $|3\rangle$ by two driving lasers of frequencies ω_j ($j = 1,2$), respectively, and at the same time the laser-driven transitions $|j\rangle \leftrightarrow |3\rangle$ are coupled to the cavity modes a_{jx} with frequencies ω_{jx} ($x = +,-$) by means of NFWM interactions. In the rotating frame with respect to the laser frequencies ω_j, the Hamiltonian of the cavity-atom system can be written as

$$V_{\text{int}} = V_{\text{ac}} + V_{\text{al}}, \tag{1}$$

$$V_{\text{ac}} = \sum_{j=1}^{2} \sum_{x\in\{+,-\}} g_{jx} a_{jx} S_{j3} e^{-ix\delta t} + \text{H.c.}, \tag{2}$$

$$V_{\text{al}} = \sum_{j=1}^{2} (-\Delta_j S_{jj}/2 + \Omega_j S_{j3}) + \text{H.c.} \tag{3}$$

where $\Delta_j = \omega_{j3} - \omega_j$, with ω_{j3} being the transition frequencies between level $|3\rangle$ and level $|j\rangle$ and $\delta = \pm(\omega_{j\pm} - \omega_j) > 0$. In the preceding Hamiltonian, S_{j3} and S_{jj} are, respectively, the atomic transition operators and population operators, g_{jx} denote the atom-cavity couplings, and Ω_j are real Rabi frequencies of the driving lasers.

By taking into account the damping of the atoms and the cavity modes in vacuum, the density operator ρ_s of the atom-field system is governed by the following master equation:

$$\frac{d}{dt}\rho_s = -i[V_{\text{int}},\rho_s] + \mathcal{L}_f\rho_s + \mathcal{L}_a\rho_s, \tag{4}$$

$$\mathcal{L}_f\rho_s = \sum_{j,x} \kappa_{jx}[a_{jx},\rho_s a_{jx}^\dagger] + \text{H.c.}, \tag{5}$$

$$\mathcal{L}_a\rho_s = \sum_{j} \gamma_j[S_{j3},\rho_s S_{3j}] + \text{H.c.}, \tag{6}$$

where κ_{jx} and γ_j are the damping rates of the cavity modes and the atoms, respectively. Using the master equation (4), the reduced density operator ρ of the cavity field can be obtained by tracing out the atomic variables. For a high-quality cavity ($\kappa_{jx} \ll \gamma_j$), the atoms approach their steady states on a time scale much faster than the cavity field. In the times after which the atomic transients have died away, the atomic ensemble can be described by a stationary process and treated as an external reservoir for the cavity field [32,36]. By taking the Markovian approximation for the atomic reservoir, the reduced master equation of the cavity field can be readily derived as [38]

$$\begin{aligned}\frac{d}{dt}\rho = \sum_{j,x} &\{\mathcal{A}_{jj}^{xx}[a_{jx}^\dagger,\rho a_{jx}] + (\mathcal{B}_{jj}^{xx} + \kappa_{jx})[a_{jx}\rho, a_{jx}^\dagger]\} \\ &+ \sum_{j}\sum_{x'\neq x} \{\mathcal{C}_{jj}^{xx'}[a_{jx}^\dagger, a_{jx'}^\dagger\rho] + \mathcal{D}_{jj}^{xx'}[\rho a_{jx}^\dagger, a_{jx'}^\dagger]\} \\ &+ \sum_{j'\neq j}\sum_{x'\neq x} \{\tilde{\mathcal{C}}_{jj'}^{xx'}[a_{jx}^\dagger, a_{j'x'}^\dagger\rho] + \tilde{\mathcal{D}}_{jj'}^{xx'}[\rho a_{jx}^\dagger, a_{j'x'}^\dagger]\} \\ &+ \sum_{j'\neq j}\sum_{x} \{\bar{\mathcal{C}}_{jj'}^{xx}[a_{jx}^\dagger, \rho a_{j'x}] + \bar{\mathcal{D}}_{jj'}^{xx}[a_{jx}\rho, a_{j'x}^\dagger]\} + \text{H.c.},\end{aligned} \tag{7}$$

where

$$\begin{aligned}\mathcal{A}_{jj}^{xx} &= \bar{g}_{jx}^2 \int_0^\infty d\tau e^{-ix\delta\tau}\langle\Delta S_{3j}(0)\Delta S_{j3}(\tau)\rangle, \\ \mathcal{B}_{jj}^{xx} &= \bar{g}_{jx}^2 \int_0^\infty d\tau e^{-ix\delta\tau}\langle\Delta S_{j3}(\tau)\Delta S_{3j}(0)\rangle, \\ \mathcal{C}_{jj}^{xx'} &= -\bar{g}_{jx}\bar{g}_{jx'} \int_0^\infty d\tau e^{-ix\delta\tau}\langle\Delta S_{j3}(\tau)\Delta S_{j3}(0)\rangle, \\ \mathcal{D}_{jj}^{xx'} &= -\bar{g}_{jx}\bar{g}_{jx'} \int_0^\infty d\tau e^{ix\delta\tau}\langle\Delta S_{j3}(0)\Delta S_{j3}(\tau)\rangle, \\ \tilde{\mathcal{C}}_{jj'}^{xx'} &= -\bar{g}_{jx}\bar{g}_{j'x'} \int_0^\infty d\tau e^{-ix\delta\tau}\langle\Delta S_{j3}(\tau)\Delta S_{j'3}(0)\rangle, \\ \tilde{\mathcal{D}}_{jj'}^{xx'} &= -\bar{g}_{jx}\bar{g}_{j'x'} \int_0^\infty d\tau e^{ix\delta\tau}\langle\Delta S_{j3}(0)\Delta S_{j'3}(\tau)\rangle, \\ \bar{\mathcal{C}}_{jj'}^{xx} &= \bar{g}_{jx}\bar{g}_{j'x} \int_0^\infty d\tau e^{-ix\delta\tau}\langle\Delta S_{3j'}(0)\Delta S_{j3}(\tau)\rangle, \\ \bar{\mathcal{D}}_{jj'}^{xx} &= \bar{g}_{jx}\bar{g}_{j'x} \int_0^\infty d\tau e^{-ix\delta\tau}\langle\Delta S_{j'3}(\tau)\Delta S_{3j}(0)\rangle.\end{aligned} \tag{8}$$

Here, $\bar{g}_{jx} = \sqrt{N}g_{jx}$, with N being the atomic number in the ensemble. The atomic noise operators $\Delta S_{kk'} = S_{kk'} - \langle S_{kk'}\rangle (k = 1,2,3)$, where $\Delta S_{kk'}(0)$ characterize the noise operators $\Delta S_{kk'}$ evaluated at a time sufficient for the steady atomic states to occur. From Eq. (8), we see that the coefficients in the master equation (7) depend only on the stationary two-time correlations of the atomic variables, as the consequence of the Markovian treatment of the external atomic reservoir. The coefficients A_{jj}^{xx} and B_{jj}^{xx} in Eq. (7) determine the gain and absorption of the cavity modes, respectively. The terms related to the coefficients $C_{jj}^{xx'}$ and $D_{jj}^{xx'}$ in Eq. (7) originate from the NFWM processes in which the atom absorbs two identical pumping photons with frequencies ω_j and emits two sideband photons in the cavity modes a_{j+} and a_{j-} via the two-level transitions $|j\rangle \leftrightarrow |3\rangle$; that is,

$$|j\rangle \xrightarrow{a_{p_j}} |3\rangle \xrightarrow{a_{j\pm}^\dagger} |j\rangle \xrightarrow{a_{p_j}} |3\rangle \xrightarrow{a_{j\mp}^\dagger} |j\rangle, \tag{9}$$

where the annihilation operators a_{p_j} and creation operators $a_{j\pm}^\dagger$, respectively, characterize the atomic absorption of a pumping photon and emission of the corresponding sideband photons via the atomic transitions. This kind of two-level NFWM process may lead to entanglement between the sideband modes a_{j-} and a_{j+} [12,13]. Meanwhile, the atom may also absorb two different pumping photons (with frequencies ω_1 and ω_2, respectively) and radiate two photons into modes a_{2+} and a_{1-} [or a_{1+} and a_{2-}] via transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, such as

$$|1\rangle \xrightarrow{a_{p_1}} |3\rangle \xrightarrow{a_{2+}^\dagger} |2\rangle \xrightarrow{a_{p_2}} |3\rangle \xrightarrow{a_{1-}^\dagger} |1\rangle, \tag{10}$$

which lead to the terms in Eq. (7) proportional to the coefficients $\tilde{C}_{jj'}^{xx'}$ and $\tilde{D}_{jj'}^{xx'}$ and may result in quantum correlations between mode a_{jx} and mode $a_{j'x'} (j \neq j', x \neq x')$ [26–28,37]. In addition, the terms related to the coefficients $\bar{C}_{jj'}^{xx}$ and $\bar{D}_{jj'}^{xx}$ are rooted from the NFWM processes in which the atom absorbs a pumping photon at frequencies ω_j and a sideband photon in mode $a_{j'+}$ (or $a_{j'-}$) and radiates a photon at pumping frequencies $\omega_{j'}$ and another one in mode a_{j+} (or a_{j-}) ($j \neq j'$), such as

$$|1\rangle \xrightarrow{a_{p_1}} |3\rangle \xrightarrow{a_{p_2}^\dagger} |2\rangle \xrightarrow{a_{2-}} |3\rangle \xrightarrow{a_{1-}^\dagger} |1\rangle, \tag{11}$$

which gives rise to linear mixings between cavity modes a_{jx} and $a_{j'x}$ [39]. Therefore, we see that each cavity mode is involved in the three kinds of NFWM processes already described, which can lead to quadripartite entanglement among the cavity modes.

The coefficients in Eq. (8), dependent on the stationary two-time correlation function of the atoms, can be determined with the help of quantum regression theorem [40]. For a high-quality cavity ($\kappa_{jx} \ll \gamma_j$) and weak effective atom-field interactions, the atoms approach their steady states on a time scale much faster than the cavity field. So, in the times after which the atomic transients have died away, we can neglect the effects of the cavity field on the stationary behavior of the atoms (as an external reservoir for the cavity field). Therefore, in the absence of the cavity field, from Eqs. (3) and (6) the equations of motion of the average values $\langle S_{kk'}\rangle$ $(k,k' = 1,2,3)$ are given by

$$\begin{aligned}
\frac{d}{dt}\langle S_{jj}\rangle &= -2\gamma_j\langle S_{11}\rangle - 2\gamma_j\langle S_{22}\rangle - i\Omega_j\langle S_{j3}\rangle \\
&\quad + i\Omega_j\langle S_{3j}\rangle + 2\gamma_j, \\
\frac{d}{dt}\langle S_{j3}\rangle &= -\bar{\gamma}_j\langle S_{j3}\rangle - 2i\Omega_j\langle S_{jj}\rangle - i\Omega_j\langle S_{j'j'}\rangle \\
&\quad - i\Omega_{j'}\langle S_{jj'}\rangle + i\Omega_j, \\
\frac{d}{dt}\langle S_{12}\rangle &= -i\Omega_2\langle S_{13}\rangle + i\Omega_1\bar{S}_{32} - 2i\Omega_2\langle S_{22}\rangle \\
&\quad - i(\Delta_1 - \Delta_2)\langle S_{12}\rangle,
\end{aligned} \tag{12}$$

where $\bar{\gamma}_j = (\gamma_1 + \gamma_2 + i\Delta_j)$ $(j \neq j')$. By introducing a column vector $\mathcal{S}(t) = [S_{13}, S_{31}, S_{23}, S_{32}, S_{11}, S_{22}, S_{12}, S_{21}]^T$, Eqs. (12) can be rewritten in the following compact form:

$$\frac{d}{dt}\langle\mathcal{S}(t)\rangle = \mathcal{L}\langle\mathcal{S}(t)\rangle + \mathcal{B}, \tag{13}$$

where the matrix $\mathcal{L}$ and the vector $\mathcal{B}$ can be easily obtained and are not presented here. Letting $\Delta\mathcal{S}(t) = \mathcal{S}(t) - \langle\mathcal{S}(0)\rangle$, where $\langle\mathcal{S}(0)\rangle = -\mathcal{L}^{-1}\mathcal{B}$, denoting the steady-state average value of $\mathcal{S}(t)$, we have

$$\frac{d}{dt}\langle\Delta\mathcal{S}(t)\rangle = \mathcal{L}\langle\Delta\mathcal{S}(t)\rangle. \tag{14}$$

According to the quantum regression theorem [40], the two-time correlation functions in Eq. (8) are governed by the equation

$$\frac{d}{d\tau}\langle\Delta\mathcal{S}(\tau)\Delta\mathcal{S}(0)^T\rangle = \mathcal{L}\langle\Delta\mathcal{S}(\tau)\Delta\mathcal{S}(0)^T\rangle, \tag{15}$$

with the solution given by

$$\langle\Delta\mathcal{S}(\tau)\Delta\mathcal{S}(0)^T\rangle = e^{\mathcal{L}\tau}\langle\Delta\mathcal{S}(0)\Delta\mathcal{S}(0)^T\rangle. \tag{16}$$

By performing the Fourier transformation on Eq. (16), we have

$$\begin{aligned}
&\int_0^\infty e^{\pm i\delta\tau}\langle\Delta\mathcal{S}(\tau)\Delta\mathcal{S}(0)^T\rangle\, d\tau \\
&\quad = -(\mathcal{L} \pm i\delta I)^{-1}\langle\Delta\mathcal{S}(0)\Delta\mathcal{S}(0)^T\rangle.
\end{aligned} \tag{17}$$

Therefore, the coefficients defined in Eq. (8) can be determined completely from Eq. (17). Since the expressions of these coefficients are quite cumbersome, we calculate them numerically in Sec. IV.

III. CORRELATION MATRIX AND SEPARABILITY CRITERION

The quadripartite entanglement that we are interested in refers to entanglement shared by photons in four different cavity modes (four-mode entanglement). The entanglement characterizes the correlations among the field's momentum and position components, which, respectively, correspond to the phase and amplitude quadratures of the light fields. For a four-mode Gaussian state, its quantum statistical properties are completely determined by the correlation matrix (CM). The entanglement properties of the four-mode Gaussian state can be analyzed by use of the negative-partial-transpose criterion [41,42].

A. Correlation matrix

For initial vacua of the cavity modes, the cavity field will evolve in a Gaussian state since the master equation (7) only contains the quadratic terms of the operators a_{jx} and $a_{jx}^\dagger$. The quantum statistical properties of the Gaussian cavity-field state are completely determined by the CM, which is defined as [43]

$$\sigma_{kk'} = \langle \mu_k \mu_{k'} + \mu_{k'}\mu_k \rangle /2, \tag{18}$$

where $\mu = (X_{1-}, P_{1-}, X_{1+}, P_{1+}, X_{2-}, P_{2-}, X_{2+}, P_{2+})$ and the quadrature operators

$$\begin{aligned} X_{jx} &= (a_{jx}e^{i\varphi_{jx}} + a_{jx}^\dagger e^{-i\varphi_{jx}}),\\ P_{jx} &= -i(a_{jx}e^{i\varphi_{jx}} - a_{jx}^\dagger e^{-i\varphi_{jx}}), \end{aligned} \tag{19}$$

with local phases φ_{jx}. To determine the CM of the cavity field, let us introduce the vector $\psi = (a_{j+}, a_{1+}^\dagger, a_{1-}, a_{1-}^\dagger, a_{2+}, a_{2+}^\dagger, a_{2-}, a_{2-}^\dagger)^T$. From the master equation (7), the equations of motion for the field operators a_{jx} can be found as

$$\frac{d}{dt}\psi = \mathcal{M}\psi + F(t), \tag{20}$$

where

$$\mathcal{M} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{pmatrix}, \tag{21}$$

and

$$\mathcal{M}_{jj'} = \begin{pmatrix} \chi_{jj'}^{++} & 0 & 0 & \chi_{jj'}^{+-} \\ 0 & (\chi_{jj'}^{++})^* & (\chi_{jj'}^{+-})^* & 0 \\ 0 & \chi_{jj'}^{-+} & \chi_{jj'}^{--} & 0 \\ (\chi_{jj'}^{-+})^* & 0 & 0 & (\chi_{jj'}^{--})^* \end{pmatrix},$$

with $\chi_{jj}^{xx} = \mathcal{A}_{jj}^{xx} - \mathcal{B}_{jj}^{xx} - \kappa_j^x$, $\chi_{jj'}^{xx} = \bar{\mathcal{C}}_{jj'}^{xx} - \bar{\mathcal{D}}_{j'j}^{xx}$, $\chi_{jj}^{xx'} = \mathcal{C}_{jj}^{xx'} - \mathcal{D}_{jj}^{xx'}$, and $\chi_{jj'}^{xx'} = \tilde{\mathcal{C}}_{jj'}^{xx'} - \tilde{\mathcal{D}}_{j'j}^{x'x}$ $(j' \neq j, x' \neq x)$. The noise operators in Eq. (20) are defined as $F(t) = [f_{1+}, f_{1+}^\dagger, f_{1-}, f_{1-}^\dagger, f_{2+}, f_{2+}^\dagger, f_{2-}, f_{2-}^\dagger]^T$, and according to the generalized Einstein relation [40], the noise operators satisfy

$$\langle F_j(t) F_{j'}^T(t') \rangle = D_{jj'}\delta(t-t'), \tag{22}$$

in the normal ordering of the field operators a_{jx}. Here, the matrix $\mathcal{D}$ is given by

$$\mathcal{D} = \begin{pmatrix} \mathcal{D}_{11} & \mathcal{D}_{12} \\ \mathcal{D}_{12}^T & \mathcal{D}_{22} \end{pmatrix}, \tag{23}$$

where

$$\mathcal{D}_{jj} = \begin{pmatrix} 0 & \alpha_j & \beta_j & 0 \\ \alpha_j & 0 & 0 & \beta_j^* \\ \beta_j & 0 & 0 & \alpha_j \\ 0 & \beta_j^* & \alpha_j & 0 \end{pmatrix},$$

$$\mathcal{D}_{12} = \begin{pmatrix} 0 & \eta & \mu & 0 \\ \eta^* & 0 & 0 & \mu^* \\ \mu & 0 & 0 & \eta \\ 0 & \mu^* & \eta^* & 0 \end{pmatrix},$$

and $\alpha_j = \mathcal{A}_j^+ + (\mathcal{A}_j^+)^*$, $\beta_j = \mathcal{C}_{jj}^{+-} + \mathcal{C}_{jj}^{-+}$, $\eta = \bar{\mathcal{C}}_{12}^{++} + (\bar{\mathcal{C}}_{21}^{++})^*$, and $\mu = \tilde{\mathcal{C}}_{12}^{+-} + (\tilde{\mathcal{C}}_{21}^{-+})^*$. Defining $\Phi = \langle \psi\psi^T \rangle$, from Eq. (20) we have

$$\frac{d}{dt}\Phi = \mathcal{M}\Phi + \Phi\mathcal{M}^T + \mathcal{D}. \tag{24}$$

For the initial vacuum of the cavity field $[\Phi(t=0)]$, the solution of Eq. (24) can be obtained as

$$\Phi(t) = \int_0^t e^{\mathcal{M}\tau}\mathcal{D}e^{\mathcal{M}^T\tau}. \tag{25}$$

Therefore, from Eq. (25) the average values $\langle a_{jx}^\dagger a_{j'x'}\rangle$ and $\langle a_{jx}a_{j'x'}\rangle$ can be obtained. Then, with the definition in Eqs. (18) and (19), the elements of the intracavity CM can be determined completely; that is,

$$\sigma_{11} = 2\Phi_{12} + \Phi_{11}e^{2i\varphi_{1+}} + \Phi_{22}e^{-2i\varphi_{1+}} + 1. \tag{26}$$

In addition, for the field outside the cavity, the spectrum of the CM $\sigma^o(\omega)$ of the output field can be obtained with the help of the intracavity spectral correlations, which are given by [44]

$$\mathcal{S}(\omega) = (\mathcal{M} + i\omega I)^{-1}\mathcal{D}(\mathcal{M}^T - i\omega I)^{-1}. \tag{27}$$

By defining the output quadrature operators X_{jx}^o and P_{jx}^o of output modes a_{jx}^o (corresponding to intracavity fields a_{jx}) as $X_{jx}^o = (a_{jx}^o e^{i\varphi_{jx}} + a_{jx}^{o\dagger}e^{-i\varphi_{jx}})$, $P_{jx}^o = -i(a_{jx}^o e^{i\varphi_{jx}} - a_{jx}^{o\dagger}e^{-i\varphi_{jx}})$, with the standard input-output relations $a_{jx}^o = \sqrt{2\kappa_{jx}}a_{jx} - a_{jx}^{\text{in}}$ [45], where a_{jx}^{in} denote the vacuum inputs of the cavity modes a_{jx}, the spectral correlations $\sigma_{kk'}^o(\omega)$ of the output field can be simply derived from Eq. (27). For example, we have

$$\begin{aligned} \langle (X_{jx}^o)^2 \rangle(\omega) &= 1 + 2k_{jx}\langle X_{jx}^2\rangle(\omega),\\ \langle X_{jx}^o X_{j'x'}^o\rangle(\omega) &= 2\sqrt{k_{jx}k_{j'x'}}\langle X_{jx}X_{j'x'}\rangle(\omega), \end{aligned} \tag{28}$$

and similarly for the P_{jx}^o quadratures.

B. Criterion for genuine quadripartite entanglement

For a physically allowable four-mode Gaussian cavity-field state with the CM σ defined in Eq. (18), the positivity of the density operator ρ of the cavity field can be equivalently expressed, in terms of the CM σ, as the uncertainty relation [41]

$$\sigma + i\Lambda \geqslant 0, \tag{29}$$

where the symplectic matrix Λ is block diagonal and defined as $\Lambda = \bigoplus_{k=1}^4 \nu_k$, with $\nu_k = \left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)$. The symplectic matrix Λ results from the commutation rules $[X_{jx}, P_{j'x'}] = \delta_{jj'}\delta_{xx'}$. To discuss bipartite entanglement in the four-mode system, we divide the system into subsystem A of n modes and subsystem B of $4-n$ modes. According to the negative-partial-transpose criterion [41,42], there exists the bipartite entanglement between subsystem A and subsystem B if the partially transposed density operator $\tilde{\rho}_A$ (or $\tilde{\rho}_B$) of the field operator ρ with respect to subsystem A (or B) is negative. Accordingly, denoting by $\tilde{\sigma}_A$ the partial transpose on the CM

σ with respect to subsystem A, from Eq. (30), subsystems A and B are entangled when

$$\tilde{\sigma}_A + i\Lambda < 0. \tag{30}$$

Note that the preceding negative-partial-transpose criterion for bipartite entanglement between subsystem A and subsystem B is sufficient and necessary for $n = 1$ and only sufficient for $n > 1$ [42]. Here, the partially transposed CM $\tilde{\sigma}_A$ can be obtained by changing the sign of the n momenta $\{P_{jx}\}$ belonging to subsystem A. Moreover, if Eq. (30) holds for all possible bipartitions of the present four-mode system, which indicates that all possible bipartite entanglements exist in the system, the state of the cavity field is fully inseparable and the genuine quadripartite entanglement among the four cavity modes can be achievable. Therefore, here we can use the negative eigenvalue E_A of matrices $\tilde{\sigma}_A + i\Lambda$ to analyze the properties of the quadripartite entanglement of the system [46]. In addition, it is shown that a smaller negative eigenvalue E_A means a stronger bipartite entanglement between subsystem A and subsystem B [42,46].

IV. RESULTS AND DISCUSSION

With the help of the negative-partial-transpose criterion [Eq. (30)], we can discuss the properties of quadripartite entanglement in the system via numerical calculation. For the sake of simplicity, in the following discussion we assume that the collective cavity-atom couplings $\bar{g}_{jx} = \bar{g} = \sqrt{N}g$, Rabi frequencies of lasers $\Omega_j = \Omega$, damping rates of cavity fields $\kappa_{jx} = \kappa$, and atomic decay rates $\gamma_j = \gamma$. In addition, the parameters are scaled by the atomic decay γ.

A. Dispersive dressed-atom/cavity interactions

For symmetric laser-atom detunings $\Delta_1 = -\Delta_2 = \Delta$, the time evolution of the intracavity bipartite entanglements for different bipartitions and the output entanglement spectra are plotted in Figs. 2(a) and 2(b), respectively. Here the negativities E_{jx} characterize the bipartite entanglements between the cavity field a_{jx} and the subsystem of the remaining cavity fields (1×3 bipartition), and the negativities $E_{jxj'x'}$ describe the bipartite entanglements between the subsystem consisting of cavity fields a_{jx} and $a_{j'x'}$ and the subsystem consisting of the remaining cavity fields (2×2 bipartition). From Fig. 2, we see that the negativities $E_{jx} < 0$ and $E_{jxj'x'} < 0$ for all possible bipartitions, which means that fully inseparable (genuine) quadripartite entanglement can be achieved inside and outside the cavity. As depicted in Fig. 2(a), bipartite entanglements inside the cavity exhibit short-time oscillations, and entanglements reach their steady values in the long-time limit. So stationary genuine quadripartite entanglement can be obtained in this scheme. Figure 2(a) also shows that the negativities $E_{j+} = E_{j'-} (j \neq j')$ and thus the bipartite entanglements between cavity field a_{j+} and the subsystem of the remaining fields are equal to those between the cavity field $a_{j'-}$ and the other fields. From Fig. 2(b), we can see that the output bipartite entanglements become maximum at cavity resonant frequencies ($\omega = 0$) and are stronger than the corresponding intracavity entanglements in the steady-state regime. Additionally, our numerical calculation reveals that the negativities E_{jx} and $E_{jxj'x'}$ (thus the quadripartite entanglement) are independent of the local phases φ_{jx} of cavity fields a_{jx}.

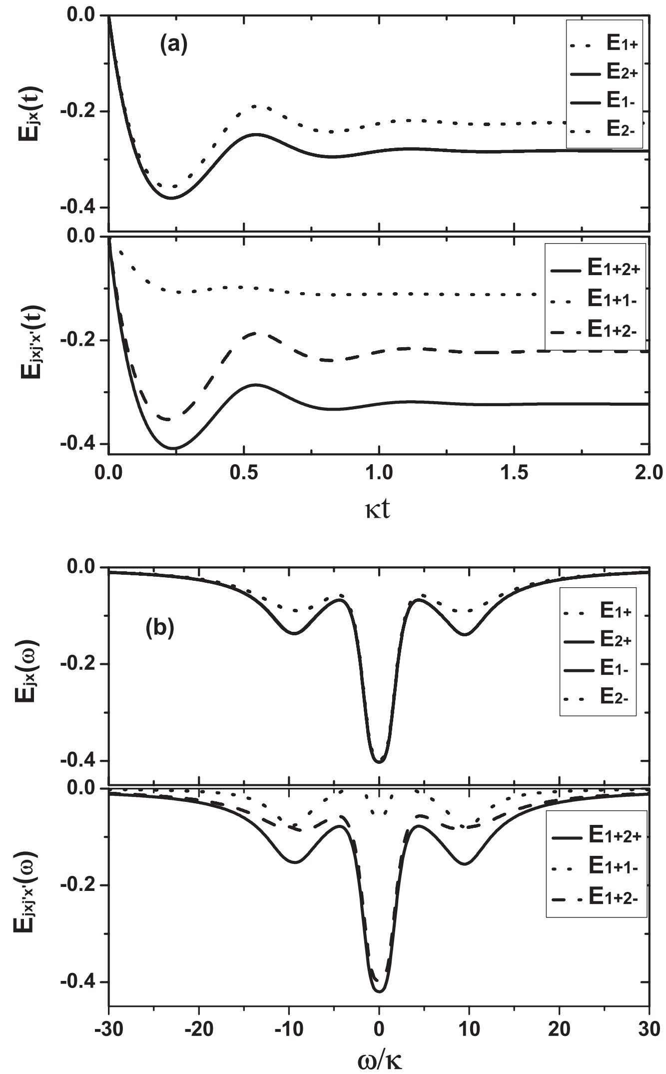

FIG. 2. Time evolution (a) and output spectra (b) of the negative eigenvalues E_{jx} and $E_{jxj'x'}$, which characterize the bipartite entanglements between field a_{jx} and the remaining fields and the bipartite entanglements between the subsystem of fields a_{jx} and $a_{j'x'}$ and the remaining fields, respectively. The parameters are chosen as $\bar{g} = 1$, $\Delta = 45$, $\Omega = 20$, $\delta = 40$, and $\kappa = 0.002$.

Generation of the genuine quadripartite entanglement shown in Fig. 2 can be understood in the dressed-state representation of the laser-driven atoms. The dressed states are the eigenstates of the laser-atom interaction Hamiltonian in Eq. (3), and in the Appendix approximate expressions of the coefficients in Eq. (8) are obtained in the dressed-state representation. Here, for the symmetric coupling case $\Delta_1 = -\Delta_2$, the dressed states can be obtained as

$$\begin{pmatrix} |1_d\rangle \\ |2_d\rangle \\ |3_d\rangle \end{pmatrix} = \begin{pmatrix} \frac{1-\sin\theta}{2} & \frac{1+\sin\theta}{2} & -\frac{\cos\theta}{\sqrt{2}} \\ -\frac{\cos\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} & \sin\theta \\ \frac{1+\sin\theta}{2} & \frac{1-\sin\theta}{2} & \frac{\cos\theta}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} |1\rangle \\ |2\rangle \\ |3\rangle \end{pmatrix}, \tag{31}$$

with the corresponding eigenvalues $\lambda_1 = -\Omega_d$, $\lambda_2 = 0$, and $\lambda_3 = \Omega_d$, where $\sin\theta = \Delta/\Omega_d$, and the generalized Rabi frequency is given by

$$\Omega_d = \sqrt{\Delta^2 + 2\Omega^2}. \tag{32}$$

For the parameters given in Fig. 2, the laser-cavity detuning δ satisfies

$$|\Omega_d \pm \delta| \gg \gamma. \tag{33}$$

Equation (33) means that far-off-resonant couplings between dressed atoms and cavity fields and dispersive interactions are formed between dressed atoms and cavity fields. Under this condition, we can neglect the effects of the spontaneous emission of atoms on the cavity fields, and then it is not difficult to find from Eqs. (A5) and (A6) that the master equation (7) becomes

$$\frac{d}{dt}\rho \approx -i[V_{\rm eff}, \rho] + \mathcal{L}_f \rho, \tag{34}$$

with the effective Hamiltonian

$$\begin{aligned} V_{\rm eff} = {} & \chi_s(a_{1+}^\dagger a_{1+} + a_{1-}^\dagger a_{1-} - a_{2+}^\dagger a_{2+} - a_{2-}^\dagger a_{2-}) \\ & + \chi_p(a_{1+}a_{1-} - a_{2+}a_{2-}) + \chi_m(a_{1+}^\dagger a_{2+} - a_{1-}^\dagger a_{2-}) \\ & + \text{H.c.}, \end{aligned} \tag{35}$$

where

$$\begin{aligned} \chi_s &= -\frac{\bar{g}^2 \sin^3\theta\,\Omega_d}{2(\Omega_d^2 - \delta^2)}\left(P_{22}^d - P_{11}^d\right), \\ \chi_p &= \frac{\bar{g}^2 \sin(2\theta)\Omega_d}{2(\Omega_d^2 - \delta^2)}\left(P_{22}^d - P_{11}^d\right), \\ \chi_m &= \frac{\bar{g}^2 \cos^2\theta\,\delta}{2(\Omega_d^2 - \delta^2)}\left(P_{22}^d - P_{11}^d\right), \end{aligned} \tag{36}$$

and the steady-state atomic populations P_{kk}^d in dressed states $|k_d\rangle$ are given by

$$\begin{aligned} P_{22}^d &= \frac{\cos^4\theta}{1 + 3\sin^4\theta}, \\ P_{11}^d &= P_{33}^d = \frac{1 - P_{22}^d}{2}. \end{aligned} \tag{37}$$

So, for the case $\Delta_1 + \Delta_2 = 0$, the present system with dispersive dressed-atom-cavity interactions reduces effectively to two independent detuned NPDC oscillators, with beam-splitter-like (BSL) interactions between cavity field a_{1x} and cavity field a_{2x}. Entanglement between cavity field a_{j+} and cavity field a_{j-} can be established directly via the relevant NPDC interactions. Furthermore, by virtue of the BSL interactions, quantum correlations between the indirection-coupling cavity fields a_{1+} and a_{2-} (and a_{2+} and a_{1-}) can also be established [48]. Consequently, genuine quadripartite entanglement can be achieved among the four cavity fields. Evidently, the short-time oscillations of bipartite entanglements, shown in Fig. 2, result from BSL interactions between the cavity fields. Meanwhile, due to the asymmetry of the Hamiltonian in Eq. (35) with respect to $a_{j+} \leftrightarrow a_{j'-} (j \neq j')$, we thus have the negativities $E_{j+} = E_{j'-}$ in Fig. 2. In addition, we see that the strengths of the effective NPDC and BSL interactions are proportional to the dressed-state population differences

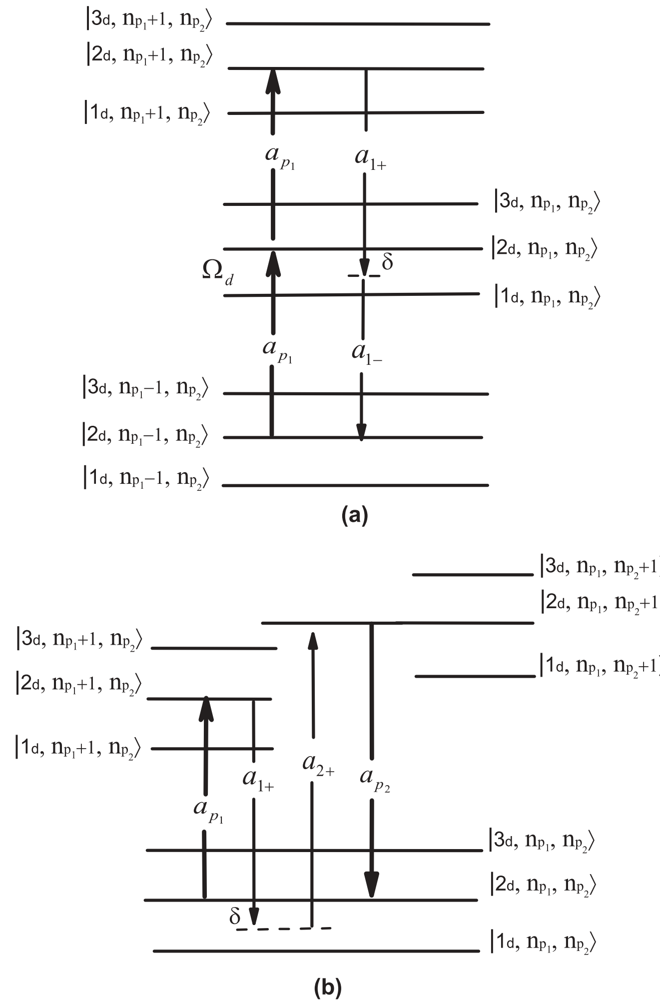

FIG. 3. (a) Possible two-photon cascaded transition channel which realizes the effective NPDC interaction between cavity field a_{1+} and cavity field a_{1-}. (b) Two-photon Raman transition of the dressed atoms which leads to the BSL interaction between cavity field a_{1+} and cavity field a_{2+} in Eq. (35).

$P_{22}^d - P_{11}^d$, indicating that the achieved quadripartite entanglement is dependent on the atomic coherence induced by the pumping lasers.

Physically, the effective NPDC and BSL interactions in Eq. (35) for dispersive dressed-atom-cavity interactions result from the two-photon cascaded transitions of the dressed atoms. In Fig. 3(a), the possible two-photon cascaded transition channel realizing the NPDC interactions between cavity fields a_{1+} and a_{1-} in Eq. (35) is plotted. It shows that, by absorbing two pumping photons of frequency ω_1, the dressed atom at first jumps from the dressed state $|2_d\rangle$ to level $|1_d\rangle$ by emitting a photon in the field a_{1+}; then, since this one-photon transition is far off resonance [by Eq. (33)], the dressed atom immediately emits another photon in field a_{1-} to level $|2_d\rangle$. In this two-photon cascaded transition, the conditions of the one-photon far-off resonance and the two-photon resonance are obeyed, which therefore leads to an effective NPDC interaction between field a_{1+} and field a_{1-}. Due to the destructive interference effect in the NFWM process [32,47], we see that the strength of the effective NPDC

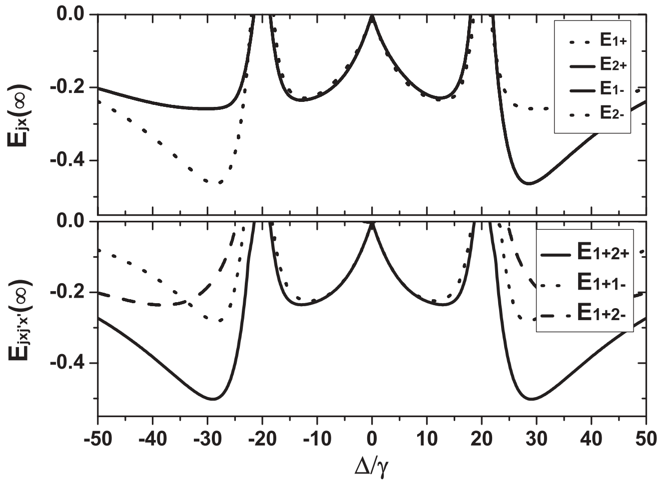

FIG. 4. Dependence of steady-state intracavity bipartite entanglements [characterized by the negative eigenvalues $E_{jx}(\infty)$ and $E_{jxj'x'}(\infty)$] on laser-atom detuning Δ. Other parameters are chosen as in Fig. 2.

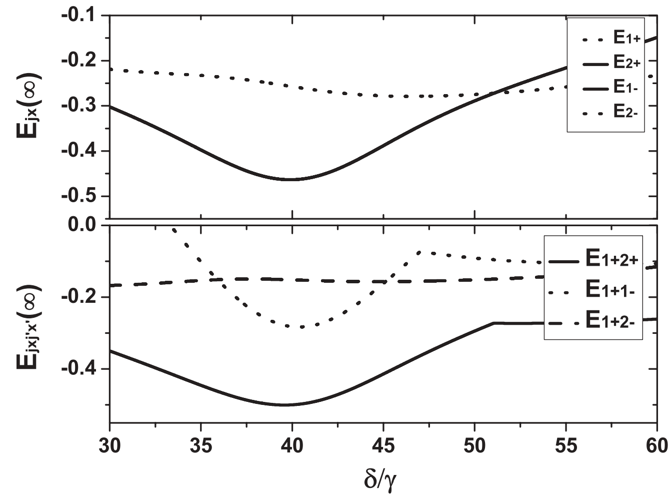

FIG. 5. Dependence of steady-state intracavity bipartite entanglements [characterized by negative eigenvalues $E_{jx}(\infty)$ and $E_{jxj'x'}(\infty)$] on laser-cavity detuning δ for laser-atom detuning $\Delta = 30$; other parameters chosen as in Fig. 2.

interactions is proportional to the dressed-state population difference $P_{22}^d - P_{11}^d$. In addition, BSL interactions between cavity fields result from two-photon Raman transitions, which are shown in Fig. 3(b). It shows that in this transition with one-photon far-off resonance and two-photon resonance, the photon in field a_{1+} is emitted and the other photon, in field a_{2+}, is absorbed, which leads to BSL interactions between cavity fields a_{1+} and a_{2+}.

In Fig. 4, we plot the dependence of stationary intracavity bipartite entanglements [characterized by negativities $E_{jx}(\infty)$ and $E_{jxj'x'}(\infty)$, respectively] on the laser-atom detuning Δ. It shows that bipartite entanglements do not occur for $\Delta = 0$, since the atom is trapped in the dark state $(|1\rangle - |2\rangle)/\sqrt{2}$, and we thus have $\sin\theta = 0$ and $\chi_{s,p} = 0$ in Eq. (35). Nevertheless, BSL interactions still exist ($\chi_m \neq 0$) for $\Delta = 0$, which essentially leads to opacity of the electromagnetically induced transparency for quantum fluctuations studied in Ref. [49]. Further, as shown in Fig. 4, the bipartite entanglements also disappear in the vicinity of the detuning $\Delta = \Omega = 20$. This is because, for $\Delta = \Omega$, the dressed-state populations $P_{kk}^d = 1/3 (k = 1,2,3)$ [see Eq. (37)] and the coupling strengths $\chi_{s,p,m} = 0$ in Eq. (35). Therefore, with the condition that detuning $\Delta = \Omega$, we see that quadripartite entanglement cannot be generated in a system with dispersive dressed-atom/cavity interactions, due to the completely destructive quantum interference in NFWM processes [47]. Also, we see that bipartite entanglements are maximal in the vicinity of the laser-atom detuning $\Delta = 30$. In Fig. 5, variations in steady intracavity bipartite entanglements with laser-cavity detuning δ are shown for the given detuing $\Delta = 30$. From Fig. 5, we can see that bipartite entanglements obtain their optimal values around the detuning $\delta = 40$. In addition, Figs. 4 and 5 show that genuine quadripartite entanglement can be achieved in a system with a wide range of control parameters.

B. Dissipative dressed-atom-cavity interactions

As shown in Figs. 4 and 5, bipartite entanglements in the system become maximum in the vicinity of detunings $\Delta = 30$ and $\delta = 40$ for the given Rabi frequency $\Omega = 20$ of the driving lasers. Evidently, in this case the condition in Eq. (33) for generating dispersive interactions between the dressed atoms and the cavity is not yet satisfied, and the laser-cavity detuning δ meets approximately

$$\delta = \Omega_d, \tag{38}$$

which indicates that the cavity fields are resonantly coupled to the dressed atoms and leads to the dissipative dressed-atom-cavity interactions. Obviously, in this situation, the quadripartite entanglement in the system is no longer determined by the effective Hamiltonian in Eq. (35) and it is fully governed by the master equation (7) with the coefficients in Eq. (8) being real [see Eq. (A6)].

In Figs. 6(a) and 6(b), intracavity and output bipartite entanglements for dissipative dressed-atom-cavity interactions ($\delta = \Omega_d$) are plotted, respectively. From these figures, we see that steady-state bipartite entanglements inside the cavity and output entanglements with frequency shift $\omega = 0$ are much stronger than the corresponding entanglements for the dispersive case shown in Fig. 2. Therefore, combined with Fig. 4, Fig. 6 shows that the genuine quadripartite entanglement becomes maximum when the cavity fields are tuned to be resonant to the dressed atoms ($\delta = \Omega_d$). Also, Fig. 6(a) shows that short-time oscillations of the bipartite entanglements are not present and the entanglements achieve steady-state values much faster than those shown in Fig. 2. This is due to the dissipative effect of the external atomic reservoir, resulting from spontaneous emission of atoms, on the cavity fields. Furthermore, Fig. 6(b) shows that output bipartite entanglements with $\omega = 0$ are much stronger than the corresponding steady entanglements in the cavity. Hence, a strong genuine quadripartite entanglement can be generated in the output field at cavity resonant frequencies with the condition $\delta = \Omega_d$. Under this condition, it is not difficult to find, in the Appendix, that when the laser-atom detuning $\Delta = 0$, at which atoms are pumped into dark states, the master

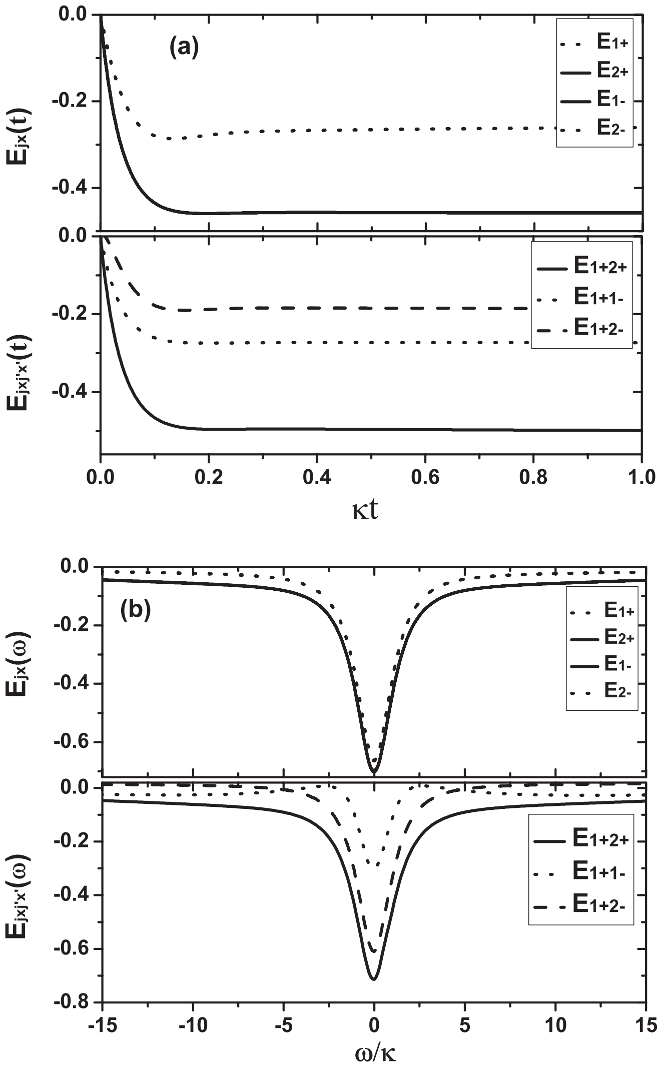

FIG. 6. Time evolution (a) and output spectra (b) of bipartite entanglement characterized by negative eigenvalues $E_{a_{jx}}$ and $E_{a_{jx}a_{j'x'}}$ for $\Delta = 30$, $\delta = \Omega_d$, and other parameters as in Fig. 2.

equation (7) becomes

$$\frac{d}{dt}\rho \approx \left(\frac{\bar{g}^2}{4\gamma} + \kappa\right) \sum_{j,x} (2a_{jx}\rho a_{jx}^\dagger - a_{jx}^\dagger a_{jx}\rho - \rho a_{jx}^\dagger a_{jx}) - \frac{\bar{g}^2}{4\gamma} \sum_{x} (2a_{1x}\rho a_{2x}^\dagger - a_{1x}a_{2x}^\dagger \rho - \rho a_{1x}a_{2x}^\dagger + \text{H.c.}). \tag{39}$$

Here, we use the fact that dressed-state population $P_{22}^d = 1$ for $\Delta = 0$. One can find that Eq. (39) is exactly the same as the evolution equation of the density matrix of a two-mode cavity field interacting with a three-level atomic system in Ref. [50]. Evidently, entanglement does not occur in this situation. Nevertheless, the linear mixings between the cavity fields described by Eq. (39) are still present, which was proposed for noise-free amplification of the quantum states of the cavity field in Ref. [50]. Again, we find that for dissipative interactions, the quadripartite entanglement also disappears when the detuning $\Delta = \Omega$, since under this condition the elements $\mathcal{M}_{jj'}$ of the drift matrix $\mathcal{M}$ in Eq. (20),

$$\mathcal{M}_{jj'} \sim \left(P_{22}^d - P_{11}^d\right), \tag{40}$$

for $\kappa = 0$. Therefore, from the preceding discussion, we find that the genuine quadripartite entanglement becomes maximum for dissipative dressed-atom-cavity interactions, and entanglement cannot be achieved when the laser-atom detuning $\Delta = \Omega$.

V. CONCLUSIONS

In conclusion, the generation of CV quadripartite entanglement of optical fields with different frequencies from a three-level Λ atomic system inside a cavity is investigated. We show that through multiply concurrent four-wave mixing processes, genuine quadripartite entanglement can be achieved inside and outside the cavity over a wide range of parameters. With the help of a dressed-state analysis, we show that for dispersive dressed-atom/cavity interactions, the generated quadripartite entanglement results from the effective NPDC and BSL interactions between the cavity fields. We find that genuine quadripartite entanglement becomes maximum when the cavity fields are tuned to be resonant with the Rabi sidebands of the driven atoms. In addition, we show that quadripartite entanglement cannot be achieved when the laser-atom detuning Δ is in the vicinity of the Rabi frequencies Ω of the driving lasers. The present system can serve as a potential source of polychromatic quadripartite entanglement which is useful in quantum communication networks.

ACKNOWLEDGMENTS

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APPENDIX: EXPRESSIONS OF THE COEFFICIENTS IN EQ. (8) DERIVED IN THE ATOMIC DRESSED-STATE REPRESENTATION

In this Appendix, we give the approximate expressions of the coefficients in Eq. (8) in the dressed-state representation of laser-driven atoms. Dressed states are eigenstates of the laser-atom Hamiltonian in Eq. (3) and given by

$$|k_d\rangle = M_{k1}|1\rangle + M_{k2}|2\rangle + M_{k3}|3\rangle \quad (k = 1,2,3), \tag{A1}$$

where

$$M_{k1} = \frac{\Omega_1\Omega_2}{D_k}, \quad M_{k2} = \frac{\lambda_k\tilde{\Delta}_1 + \Omega_1^2}{D_k}, \quad M_{k3} = \frac{\Omega_2\tilde{\Delta}_1}{D_k}, \tag{A2}$$

$$D_k = \sqrt{\left(\lambda_k\tilde{\Delta}_1 - \Omega_1^2\right)^2 + \Omega_2^2\tilde{\Delta}_1^2 + \Omega_1^2\Omega_2^2}, \tag{A3}$$

with $\tilde{\Delta}_j = \Delta_j + \lambda_j$ ($j = 1,2$) and λ_k being the roots of the equation

$$(\Delta_1 + \lambda)\left[\lambda(\Delta_2 + \lambda) - \Omega_2^2\right] - \Omega_1^2(\Delta_2 + \lambda) = 0. \tag{A4}$$

By defining the dressed operators $R_{kk'} = |k_d\rangle\langle k'_d|$, and with the same method used to derive Eq. (17), the coefficients in Eq. (8) can be derived with the help of the secular approximation [38], and we have

$$A_{jj}^{xx} = Ng_{jx}^2 \sum_{k\neq k'=1}^{3} P_{kk}^d \left(\xi_{kk'}^j\right)^2 F_{kk'}(\delta),$$

$$B_{jj}^{xx} = Ng_{jx}^2 \sum_{k\neq k'=1}^{3} P_{k'k'}^d \left(\xi_{kk'}^j\right)^2 F_{kk'}(\delta),$$

$$C_{jj}^{xx'} = -Ng_{jx}g_{jx'} \sum_{k=1}^{3} P_{kk}^d \sum_{k'\neq k} \xi_{kk'}^j \xi_{k'k}^j F_{kk'}^*(-\delta), \quad \text{(A5)}$$

$$\tilde{C}_{jj'}^{xx'} = -Ng_{jx}g_{j'x'} \sum_{k=1}^{3} P_{kk}^d \sum_{k'\neq k} \xi_{k'k}^j \xi_{kk'}^{j'} F_{kk'}^*(-\delta),$$

$$\bar{C}_{jj'}^{xx} = Ng_{jx}g_{j'x} \sum_{k=1}^{3} P_{kk}^d \sum_{k'\neq k} \xi_{kk'}^j \xi_{kk'}^{j'} F_{kk'}(\delta),$$

$D_{jj}^{xx'} = (C_{jj}^{xx'})^*[\delta \to -\delta]$, $\tilde{D}_{jj}^{xx'} = (\tilde{C}_{jj}^{xx'})^*$, and $\bar{D}_{jj}^{xx'} = (\bar{C}_{jj}^{xx'})^*$. Here, P_{kk}^d are the steady atomic populations in dressed states $|k_d\rangle$, and

$$F_{kk'}(\delta) = \frac{1}{\Gamma_{kk'} + i\Omega_{kk'} + ix\delta}, \quad \text{(A6)}$$

where $\Omega_{kk'} = \lambda_k - \lambda_{k'}$ is the energy difference between the dressed states $|k_d\rangle$ and $|k'_d\rangle$. The parameters $\xi_{kk'}^j$ and decay rates $\Gamma_{kk'}$ are given by

$$\{\xi_{kk'}^j\} = \begin{pmatrix} c_{31}c_{j1} & c_{31}c_{j2} & c_{31}c_{j3} \\ c_{32}c_{j1} & c_{32}c_{j2} & c_{32}c_{j3} \\ c_{33}c_{j1} & c_{33}c_{j2} & c_{33}c_{j3} \end{pmatrix}, \quad \text{(A7)}$$

with

$$\{c_{kk'}\} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}^{-1}, \quad \text{(A8)}$$

and

$$\Gamma_{12} = \sum_{j=1}^{2} \gamma_j \left(\xi_{11}^j + \xi_{22}^j + \xi_{12}^j + \xi_{21}^j + \xi_{13}^j + \xi_{23}^j\right),$$

$$\Gamma_{13} = \sum_{j=1}^{2} \gamma_j \left(\xi_{11}^j + \xi_{33}^j + \xi_{13}^j + \xi_{31}^j + \xi_{12}^j + \xi_{32}^j\right), \quad \text{(A9)}$$

$$\Gamma_{23} = \sum_{j=1}^{2} \gamma_j \left(\xi_{22}^j + \xi_{33}^j + \xi_{23}^j + \xi_{32}^j + \xi_{21}^j + \xi_{31}^j\right).$$

It should be noted that here we take into account only the sideband transitions of the dressed atoms and neglect the central-peak transitions since the expressions from these processes are very complicated, and moreover, they contribute little to the entanglement of the cavity field for the case $\delta \gg \gamma_j$ [32,34].

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