

# Complete hyperentangled-Bell-state analysis for quantum communication

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It is impossible to unambiguously distinguish the four Bell states in polarization, resorting to linear optical elements only. Recently, the hyperentangled Bell state, the simultaneous entanglement in more than one degree of freedom, has been used to assist in the complete Bell-state analysis of the four Bell states. However, if the additional degree of freedom is qubitlike, one can only distinguish 7 from the group of 16 states. Here we present a way to distinguish the hyperentangled Bell states completely with the help of cross-Kerr nonlinearity. Also, we discuss its application in the quantum teleportation of a particle in an unknown state in two different degrees of freedom and in the entanglement swapping of hyperentangled states. These applications will increase the channel capacity of long-distance quantum communication.

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## I. INTRODUCTION

Quantum teleportation [1,2], quantum dense coding [3], quantum superdense coding [4], and some important quantum cryptographic schemes [5–8], which are the completion of the most fundamental quantum communication processes involving bipartite entanglement, need the complete and deterministic analysis of the Bell states. For instance, in the well-known teleportation protocol [1], an unknown qubit state may be teleported between two parties over a long distance as long as each of them possesses one particle in an entangled photon pair and then the sender, Alice, makes an appropriate joint measurement on her particle in an unknown state and one of the pair shared with the receiver, Bob. Unfortunately, with only linear optical elements, a complete Bell-state analysis (BSA) is impossible and one can only get the optimal success probability of  $\frac{1}{2}$  [9–11]. In the optimal optical BSA scheme realized experimentally, its success probability is only 50% [12–14].

Hyperentanglement [15–17], the simultaneous entanglement in more than one degree of freedom, such as polarization-momentum, polarization-time-bin, and polarization- and spatial modes-energy-time, can be used to assist the complete Bell-state discrimination [18–23]. For instance, Kwiat and Weinfurter [18] first discussed the way to distinguish the four orthogonal Bell states with both momentum entanglements and polarization entanglements. In 2003, Walborn *et al.* [19] proposed a simple scheme for complete Bell-state measurement for photons using hyperentangled states. In their protocol, they performed the polarization BSA by using momentum entanglement as an ancilla. They also showed that the polarization states can be used to distinguish the momentum Bell states. The experiments of a complete BSA have also been reported with polarization-time-bin hyperentanglement [20] and polarization-momentum hyperentanglement [21]. In 2008, Kwiat *et al.* beat the channel capacity limit for linear

photonic superdense coding with polarization–orbital-angular-momentum hyperentanglement [22]. In essence, a BSA in the polarization degree of freedom with hyperentanglement works in a larger Hilbert space by introducing other degrees of freedom.

If we consider a large Hilbert space with an additional degree of freedom, the case is quite different. For a quantum system in a hyperentangled state in two degrees of freedom, it has 16 orthogonal Bell states. With linear optics only, one cannot distinguish them completely. For instance, the momentum entanglement can be used to distinguish the four polarization Bell states completely, but the momentum entanglement itself cannot be distinguished well. In 2007, Wei *et al.* [23] pointed out that one can only distinguish 7 states in the group of 16 orthogonal Bell states. If we consider each photon in  $n$  qubitlike degrees of freedom, there are, in total,  $4^n$  Bell-like states for each two-photon quantum system. Wei *et al.* [23] also extended their result and showed that the upper bound of the maximal number of mutually distinguishable Bell-like states is  $2^{n+1} - 1$ , as is true for  $n = 1$  and  $n = 2$ .

In this article, we present a complete hyperentanglement BSA (HBSA) with cross-Kerr nonlinearity. The hyperentangled state we discussed comprises the entanglement with two different kinds of degrees of freedom, that is, polarization and spatial modes. This HBSA scheme can be divided into two steps. The first step is used to distinguish the Bell states in spatial modes but not destroy the two photons. With this step, one can not only read the information about the Bell states in spatial modes, but also confirm the outputs of the two photons, which provides useful information for the BSA in polarization. This task should resort to quantum nondemolition detectors (QNDs). In the second step, one can first divide the four Bell states in polarization into two groups according to their parities, that is, the even-parity states and the odd-parity states with QND, and then distinguish the two Bell states in each group with two Hadamard operations and two parity-check measurements. This HBSA protocol can be used in quantum teleportation of a single-particle system in an unknown state in the polarization- and spatial-mode degrees of freedom. Also,

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it has a good application in hyperentanglement swapping of a hyperentangled state in two different degrees of freedom. These applications will increase the channel capacity in long-distance quantum communication.

This article is organized as follows: In Sec. II, we explain the principle of the present HBSA scheme. It is divided into two steps. One is used to distinguish the four Bell states in the spatial-mode degree of freedom with QNDs, which is discussed in Sec. II A. The other is used to analyze the four Bell states in the polarization degree of freedom, including a parity-check measurement with QND and another one with linear optical elements, shown in Sec. II B. In Sec. III, we discuss the applications of the present HBSA scheme in long-distance quantum communication; that is, a hyperentanglement teleportation scheme and a hyperentanglement-swapping scheme are discussed in Secs. III A and III B, respectively. A discussion and a summary are given in Sec. IV.

## II. COMPLETE HYPERENTANGLEMENT STATE BELL-STATE ANALYSIS

### A. HBSA protocol for Bell states in spatial modes

The previous works indicated that QNDs with a cross-Kerr medium and a coherent state can be used for operating the controlled-NOT (CNOT) gate [24,25] and single-photon logic gates with minimal sources [26], entanglement purification, and concentration [27,28]; generating high-quality entanglement [29,30] and qubits [31–33]; and analyzing the Bell states with the polarization degree of freedom [34]. In general, cross-Kerr nonlinearities can be described with the Hamiltonian as [24,34]

$$H_{\text{ck}} = \hbar \chi a_s^\dagger a_s a_p^\dagger a_p. \quad (1)$$

Here  $a_s$  and  $a_p$  are the annihilation operations and  $a_s^\dagger$  and  $a_p^\dagger$  denote the creation operations.  $\hbar \chi$  is the coupling strength of the nonlinearity and it is decided by the property of the nonlinear material. If we consider a single-photon state  $|\varphi\rangle = a|0\rangle + b|1\rangle$  and a coherent state  $|\alpha\rangle$ , the cross-Kerr interaction causes the combined system composed of a single photon and a coherent state evolve as

$$U_{\text{ck}}|\varphi\rangle|\alpha\rangle = e^{iH_{\text{QND}}t/\hbar}(a|0\rangle + b|1\rangle)|\alpha\rangle = a|0\rangle|\alpha\rangle + b|1\rangle|\alpha e^{i\theta}\rangle, \quad (2)$$

where  $|0\rangle$  and  $|1\rangle$  are the Fock states, which means the states contain 0 and 1 photon, respectively. Here the phase shift  $\theta = \chi t$  and  $t$  is the interaction time which is directly proportional to the number of photons with the single-photon state being unaffected. In 2005, Barrett *et al.* performed a polarization BSA with QND [34]. They first used the QND to construct the parity-check measurement.

Now we will work with a hyperentangled two-photon state with the form

$$|\Phi_{ab}^+\rangle_{PS} = \frac{1}{2}(|HH\rangle + |VV\rangle)_{ab} \otimes (|a1b1\rangle + |a2b2\rangle)_{ab}. \quad (3)$$

$|H\rangle$  and  $|V\rangle$  are the horizontal and the vertical polarizations, respectively. The subscripts  $a$  and  $b$  represent the two photons in the hyperentangled state. The subscript  $P$  denotes the polarization degree of freedom and  $S$  is the spatial-mode degree of freedom.  $a1(b1)$  and  $a2(b2)$  are the different spatial

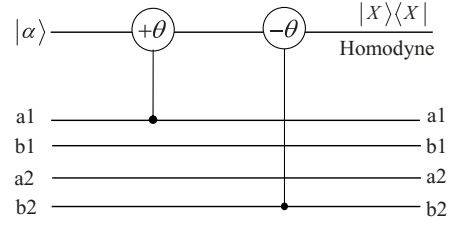


FIG. 1. Schematic diagram of the present HBSA protocol for spatial-mode entangled Bell states. This QND is used to distinguish the even-parity states  $\{|\phi^\pm\rangle_S\}$  from the odd-parity states  $\{|\psi^\pm\rangle_S\}$ . The action of first cross-Kerr nonlinearity puts a phase shift  $\theta$  on the coherent probe beam if a photon appears in the mode coupled. The second cross-Kerr nonlinearity puts a phase shift with  $-\theta$ . After the nonlinear interactions, the probe beam picks up the phase shift  $\theta$  or  $-\theta$  if the state is  $|a1b1\rangle$  or  $|a2b2\rangle$ , respectively. Otherwise, the states  $|a1b2\rangle$  and  $|a2b1\rangle$  pick up no phase shifts.

modes for photon  $a$  ( $b$ ), shown in Fig. 1. The state of Eq. (3) can be easily produced with a parametric down-conversion source. For example, in Ref. [35], a pump pulse comes from below and traverses a nonlinear  $\beta$  barium borate (BBO) crystal, where it can produce the entangled state into the modes  $a1b1$ ; then it is reflected and traverses the BBO crystal twice to produce entangled pairs in the modes  $a2b2$ . The photon pairs also are entangled in the polarization degree of freedom. That is, each photon pair is in the hyperentangled state  $|\Phi_{ab}^+\rangle_{PS}$ .

We denote four Bell states in the polarization degree of freedom as

$$\begin{aligned} |\phi^\pm\rangle_P &= \frac{1}{\sqrt{2}}(|HH\rangle \pm |VV\rangle), \\ |\psi^\pm\rangle_P &= \frac{1}{\sqrt{2}}(|HV\rangle \pm |VH\rangle), \end{aligned} \quad (4)$$

and four Bell states in the spatial-mode degree of freedom as

$$\begin{aligned} |\phi^\pm\rangle_S &= \frac{1}{\sqrt{2}}(|a1b1\rangle \pm |a2b2\rangle), \\ |\psi^\pm\rangle_S &= \frac{1}{\sqrt{2}}(|a1b2\rangle \pm |a2b1\rangle). \end{aligned} \quad (5)$$

Sometimes we refer to the states  $|\phi^\pm\rangle_P$  and  $|\phi^\pm\rangle_S$  as the even-parity states and  $|\psi^\pm\rangle_P$  and  $|\psi^\pm\rangle_S$  as the odd-parity states.

An appealing advantage of the hyperentangled product states with the form of Eq. (3) is that state in the two degrees of freedom can be well operated independently. For example, if we manipulate the polarization Bell states using some local operations, such as a polarization beam splitter (PBS) or a half-wave plate, we will leave the spatial-mode entangled state unchanged. Meanwhile, if we operate the spatial-mode Bell states, the polarization Bell states remain unchanged too. This feature provides us an effective way of achieving HBSA. That is, one can perform the polarization BSA and spatial-mode BSA independently. The spatial-mode entanglement no longer acts as an ancilla for the polarization BSA. Therefore, a HBSA can be divided into two steps. The first step is used for the spatial-mode BSA and the second is for polarization. The precondition for this scheme being well realized is that of nondestructive measurement, as one cannot operate polarization BSA anymore if the photons are detected

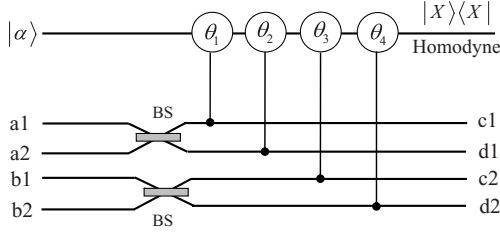


FIG. 2. The second QND for the BSA in spatial modes. 50:50 BS acts as a Hadamard operation.  $\theta_i$  ( $i = 1, 2, 3, 4$ ) represent four different cross-Kerr nonlinear media with the phase shifts  $\theta_i$ . This QND is used to distinguish the four spatial modes  $c1c2$ ,  $d1d2$ ,  $c1d2$ , and  $d1c2$  with the phase shifts  $\theta_1 + \theta_3$ ,  $\theta_2 + \theta_4$ ,  $\theta_1 + \theta_4$ , and  $\theta_2 + \theta_3$ , respectively. When the two photons appear at the spatial modes  $c1c2$  or  $d1d2$  ( $c1d2$  or  $d1c2$ ), they are originally in the Bell state  $|\phi^+\rangle_s$  ( $|\phi^-\rangle_s$ ) if the phase shift in the first QND is  $\theta$  or  $-\theta$ . The result is kept for the state  $|\psi^+\rangle_s$  ( $|\psi^-\rangle_s$ ) when the phase shift in the first QND is zero. Completing the BSA with the first and the second QNDs, the outputs of the two photons are determinate, which provides useful information for the analysis of the Bell states in the polarization degree of freedom.

and destroyed. Fortunately, cross-Kerr nonlinearities provide us a powerful tool for accomplishing QNDs.

The principle of the present HBSA protocol for spatial-mode entangled Bell states is shown in Fig. 1 and Fig. 2. With the first QND shown in Fig. 1, the state  $|\phi^\pm\rangle_s$  with the coherent state  $|\alpha\rangle$  evolves as

$$\begin{aligned} |\phi^\pm\rangle_s |\alpha\rangle &= \frac{1}{\sqrt{2}}(|a1b1\rangle \pm |a2b2\rangle) |\alpha\rangle \\ &\rightarrow \frac{1}{\sqrt{2}}(|a1b1\rangle |\alpha e^{i\theta}\rangle \pm |a2b2\rangle |\alpha e^{-i\theta}\rangle), \end{aligned} \quad (6)$$

but the state  $|\psi^\pm\rangle_s$  with the coherent state  $|\alpha\rangle$  evolves as

$$\begin{aligned} |\psi^\pm\rangle_s |\alpha\rangle &= \frac{1}{\sqrt{2}}(|a1b2\rangle \pm |a2b1\rangle) |\alpha\rangle \\ &\rightarrow \frac{1}{\sqrt{2}}(|a1b2\rangle |\alpha\rangle \pm |a2b1\rangle |\alpha\rangle) \\ &= |\psi^\pm\rangle_s |\alpha\rangle. \end{aligned} \quad (7)$$

One can observe immediately that the states  $|a1b2\rangle$  and  $|a2b1\rangle$  pick up no phase shifts and maintain the coherence with respect to each other. However, the states  $|a1b1\rangle$  and  $|a2b2\rangle$  pick up the phase shifts  $\theta$  and  $-\theta$ , respectively. If we choose an  $X$ -quadrature measurement on the coherent beam, with which the states  $|\alpha e^{-i\theta}\rangle$  and  $|\alpha e^{i\theta}\rangle$  cannot be distinguished [24], we can distinguish  $|\phi^\pm\rangle_s$  and  $|\psi^\pm\rangle_s$  with different phase shifts by homodyne-heterodyne measurements. This QND detector is a parity-checking device which can distinguish the even-parity states  $|\phi^\pm\rangle_s$  from the odd-parity states  $|\psi^\pm\rangle_s$ .

With the first QND, the four Bell states in spatial modes are divided into two groups  $|\phi^\pm\rangle_s$  and  $|\psi^\pm\rangle_s$ . The next task of BSA in spatial modes is to distinguish the different phases in each group. The second QND is used to distinguish the Bell state with the phase zero and the Bell state with the phase  $\pi$ ,

shown in Fig. 2. A 50:50 beam splitter (BS) can accomplish the following transformation in the spatial modes:

$$\begin{aligned} |a1\rangle &\rightarrow \frac{1}{\sqrt{2}}(|c1\rangle + |d1\rangle), \\ |a2\rangle &\rightarrow \frac{1}{\sqrt{2}}(|c1\rangle - |d1\rangle), \\ |b1\rangle &\rightarrow \frac{1}{\sqrt{2}}(|c2\rangle + |d2\rangle), \\ |b2\rangle &\rightarrow \frac{1}{\sqrt{2}}(|c2\rangle - |d2\rangle). \end{aligned} \quad (8)$$

For the group  $|\phi^\pm\rangle_s$ , the state  $\frac{1}{\sqrt{2}}(|a1b1\rangle + |a2b2\rangle)$  will become  $\frac{1}{\sqrt{2}}(|c1c2\rangle + |d1d2\rangle)$  after the BSs, but  $\frac{1}{\sqrt{2}}(|a1b1\rangle - |a2b2\rangle)$  will become  $\frac{1}{\sqrt{2}}(|c1d2\rangle + |d1c2\rangle)$ . From Fig. 2, one can see states  $c1c2$ ,  $d1d2$ ,  $c1d2$ , and  $d1c2$  with the phase shifts  $\theta_1 + \theta_3$ ,  $\theta_2 + \theta_4$ ,  $\theta_1 + \theta_4$ , and  $\theta_2 + \theta_3$ , respectively. With an  $X$ -quadrature measurement on the coherent beam, one can read out the information about the phases in the group  $|\phi^\pm\rangle_s$ . That is, the two entangled photons are originally in the Bell state  $|\phi^+\rangle_s$  when they appear at the outputs  $c1c2$  or  $d1d2$ ; otherwise, they are originally in the state  $|\phi^-\rangle_s$ .

The states  $|\psi^\pm\rangle_s$  can also be distinguished with the same method discussed previously. That is, the state  $|\psi^+\rangle_s$  will become  $\frac{1}{\sqrt{2}}(|c1c2\rangle - |d1d2\rangle)$  and the state  $|\psi^-\rangle_s$  will become  $\frac{1}{\sqrt{2}}(|c1d2\rangle - |d1c2\rangle)$  after the two BSs. If the two entangled photons appear at output  $c1c2$  or  $d1d2$ , they are originally in the Bell state  $|\psi^+\rangle_s$ ; otherwise, they are originally in the Bell state  $|\psi^-\rangle_s$ .

From the preceding analysis, one can see that the role of the two QNDs is to accomplish the task of parity check. The first QND can distinguish the two even-parity states in spatial modes  $|\phi^\pm\rangle_s$  from the two odd-parity states  $|\psi^\pm\rangle_s$ . With two BSs, the two states with two different relative phases are transformed into two states with different parities. With the second QND, one can, in principle, distinguish the four Bell states in spatial modes without destroying the two photons. Moreover, the  $X$ -quadrature measurement on the coherent beam will give useful information about the outputs of the two entangled photons, which will make the HBSA for the four Bell states in polarization more convenient.

## B. HBSA protocol for Bell states in polarization

Now let us move our attention to distinguish the four Bell states  $|\phi^\pm\rangle_P$  and  $|\psi^\pm\rangle_P$  in polarization. From the preceding analysis, the spatial-mode entangled states have been deterministically discriminated. Suppose that one gets the entangled state in spatial modes  $|\phi^+\rangle_s = \frac{1}{\sqrt{2}}(|c1c2\rangle + |d1d2\rangle)$  with QNDs shown in Figs. 1 and 2. In fact, we need only to discuss the case that the photon pair is in the spatial modes  $c1c2$  (the spatial modes  $a1b1$  in Fig. 3), as the case in the modes  $d1d2$  is similar to it. The analysis of the four Bell states in polarization in the present protocol is similar to that in Ref. [34].

The setup for the discrimination of the four Bell states in polarization is shown in Fig. 3. Cross-Kerr nonlinearities with

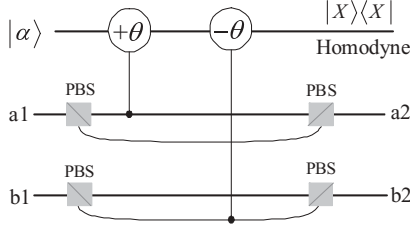


FIG. 3. Schematic diagram of the present HBSA protocol for Bell states in polarization. The state  $|HH\rangle$  picks up the phase shift  $\theta$  and  $|VV\rangle$  picks up the phase shift  $-\theta$ . The states  $|HV\rangle$  and  $|VH\rangle$  pick up no phase shifts. That is, this setup is a parity-check device for the four Bell states in polarization.

the combined system  $|\phi^\pm\rangle_P |\alpha\rangle$  will evolve as

$$\begin{aligned} |\phi^\pm\rangle_P |\alpha\rangle &= \frac{1}{\sqrt{2}}(|HH\rangle \pm |VV\rangle)|\alpha\rangle \\ &\rightarrow \frac{1}{\sqrt{2}}(|HH\rangle |\alpha e^{i\theta}\rangle \pm |VV\rangle |\alpha e^{i\theta}\rangle). \end{aligned} \quad (9)$$

$|\psi^\pm\rangle_P |\alpha\rangle$  will evolve as

$$\begin{aligned} |\psi^\pm\rangle_P |\alpha\rangle &= \frac{1}{\sqrt{2}}(|HV\rangle \pm |VH\rangle)|\alpha\rangle \\ &\rightarrow \frac{1}{\sqrt{2}}(|HV\rangle |\alpha\rangle \pm |VH\rangle |\alpha\rangle) \\ &= |\psi^\pm\rangle_P |\alpha\rangle. \end{aligned} \quad (10)$$

In these evolutions,  $|HV\rangle$  and  $|VH\rangle$  pick up no phase shifts and maintain the coherent state with respect to each other, but  $|HH\rangle$  and  $|VV\rangle$  pick up the phase shifts  $\theta$  and  $-\theta$ , respectively. Similar to the preceding case for spatial-mode entangled states, we choose an  $X$ -quadrature measurement on the coherent beam to make  $|\alpha e^{\pm i\theta}\rangle$  not be distinguished.

With the QND shown in Fig. 3, the four Bell states in polarization are divided into two groups, that is, the even-parity states  $|\phi^\pm\rangle_P$  and the odd-parity states  $|\psi^\pm\rangle_P$ . The next step is to distinguish the different relative phases in each group. This task can be accomplished with linear optical elements, shown in Fig. 4. One can use  $\lambda/4$  wave plates  $R_{45}$  to rotate the two photons  $a$  and  $b$  by  $45^\circ$ . The unitary transformation of  $45^\circ$

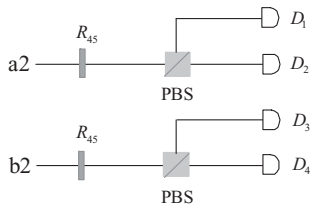


FIG. 4. The setup for distinguishing the relative phases of the Bell states in polarization. PBS, polarization beam splitter which is used to pass through  $|H\rangle$  polarization photons and reflect  $|V\rangle$  polarization photons. The wave plates  $R_{45}$  rotate the horizontal and vertical polarizations by  $45^\circ$ , which accomplishes a Hadamard operation on polarization.

rotations can be described as

$$\begin{aligned} |H\rangle_{a2} &\rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{a2} + |V\rangle_{a2}), \\ |H\rangle_{b2} &\rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{b2} + |V\rangle_{b2}), \\ |V\rangle_{a2} &\rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{a2} - |V\rangle_{a2}), \\ |V\rangle_{b2} &\rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{b2} - |V\rangle_{b2}). \end{aligned} \quad (11)$$

The state  $|\phi^+\rangle_P$  ( $|\psi^-\rangle_P$ ) is kept unchanged after the two rotations  $R_{45}$  and the two photons will click the detectors  $D_1 D_3$  or  $D_2 D_4$  ( $D_1 D_4$  or  $D_2 D_3$ ). The state  $|\phi^-\rangle_P$  ( $|\psi^+\rangle_P$ ) will become  $|\psi^+\rangle_P$  ( $|\phi^-\rangle_P$ ) and the two photons will click the detectors  $D_1 D_4$  or  $D_2 D_3$  ( $D_1 D_3$  or  $D_2 D_4$ ).

In fact, the BSA in polarization essentially equals the BSA in spatial modes. In Fig. 3, the polarization-entangled states in the modes  $a1b1$  are divided into four different spatial modes by two PBSs. The QND serves the same purpose as those in Fig. 1 and Fig. 2. The process of the discrimination of the Bell states can be divided into two steps, with each step being a parity-check measurement. Thus, the present HBSA protocol can also be served as parity-check measurements for different degrees of freedom of photons. The distinct feature of the hyperentangled state is that the different degrees of freedom are relatively independent of each other, which ensures that one can manipulate each degree of freedom independently. In theory, one can also distinguish the Bell states in polarization mode first and then the BSA in spatial mode modestly. However, this strategy will make the whole discrimination scheme more complicated as the spatial mode is uncertain, which will lead us to add more QNDs to accomplish the parity-check measurements.

### III. APPLICATIONS OF HBSA IN QUANTUM COMMUNICATION

So far, we have described a full HBSA with QNDs. It is interesting to discuss the applications of HBSA in quantum communication. There are two unique techniques in long-distance quantum communication, that is, quantum teleportation and entanglement swapping. The former can be used to transmit an unknown state of a particle to a remote point without distributing the particle itself. The latter provides a good tool for quantum repeaters in long-distance quantum communication. We discuss the way to teleport an unknown state in two degrees of freedom of photons and the way to swap two hyperentangled states in what follows.

#### A. Teleportation with a hyperentanglement-state channel

A Bell state shared enables the teleportation of an unknown single-qubit state. In a quantum teleportation protocol [1], two parties, say Alice and Bob, in distant locations are in possession of one photon of a polarization-entangled pair, which is prepared in a Bell state. Alice wants to teleport another photon to Bob but he does not know any information about its state; otherwise, she only needs to tell Bob to prepare it with classical communication. For this end, Alice first makes a Bell-state measurement on her teleported photon and



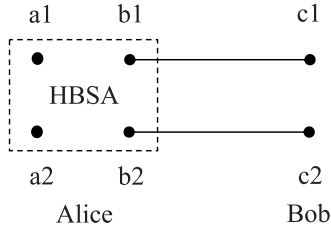


FIG. 5. The procedure of teleportation of an unknown single-particle state in the polarization- and spatial-mode degrees of freedom with a hyperentanglement-state channel, resorting to HBSA.

the photon in the entangled pair shared with Bob and then tells her result to Bob with classical communication. Finally, Bob can recover the state of Alice's photon according to her results with some local unitary operations. In 1997, the teleportation protocol based on polarization entanglement was demonstrated experimentally [36] and the teleportation was also realized using path-entangled (spatial-mode) photons in 1998 [37].

With linear optics only, the complete BSA of a two-photon polarization state alone is impossible; protocols resorting to additional degrees of freedom have been proposed. Our analysis shows that HBSA enables the teleportation of an arbitrary state encoded in both polarization and spatial mode with a success probability of 100%.

Suppose that the photon *A* teleported in Alice's laboratory is in an arbitrary state  $|\varphi\rangle_A$  in both polarization- and spatial-mode degrees of freedom; that is,

$$|\varphi\rangle_A = (\alpha|H\rangle + \beta|V\rangle) \otimes (\gamma|a1\rangle + \delta|a2\rangle), \quad (12)$$

where  $|\alpha|^2 + |\beta|^2 = 1$  and  $|\gamma|^2 + |\delta|^2 = 1$ . Alice and Bob share a hyperentanglement-state channel *BC* with the form

$$|\Phi^+\rangle_{BC} = \frac{1}{2}(|HH\rangle + |VV\rangle) \otimes (|b1c1\rangle + |b2c2\rangle). \quad (13)$$

The principle of teleportation of an unknown single-particle state in two degrees of freedom with hyperentanglement is shown in Fig. 5. If Alice performs a HBSA on photons *A* and *B*, the whole system will evolve as

$$\begin{aligned} |\varphi\rangle_A \otimes |\Phi^+\rangle_{BC} &= \frac{1}{2}(\alpha|HHH\rangle + \alpha|HVV\rangle + \beta|VHH\rangle \\ &\quad + \beta|VVV\rangle)_{ABC} \otimes (\gamma|a1b1c1\rangle + \gamma|a1b2c2\rangle \\ &\quad + \delta|a2b1c1\rangle + \delta|a2b2c2\rangle)_{ABC} \\ &= \frac{1}{4}[(|\phi^+\rangle_P(\alpha|H\rangle + \beta|V\rangle) + |\phi^-\rangle_P(\alpha|H\rangle - \beta|V\rangle)) \\ &\quad + |\psi^+\rangle_P(\alpha|V\rangle + \beta|H\rangle) + |\psi^-\rangle_P(\alpha|V\rangle - \beta|H\rangle)]_{ABC} \\ &\quad \otimes [(\phi^+\rangle_S(\gamma|c1\rangle + \delta|c2\rangle) + |\phi^-\rangle_S(\gamma|c1\rangle - \delta|c2\rangle) \\ &\quad + |\psi^+\rangle_S(\gamma|c2\rangle + \delta|c1\rangle) + |\psi^-\rangle_S(\gamma|c2\rangle - \delta|c1\rangle)]_{ABC}. \end{aligned} \quad (14)$$

If Alice obtains the outcomes  $|\phi^\pm\rangle_P|\phi^\pm\rangle_S$ ,  $|\phi^\pm\rangle_P|\psi^\pm\rangle_S$ ,  $|\psi^\pm\rangle_P|\phi^\pm\rangle_S$ , or  $|\psi^\pm\rangle_P|\psi^\pm\rangle_S$ , the photon in Bob's hand will be in the states  $(\alpha|H\rangle \pm \beta|V\rangle)_P(\gamma|c1\rangle \pm \delta|c2\rangle)_S$ ,  $(\alpha|H\rangle \pm \beta|V\rangle)_P(\gamma|c2\rangle \pm \delta|c1\rangle)_S$ ,  $(\alpha|V\rangle \pm \beta|H\rangle)_P(\gamma|c1\rangle \pm \delta|c2\rangle)_S$ , or  $(\alpha|V\rangle \pm \beta|H\rangle)_P(\gamma|c2\rangle \pm \delta|c1\rangle)_S$ , respectively. With the results published by Alice, Bob can recover the unknown state  $|\varphi\rangle_A$  with two local unitary operations on his photon *C*. For example, if Bob obtains the state  $(\alpha|V\rangle -$

$\beta|H\rangle)_P(\gamma|c2\rangle - \delta|c1\rangle)_S$ , he first performs a unitary operation  $-i\sigma_y \equiv |H\rangle\langle V| - |V\rangle\langle H|$  on the photon *B* in the polarization degree of freedom (each of the two spatial modes *c1* and *c2*) and then introduces a relative phase  $\pi$  in the spatial mode *c1* which can be accomplished with a  $\lambda/2$  wave plate. With the exchange of the two spatial modes *c1* and *c2*, Bob can obtain the unknown single-particle state in two degrees of freedom  $|\varphi\rangle_B = (\alpha|H\rangle + \beta|V\rangle) \otimes (\gamma|c1\rangle + \delta|c2\rangle)$ . The other cases are similar to this one with or without a little modification.

To date, quantum teleportation protocols have been demonstrated with a success probability of 50%, resorting to linear optical Bell-state measurements, such as two-photon interference, single-photon detection, and polarization analysis. Walborn *et al.* [19] also discussed quantum teleportation protocol with hyperentanglement in 2003. However, in their protocol, they can teleport an arbitrary state encoded in either the polarization or the momentum degree of freedom with a success probability of 50%, that is, the same probability as the protocol with two-photon polarization BSA based on linear optics. Compared with the conventional polarization teleportation protocols, it does not offer more advantages. The present teleportation scheme with hyperentanglement provides us a way of teleporting a quantum state in both polarization- and spatial-mode degrees of freedom. Compared with the conventional teleportation protocols, more quantum information can be transmitted.

## B. Entanglement swapping with hyperentangled states

Another interesting application of HBSA is quantum entanglement swapping, which enables one to entangle two quantum systems that have never being interacted with each other [38,39]. Entanglement swapping has been widely applied in quantum repeaters [40–47]. In a practical transmission for long-distance quantum communication, the photon losses increase exponentially with the length of the communication channel [40]. In order to overcome the photon losses, the whole transmission channel is usually divided into many segments and the length of each segment is comparable to the channel attenuation length. Entanglement is first generated in each segment and then extended to a greater length by connecting two adjacent segments with entanglement swapping.

The existing entanglement-swapping protocols usually focused on the Bell states with only one degree of freedom, that is, the polarization of photons. Here we show that we can also perform the entanglement swapping with hyperentangled states, which will improve largely the channel capacity in long-distance quantum communication.

Let pairs *AB* and *CD* be in the following hyperentangled states:

$$\begin{aligned} |\Phi^+\rangle_{AB} &= |\phi^+\rangle_P^{AB} \otimes |\phi^+\rangle_S^{AB} \\ &= \frac{1}{2}(|HH\rangle + |VV\rangle) \otimes (|a1b1\rangle + |a2b2\rangle), \end{aligned} \quad (15)$$

$$\begin{aligned} |\Phi^+\rangle_{CD} &= |\phi^+\rangle_P^{CD} \otimes |\phi^+\rangle_S^{CD} \\ &= \frac{1}{2}(|HH\rangle + |VV\rangle) \otimes (|c1d1\rangle + |c2d2\rangle). \end{aligned} \quad (16)$$

The subscript *P* denotes the polarization part of the hyperentangled state and *S* is the spatial-mode part. The superscripts *A* and *B* denote that the particles are in nodes *A* and *B*,

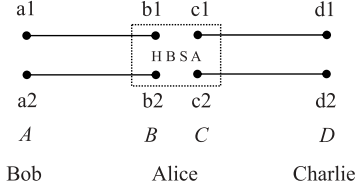


FIG. 6. Schematic diagram of hyperentanglement swapping in the polarization- and spatial-mode degrees of freedom. The initial hyperentangled states are prepared in nodes  $AB$  and  $CD$  (also the four photons). After Alice performs the HBSA on the two photons  $BC$ , Bob and Charlie can get the hyperentangled state between nodes  $A$  and  $D$ .  $a1b1$  ( $a2b2$ ) are the different spatial modes for each hyperentangled state.

respectively, as shown in Fig. 6. That is, Alice shares a photon pair  $AB$  with Bob. Also, she shares a photon pair  $CD$  with Charlie. The task of this entanglement-swapping protocol is to entangle the two photons  $A$  and  $D$  in both polarization- and spatial-mode degrees of freedom.

For entanglement swapping of a hyperentangled state, Alice performs HBSA on the two particles  $B$  and  $C$ , as shown in Fig. 6. The whole system evolves as

$$\begin{aligned}
 & |\Phi^+\rangle_{AB} \otimes |\Phi^+\rangle_{CD} \\
 &= \frac{1}{4}(|HHHH\rangle + |HHVV\rangle + |VVHH\rangle + |VVVV\rangle) \\
 &\quad \otimes (|a1b1c1d1\rangle + |a1b1c2d2\rangle + |a2b2c1d1\rangle \\
 &\quad + |a2b2c2d2\rangle) \\
 &= \frac{1}{4}(|HHHH\rangle + |HVVH\rangle + |VHVV\rangle + |VVVV\rangle) \\
 &\quad \otimes (|a1d1b1c1\rangle + |a1d2b1c2\rangle + |a2d1b2c1\rangle \\
 &\quad + |a2d2b2c2\rangle) \\
 &= \frac{1}{4}[(|\phi^+\rangle_P^{AD}|\phi^+\rangle_P^{BC} + |\phi^-\rangle_P^{AD}|\phi^-\rangle_P^{BC} \\
 &\quad + |\psi^+\rangle_P^{AD}|\psi^+\rangle_P^{BC} + |\psi^-\rangle_P^{AD}|\psi^-\rangle_P^{BC}) \\
 &\quad \otimes (|\phi^+\rangle_S^{AD}|\phi^+\rangle_S^{BC} + |\phi^-\rangle_S^{AD}|\phi^-\rangle_S^{BC} \\
 &\quad + |\psi^+\rangle_S^{AD}|\psi^+\rangle_S^{BC} + |\psi^-\rangle_S^{AD}|\psi^-\rangle_S^{BC})]. \quad (17)
 \end{aligned}$$

If Alice obtains the outcome  $|\Phi^+\rangle_{BC} = |\phi^+\rangle_P^{BC}|\phi^+\rangle_S^{BC}$ , the two photons located in nodes  $A$  and  $D$  will be in the hyperentangled state  $|\Phi^+\rangle_{AD} = |\phi^+\rangle_P^{AD}|\phi^+\rangle_S^{AD}$ . The outcomes will lead to the other hyperentangled states, such as  $|\phi^+\rangle_P^{AD}|\phi^-\rangle_S^{AD}$ ,  $|\phi^-\rangle_P^{AD}|\phi^+\rangle_S^{AD}$ ,  $|\phi^-\rangle_P^{AD}|\phi^-\rangle_S^{AD}$ ,  $|\psi^+\rangle_P^{AD}|\phi^+\rangle_S^{AD}$ ,  $|\psi^-\rangle_P^{AD}|\phi^+\rangle_S^{AD}$ ,  $|\psi^+\rangle_P^{AD}|\phi^-\rangle_S^{AD}$ ,  $|\psi^-\rangle_P^{AD}|\phi^-\rangle_S^{AD}$ ,  $|\psi^+\rangle_S^{AD}|\psi^+\rangle_P^{AD}$ ,  $|\psi^-\rangle_S^{AD}|\psi^+\rangle_P^{AD}$ ,  $|\psi^+\rangle_S^{AD}|\psi^-\rangle_P^{AD}$ , and  $|\psi^-\rangle_S^{AD}|\psi^-\rangle_P^{AD}$ . Moreover, it is, in principle, not difficult for Bob and Charlie to transform their hyperentangled states into the form  $|\Phi^+\rangle_{AD}$ . For instance, if Bob and Charlie obtain the state  $|\psi^-\rangle_P^{AD}|\psi^-\rangle_S^{AD} = \frac{1}{2}(|HV\rangle - |VH\rangle)(|a1d2\rangle - |a2d1\rangle)$ , they can obtain the state  $|\Phi^+\rangle_{AD} = \frac{1}{2}(|HH\rangle + |VV\rangle)(|a1d1\rangle + |a2d2\rangle)$  in the way that Charlie performs an operation  $-i\sigma_y$  in polarization (the two spatial modes, i.e., the two paths  $d1$  and  $d2$ ) and then exchanges the two spatial modes after he introduces a phase  $\pi$  in the spatial mode  $d1$  with a  $\lambda/2$  wave plate.

In essence, the present hyperentanglement-swapping protocol can be divided into two processes, that is the entangled swapping for entanglement states in polarization and that in spatial modes. Each entanglement-swapping process is independent of the other. Meanwhile, they must be swapped simultaneously. If we only perform the Bell-state measurement

on photons  $B$  and  $C$  in the polarization degree of freedom, photons  $A$  and  $D$  will be entangled in the polarization degree of freedom but their state in spatial modes may be a mixed one.

#### IV. DISCUSSION AND SUMMARY

A hyperentangled BSA is far different from a hyperentanglement-assisted BSA as the latter is used to only analyze the Bell states in the polarization degree of freedom, and the other degree of freedom, such as momentum and time-bin, is an additional auxiliary system and is consumed in the analysis. This HBSA protocol shows that with cross-Kerr nonlinearity, one can also perform a complete HBSA. During a HBSA process, the entanglements in different degrees of freedom can be manipulated independently. The key element in this scheme is QND as it provides us the way to manipulate the photons but not destroy them. This protocol also reveals that complete HBSA with only linear optical elements is also impossible.

In the process of describing the principle of our HBSA scheme, we mainly exploit the cross-Kerr nonlinearity to construct the parity-check gate. We should acknowledge that although a lot of works have been studied on cross-Kerr nonlinearity [48], a clean cross-Kerr nonlinearity in the optical single-photon regime is still quite a controversial assumption, especially with current technology. In Ref. [49], Kok *et al.* showed that, operating in the optical single-photon regime, the Kerr phase shift is only  $\tau \approx 10^{-18}$ . With electromagnetically induced transparent materials, cross-Kerr nonlinearities of  $\tau \approx 10^{-5}$  can be obtained. As pointed out by Gea-Banacloche [50] recently, the large phase shifts via the giant Kerr effect with single-photon wave packets is impossible at present. These results also agree with the previous works by Shapiro and Razavi [51,52]. The weak cross-Kerr nonlinearity will make the phase shifts  $\theta_1 + \theta_3$ ,  $\theta_2 + \theta_4$ ,  $\theta_1 + \theta_4$ , and  $\theta_2 + \theta_3$  of the coherent state become extremely small, which will be hard to detect. That is to say, using homodyne detector, it is difficult to determine the phase shift due to the impossible discrimination of two overlapping coherent states, which will decrease the success probability of the present HBSA scheme. In 2003, Hofmann *et al.* [53] showed that a phase shift of  $\pi$  can be achieved with a single two-level atom in a one-sided cavity. In 2010, Wittmann *et al.* [54] investigated quantum measurement strategies capable of discriminating two coherent states using a homodyne detector and a photon-number-resolving (PNR) detector. In order to lower the error probability, the postselection strategy is applied to the measurement data of homodyne detector as well as a PNR detector. They showed that the performance of the new displacement-controlled PNR is better than that of a homodyne receiver. That is, the present HBSA scheme may be feasible if we choose a suitable Kerr nonlinear media and some good quantum measurement strategies on coherent beams. Moreover, quantum gates based on optical nonlinearities have attracted a lot of attention [55–58] in recent years, which will impel the development of optical nonlinearity techniques.

In fact, the present HBSA scheme requires that the positive and negative signs in the phase shifts of the coherent beam cannot be distinguished. That is, it is unnecessary for us to have interaction-induced phases of both positive and negative

sign in the present scheme. Here a cross-Kerr nonlinearity in QNDs is only used to make a parity check for two photons, and other elements can also be used to construct QNDs [59–61] for this HBSA scheme.

In summary, we have proposed a complete HBSA scheme with cross-Kerr nonlinearity. We use the cross-Kerr nonlinearity to construct parity-check measurements and analyze Bell states in different degrees of freedom of photons. We also discussed its applications in quantum teleportation and entanglement swapping in two different degrees of freedom simultaneously. We concluded that one can teleport a particle in more than one degree of freedom if a hyperentanglement channel is set up and a hyperentanglement Bell-state measurement is permitted perfectly. We also revealed that quantum communication based on hyperentanglement is possible as

we can set up a hyperentanglement quantum channel for a long-distance quantum communication with hyperentanglement swapping. All these results may be useful in practical applications in quantum information.

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- [1] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, *Phys. Rev. Lett.* **70**, 1895 (1993).
  - [2] A. Karlsson and M. Bourennane, *Phys. Rev. A* **58**, 4394 (1998); F. G. Deng, C. Y. Li, Y. S. Li, H. Y. Zhou, and Y. Wang, *ibid.* **72**, 022338 (2005).
  - [3] C. H. Bennett and S. J. Wiesner, *Phys. Rev. Lett.* **69**, 2881 (1992).
  - [4] X. S. Liu, G. L. Long, D. M. Tong, and L. Feng, *Phys. Rev. A* **65**, 022304 (2002).
  - [5] A. K. Ekert, *Phys. Rev. Lett.* **67**, 661 (1991).
  - [6] C. H. Bennett, G. Brassard, and N. D. Mermin, *Phys. Rev. Lett.* **68**, 557 (1992).
  - [7] G. L. Long and X. S. Liu, *Phys. Rev. A* **65**, 032302 (2002).
  - [8] F. G. Deng and G. L. Long, *Phys. Rev. A* **68**, 042315 (2003).
  - [9] L. Vaidman and N. Yoran, *Phys. Rev. A* **59**, 116 (1999).
  - [10] N. Lütkenhaus, J. Calsamiglia, and K. A. Suominen, *Phys. Rev. A* **59**, 3295 (1999).
  - [11] J. Calsamiglia, *Phys. Rev. A* **65**, 030301(R) (2002).
  - [12] K. Mattle, H. Weinfurter, P. G. Kwiat, and A. Zeilinger, *Phys. Rev. Lett.* **76**, 4656 (1996).
  - [13] J. A. W. van Houwelingen, N. Brunner, A. Beveratos, H. Zbinden, and N. Gisin, *Phys. Rev. Lett.* **96**, 130502 (2006).
  - [14] R. Ursin, T. Jennewein, M. Aspelmeyer, R. Kaltenbaek, M. Lindenthal, P. Walther, and A. Zeilinger, *Nature (London)* **430**, 849 (2004).
  - [15] J. T. Barreiro, N. K. Langford, N. A. Peters, and P. G. Kwiat, *Phys. Rev. Lett.* **95**, 260501 (2005).
  - [16] M. Barbieri, C. Cinelli, P. Mataloni, and F. De Martini, *Phys. Rev. A* **72**, 052110 (2005).
  - [17] G. Vallone, R. Ceccarelli, F. De Martini, and P. Mataloni, *Phys. Rev. A* **79**, 030301(R) (2009).
  - [18] P. G. Kwiat and H. Weinfurter, *Phys. Rev. A* **58**, R2623 (1998).
  - [19] S. P. Walborn, S. Pádua, and C. H. Monken, *Phys. Rev. A* **68**, 042313 (2003).
  - [20] C. Schuck, G. Huber, C. Kurtsiefer, and H. Weinfurter, *Phys. Rev. Lett.* **96**, 190501 (2006).
  - [21] M. Barbieri, G. Vallone, P. Mataloni, and F. De Martini, *Phys. Rev. A* **75**, 042317 (2007).
  - [22] J. T. Barreiro, T. C. Wei, and P. G. Kwiat, *Nat. Phys.* **4**, 282 (2008).
  - [23] T. C. Wei, J. T. Barreiro, and P. G. Kwiat, *Phys. Rev. A* **75**, 060305(R) (2007).
  - [24] K. Nemoto and W. J. Munro, *Phys. Rev. Lett.* **93**, 250502 (2004).
  - [25] Q. Lin and J. Li, *Phys. Rev. A* **79**, 022301 (2009).
  - [26] Q. Lin and B. He, *Phys. Rev. A* **80**, 042310 (2009).
  - [27] Y. B. Sheng, F. G. Deng, and H. Y. Zhou, *Phys. Rev. A* **77**, 042308 (2008).
  - [28] Y. B. Sheng, F. G. Deng, and H. Y. Zhou, *Phys. Rev. A* **77**, 062325 (2008).
  - [29] B. He, Y. Ren, and J. A. Bergou, *Phys. Rev. A* **79**, 052323 (2009).
  - [30] B. He, Y. Ren, and J. A. Bergou, *J. Phys. B* **43**, 025502 (2010).
  - [31] Y. M. Li, K. S. Zhang, and K. C. Peng, *Phys. Rev. A* **77**, 015802 (2008).
  - [32] H. Jeong and N. B. An, *Phys. Rev. A* **74**, 022104 (2006).
  - [33] G. S. Jin, Y. Lin, and B. Wu, *Phys. Rev. A* **75**, 054302 (2007).
  - [34] S. D. Barrett, P. Kok, K. Nemoto, R. G. Beausoleil, W. J. Munro, and T. P. Spiller, *Phys. Rev. A* **71**, 060302(R) (2005).
  - [35] C. Simon and J. W. Pan, *Phys. Rev. Lett.* **89**, 257901 (2002).
  - [36] D. Bouweester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, *Nature (London)* **390**, 575 (1997).
  - [37] D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, *Phys. Rev. Lett.* **80**, 1121 (1998).
  - [38] M. Zukowski, A. Zeilinger, M. A. Horne, and A. K. Ekert, *Phys. Rev. Lett.* **71**, 4287 (1993).
  - [39] S. Bose, V. Vedral, and P. L. Knight, *Phys. Rev. A* **57**, 822 (1998).
  - [40] H. J. Briegel, W. Dür, J. I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **81**, 5932 (1998).
  - [41] W. Dür, H. J. Briegel, J. I. Cirac, and P. Zoller, *Phys. Rev. A* **59**, 169 (1999).
  - [42] L. M. Duan, M. D. Lukin, J. T. Cirac, and P. Zoller, *Nature (London)* **414**, 413 (2001).
  - [43] B. Zhao, Z. B. Chen, Y. A. Chen, J. Schmiedmayer, and J. W. Pan, *Phys. Rev. Lett.* **98**, 240502 (2007).
  - [44] P. van Loock, T. D. Ladd, K. Sanaka, F. Yamaguchi, K. Nemoto, W. J. Munro, and Y. Yamamoto, *Phys. Rev. Lett.* **96**, 240501 (2006).
  - [45] W. J. Munro, R. Van Meter, S. G. R. Louis, and K. Nemoto, *Phys. Rev. Lett.* **101**, 040502 (2008).
  - [46] J. W. Pan, D. Bouweester, H. Weinfurter, and A. Zeilinger, *Phys. Rev. Lett.* **80**, 3891 (1998).

- [47] C. Y. Lu, T. Yang, and J. W. Pan, *Phys. Rev. Lett.* **103**, 020501 (2009).
- [48] P. Kok, W. J. Munro, K. Nemoto, T. C. Ralph, J. P. Dowling, and G. J. Milburn, *Rev. Mod. Phys.* **79**, 135 (2007).
- [49] P. Kok, H. Lee, and J. P. Dowling, *Phys. Rev. A* **66**, 063814 (2002).
- [50] J. Gea-Banacloche, *Phys. Rev. A* **81**, 043823 (2010).
- [51] J. H. Shapiro, *Phys. Rev. A* **73**, 062305 (2006).
- [52] J. H. Shapiro and M. Razavi, *New J. Phys.* **9**, 16 (2007).
- [53] H. F. Hofmann, K. Kojima, S. Takeuchi, and K. Sasaki, *J. Opt. B* **5**, 218 (2003).
- [54] C. Wittmann, U. L. Andersen, M. Takeoka, D. Sych, and G. Leuchs, *Phys. Rev. A* **81**, 062338 (2010).
- [55] I. Friedler, D. Petrosyan, M. Fleischhauer, and G. Kurizki, *Phys. Rev. A* **72**, 043803 (2005).
- [56] I. Friedler, G. Kurizki, and D. Petrosyan, *Phys. Rev. A* **71**, 023803 (2005).
- [57] I. Friedler, G. Kurizki, and D. Petrosyan, *Europhys. Lett.* **68**, 625 (2004).
- [58] P. Kok, *Phys. Rev. A* **77**, 013808 (2008).
- [59] G. J. Pryde, J. L. O'Brien, A. G. White, S. D. Bartlett, and T. C. Ralph, *Phys. Rev. Lett.* **92**, 190402 (2004).
- [60] T. C. Ralph, S. D. Bartlett, J. L. O'Brien, G. J. Pryde, and H. M. Wiseman, *Phys. Rev. A* **73**, 012113 (2006).
- [61] G. J. Pryde, J. L. O'Brien, A. G. White, T. C. Ralph, and H. M. Wiseman, *Phys. Rev. Lett.* **94**, 220405 (2005).