Gedanken experiments with Casimir forces and vacuum energy

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Gedanken experiments are used to explore properties of quantum vacuum energy that are currently challenging to explore experimentally. A constant lateral Casimir force is predicted to exist between two overlapping finite parallel plates at 0 K; otherwise it would be possible to extract an arbitrary amount of energy from the quantum vacuum. A rigid unpowered object cannot be accelerated by the quantum vacuum because of the translational symmetry of space. By considering systems in which vacuum energy and other forms of energy are exchanged, we demonstrate that a change ΔE in vacuum energy, whether positive or negative with respect to the free field, corresponds to an equivalent inertial mass and equivalent gravitational mass $\Delta M = \Delta E/c^2$. We consider the possibility of a gravitational shield and show that, if it exists, the energy to operate it would have to cancel the net energy extracted from the gravitational field; otherwise we could extract an arbitrary amount of energy from the field.

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I. INTRODUCTION

The Casimir force per unit area between two infinite, parallel, perfectly conducting plates is given by $F(a) = -K/a^4$, where $K = \pi^2\hbar c/240 = 1.3 \times 10^{-27}$ N m^2, and $\hbar$ is Planck's constant and c is the speed of light in vacuum. This attractive force arises because the plates change the vacuum energy density between the plates from the free-field energy density. Although the force was predicted by Casimir in 1948, it is so small, even at distances of several tenths of a micrometer, that a quantitative measurement was not made until 1998, when an atomic force microscope (AFM) was used to measure the force between a sphere and a plate to an accuracy of 1% [1]. The challenge of securing parallelism between plates with submicrometer separations has limited the accuracy of force measurements between two plates to about 15% [2].

We are interested in considering several aspects of vacuum energy and Casimir forces, including the inertial mass associated with vacuum energy, the interaction of vacuum energy and gravity, and the possibilities of utilizing vacuum energy for propulsion or other purposes. There are three conceptual types of mass: inertial mass that resists acceleration, active gravitational mass that generates a gravity field around it, and passive gravitational mass that reacts to a gravitational field. These terms arise in the parametrized post-Newtonian expressions for gravitational energy and force and are discussed by Will [3]. These terms all can conceivably be positive, negative, imaginary, complex, position dependent, or anisotropic. Some of them can be conceivably identical. Newtonian mechanics and general relativity assume that inertial mass, active gravitational mass, and passive gravitational mass are identical, positive, and isotropic, and no experiments to date have contradicted these assumptions. The equivalence principle assumes that inertial mass and passive gravitational mass are identical and independent of material, and the measurements to date have not contradicted this assumption [3]. The notion of inertial mass arises in special relativity as the Lorentz-invariant norm $p^\mu p_\mu$ of the energy-momentum four-vector (E, pc), namely, $p^\mu p_\mu = E^2 - p^2c^2 = m^2c^4$. In the rest frame of the particle, the momentum p is zero, so the energy is mc^2 and m is called the rest mass of the particle. The existence of inertial mass can be seen as a consequence of the four-dimensional symmetry of space-time.

Experimentalists measuring Casimir forces have looked for a modification to the usual force of gravity at short distances as proposed by Fischbach *et al.* [4], but to date no such modifications have been found [5].

There is no generally accepted theory of inertial mass [3]. A recent proposal based on interactions with the vacuum field is controversial [6], but nevertheless the vacuum field does seem to be a factor. In conventional quantum electrodynamics, radiative shifts arise from the interactions of a particle with the zero-point fluctuations of the vacuum electromagnetic field. The real part of the shift is a mass shift. The vacuum field can be interpreted as jostling an idealized point particle, giving it kinetic energy and an equivalent mass [7]. The amplitude of the motion is too small to be observed directly, but changes in the vacuum field can result in measurable changes in the mass. Thus if the vacuum field is altered from the free field, mass shifts occur. For example, a spinless particle near a surface experiences a different vacuum field than if very far from the surface, which results in a shift in the effective mass [8]. If there were a generally accepted theory of inertial mass, then it might be somehow "different" in definition and perhaps behavior than gravitational mass. Even so, it is highly probable that active gravitational mass and passive gravitational mass are identical and positive for all ordinary matter we know. Very puzzling dynamics can occur if they are not equal.

Gravity is generated by the local energy-momentum tensor source term in the Einstein gravity equation, which is a function of mass, energy, linear momentum, angular momentum, stress, charge, spin, etc. Some contributions, such as the "gravitational twist" that angular momentum makes, are gravity fields that are distinguishable from the radial Newtonian gravity field of the rest mass. If the ground-state electromagnetic energy in the quantum vacuum were treated like any other form of energy, it would be expected to produce a corresponding gravitational field, and changes in the energy would be expected to produce changes in the gravitational

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field. There are, of course, some severe problems reconciling quantum field theory with general relativity: The current theory would regard the infinite quantum vacuum energy density as a gravitational source term, the effect of which would be to rip apart the universe [9]. Although this inconsistency between two widely accepted theories has not been resolved, the general consensus is that only changes in vacuum energy act as a source of a gravitational field.

Calculations of vacuum stresses for a variety of geometric shapes, such as spheres, cylinders, rectangular parallelepipeds, and wedges, are reviewed in [10–12]. There are complications and problems with the computation of vacuum energies of objects and surfaces, especially divergences in the energy which arise from curvature in the surfaces [13], such as right angles, or from ideal boundary conditions, such as perfect conductors [14,15]. The material properties, such as the frequency-dependent dielectric constant and plasma frequency of the metal and the surface roughness, affect the vacuum forces. In addition, in the usual calculations only a spatial average of the force for a given area for the ground state of the quantum vacuum field is computed, and material properties, such as binding energies, are ignored, a procedure which Barton has questioned [16]. Difficulties is defining vacuum energies for spheres and other shapes have been discussed by Graham *et al.* [17]. We will consider the vacuum force for a very simple case of two finite plates sliding over each other with a fixed separation.

There are other less dramatic difficulties in distinguishing the energy and mass of particles in gravitational fields, for example the difficulties of including the effect of gravity on the mass of a particle, as in a system of two particles with a gravitational potential energy between the masses that results in a gravitational binding energy which changes the effective particle mass [18]. There are even difficulties with the notion of a particle in curved spaces [19].

From quantum field theory, it appears that only the changes in vacuum energy from the free-field values are meaningful experimentally, since we can compute these changes only, not the absolute value of the energy, which generally is divergent. For example, the Casimir energy density for a parallel plate geometry is actually the difference between the free-field vacuum energy density and the vacuum energy density between the plates. Similarly, the Lamb shift in the hydrogen atom can be computed as the real part of the shift in the atomic energy level that arises when the atom is placed in the quantum vacuum. It can also be computed as the corresponding shift in the energy of the quantum vacuum that occurs when hydrogen atoms are put in the quantum vacuum [12]. The latter shift arises from the change in the frequencies of the vacuum field due to the change in the index of refraction from the hydrogen atoms. It makes sense to propose that a shift in vacuum energy corresponds to a shift in the inertia of an object [20]. To verify this hypothesis, Reynaud calculated the effective inertial mass of two parallel mirrors that are coupled by the vacuum field and found that this mass included the shift in the vacuum energy between the two plates [21]. Changes in vacuum energy density result in changes in the effective mass.

When we consider vacuum energy, there are three questions that we will address: (1) Is a change in inertia of a system associated with a change in the vacuum energy of the system? (2) Is a gravitational field generated by the change in vacuum energy (equivalent active gravitational mass)? (3) If an external gravitational field is present, is there a change in the gravitational energy of the system that is associated with the change of vacuum energy (equivalent passive gravitational mass)? We will consider these questions and several other questions using a variety of gedanken experiments [22].

The benefit of this approach is that we can assume a vastly simpler landscape than the formal approach using quantum electrodynamics and general and special relativity, omitting details of specific systems and the explicit consideration of divergences. This simplification can highlight the role of basic concepts and clarify and generalize the essential physics. On the other hand, what is lost are the details, for example the presence of divergences and the explicit consideration of the Hamiltonian of the vacuum field. It is therefore very reassuring that for the cases in which formal calculations have been attempted, there is agreement with our gedanken results. For the parallel plate geometry, Milton *et al.* computed that changes in vacuum energy correspond to changes in inertial mass and couple to gravity in the same way as conventional forms of energy [23,24].

The gedanken experiment was a powerful tool in the hands of Einstein. He described a gedanken experiment in which he said he "demonstrated that the mass of a body is a measure of its energy content; if the energy changes by E, the mass changes by E/c^2." Probably his most famous gedanken experiment was the Einstein-Podolsky-Rosen description of entangled states. Unfortunately, he never considered vacuum energy.

Another benefit of gedanken experiments is that we consider systems that may be very difficult or impossible to investigate experimentally. Many of the effects related to vacuum energy are extremely small. Nevertheless, they are of fundamental interest. For example, the Schornhorst effect [25,26] predicts that light moving transversely between parallel plates propagates faster than c, but the effect is so small it cannot be directly measured [27]. Negative vacuum energy acts like a negative mass. Calloni *et al.* [28] have considered the repulsive gravitational force due to the negative vacuum energy in a stack of 10^6 parallel plate capacitors and have found that it is slightly beyond the current capability for measurement using the most sophisticated gravitational wave detection technology.

In addition to the fundamental understanding of vacuum energy and gravitation, we are interested in the potential role of vacuum energy in space travel [29–31]. Hence several space-motivated gedanken experiments are included. Science fiction author Arthur C. Clark, who proposed geosynchronous communications satellites in 1945, described a "quantum ramjet drive" in 1985 in *Songs of Distant Earth* and observed, "If vacuum fluctuations can be harnessed for propulsion by anyone besides science-fiction writers, the purely engineering problems of interstellar flight would be solved" [32]. Recently, Australian writer Ken Ingle described a quantum-vacuum-powered engine [33]. There have been numerous papers on space warps and drives that often presuppose the ability to generate material with negative mass, or generate macroscopic gravitational fields by manipulation of vacuum energy [34]. One proposal is to employ the Casimir effect to reduce the vacuum energy density below the free-field value, but this effect is very small, as Calloni computed [28]. The effect of gravitational shielding on the kinematics of rockets was considered in Ref. [35]. Unfortunately, these interesting ideas are well beyond any technology that we can foresee. Without some breakthrough, such as a new boundary condition on the vacuum that causes much greater energy shifts, interstellar exploration appears impossible.

For simplicity in this paper we will only consider the free quantum vacuum field at zero absolute temperature. In the gedanken experiments, we will assume that for ordinary matter the active, passive, and inertial masses are identical and that for ordinary energy E, such as chemical or mechanical, the contribution to inertial mass is given by $E = mc^2$.

The first gedanken experiment demonstrates that there must be a lateral Casimir force acting when one finite flat plate slides over another perfectly parallel finite flat plate. If this lateral Casimir force were not present, it would be possible to extract an arbitrary amount of energy from the quantum vacuum. (In Casimir research, the same phrase, "lateral force," has been used to describe transverse vacuum forces between corrugated surfaces [36].)

The second gedanken experiment shows that a rigid unpowered object cannot be accelerated in the quantum vacuum unless some of the mass of the object is being converted to energy directly, as in radioactive decay. If an unpowered object could be accelerated by the quantum vacuum, it would, in principle, be possible to extract an unlimited amount of energy from the vacuum, and we would have a continuous acceleration for a spacecraft with no expenditure of energy.

The third gedanken experiment demonstrates that we need to associate an inertial mass Δm with changes ΔE in vacuum energy according to $\Delta E = \Delta mc^2$. The fourth, fifth, and sixth experiments show that changes in vacuum energy correspond to equivalent active and passive gravitational masses. Changes in vacuum energy couple to the gravitational field like other forms of energy; otherwise one could continuously extract energy from a gravitational field.

The seventh gedanken experiment is motivated by science fiction writers proposing gravitational shields. We consider the existence of a box which would insulate the mass inside from the effects of an external gravitational force. This device would lead to the paradox of being able to extract energy continuously from the gravitational field, unless the energy required to open and close the box just canceled the extracted energy.

II. GEDANKEN EXPERIMENTS

A. Gedanken experiment 1: Existence of a finite-flat-plate lateral Casimir force

In this gedanken experiment we consider the energy balance when we move parallel conducting plates through a cycle of both lateral and transverse motions [37]. Initially we have two perfectly conducting, completely overlapping ($x = L$), square, parallel plates, a distance L on each side, that are a distance a apart, with $a \ll L$. If we allow the upper plate to approach the lower (fixed) plate quasistatically, then the attractive Casimir force $F_C(a) = -KL^2/a^4$ does positive mechanical work during this reversible thermodynamic transformation. We are neglecting all edge effects by assuming that the force is proportional to the exact area of overlap. During the transformation, the vacuum energy $E_C(a) = -KL^2/3a^3$ between the plates will be reduced, conserving the total energy in the system. If the separation decreases from a_i to a_f, then the energy balance is

$$E_C(a_f) - E_C(a_i) = -\int_{a_f}^{a_i} F_C(a)\, da. \qquad (1)$$

If we then separate the plates quasistatically, letting a increase from a_f to a_i, we do work on the system to restore it to its initial state. Over the entire cycle, no net work is done, and there is no net change in the vacuum energy.

Consider an alternative cycle that has been proposed in order to extract energy from the vacuum fluctuations: After the plates have reached the point of minimum separation, slowly slide the upper plate laterally until it no longer is opposite the lower plate ($x = 0$), which eliminates the normal Casimir force, then raise the upper plate to its original height, and slowly slide it laterally over the lower fixed plate ($x = L$). Finally, we allow the plates to come together as before, extracting energy from the vacuum fluctuations and doing mechanical work. If no energy was expended in moving the plates laterally, then this cycle would indeed result in net positive work equal to the energy extracted from the vacuum. Although no one has yet computed in detail the lateral forces between offset finite parallel plates, it is highly probable that such forces exist and that no net extraction of energy occurs for this cycle.

We can verify this by a simple approximate calculation. We neglect Casimir energy "fringing fields," and we assume that the energy density differs from the free-field density only in the region in which the two square ($L \times L$) plates overlap an amount x, where $0 < x < L$ (see Fig. 1). Then we can compute the lateral force F_{L2} between the two plates using the conservation of energy (principle of virtual work):

$$F_{L2}(x) = -d(-KLx/3a^3)/dx = KL/3a^3, \qquad (2)$$

where a ($a \ll L$) is the perpendicular distance between the plates. This constant (independent of x) attractive lateral force tends to increase x or pull the plates toward each other so they have the maximum amount of overlap and minimum vacuum energy. In fact, the positive work done to move one plate laterally a distance L exactly cancels the work extracted from the vacuum fields in moving the plates from a large separation to a distance a apart, so there is no net change in total energy (mechanical plus field) in the complete cycle, as expected.

The normal Casimir force between these $L \times L$ plates when they are directly opposite, with complete overlapping

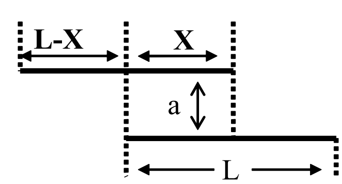

FIG. 1. Two square $L \times L$ parallel perfectly conducting plates with an overlapping area $L \times X$, separated by a distance a. Only lateral motion is permitted.

$(x = L)$, is $L/3a$ times larger than the constant lateral force given by Eq. (2). Experimental verification of lateral forces on flat plates is challenging but may be possible. This force could be determined by a measurement in the shift of the average position $\langle x \rangle$ of a vibrating surface opposite the edge of a fixed surface. One experimental arrangement would be to have a vertical cantilever with a flat horizontal plate on the top. This vertical cantilever could be vibrated horizontally in the x direction and its deflection measured. Then a second (horizontal) cantilever with a ball on the end would be mounted on an xyz piezostage allowing it to move in the y direction (up and down), as well in the x direction (in and out). The ball would be brought down so its center would be directly above the edge of the plate on the vertical cantilever. The ball has an approximately fixed x coordinate, and the y coordinate must be close enough so the Casimir force is measurable. Then the plate will experience a lateral force. If the vertical cantilever is vibrating, then the lateral force from passing over one edge of the plate will result in a net force that will shift the mean location of the plate as the ball moves closer to the plate. The vertical Casimir force will also be changing.

Numerous investigators have considered from a theoretical perspective the situation of two infinite plates at 0 K sliding at constant velocity over one another. Some researchers have concluded that a quantum friction is present and some have not. These efforts were recently reviewed by Philbin and Leonhart [38], who computed that there is no quantum friction for this situation, although there is a higher-order modification of the transverse Casimir force due to the velocity. Not all researchers agree with their conclusions. As they mention, the situation for finite plates is quite different, which our gedanken experiment confirms. In the gedanken experiment we assumed the motion was slow, and we neglected velocity corrections to the transverse Casimir force.

B. Gedanken experiment 2: No quantum vacuum sails

This gedanken experiment shows that no rigid, unpowered object can experience a net acceleration in the quantum vacuum, unless its mass is being directly converted to energy, as in radioactive decay. Imagine an object in the free-field isotropic vacuum, distant from any other objects, whose geometry is fixed. The object might be composed of various materials, with various dielectric coefficients, in thermal equilibrium, and with a fixed arbitrary shape. Assume the object does not contain any power supply, mechanical or electric. It is generally quite difficult to explicitly compute the vacuum stress on such an object; however, if the object did experience a net acceleration in the vacuum, then one could, in principle, use the movement of the object to operate a machine and extract an arbitrary amount of energy from the vacuum.

First consider a plane surface, because a variety of sail concepts have been proposed [39]. We can view the vacuum as a source of radiation pressure from virtual photons [12]. The challenge is to design surfaces that alter the symmetry of the free vacuum and produce a net force. Consider, for example, a sail made of two different materials on opposite sides, that absorb electromagnetic radiation differently. Can we expect a net force on the sail? A simple classical analysis as shown in Fig. 2 suggests the answer to this question [40].

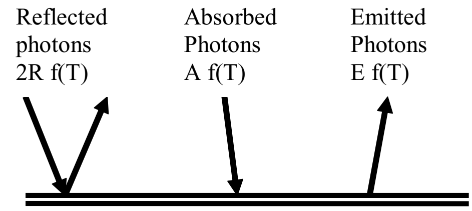

FIG. 2. Schematic of the momentum transfer from electromagnetic radiation to a sail made of different materials on the top and bottom.

For a given frequency, assume the radiation energy density is proportional to $cf(\omega,T)$, so the net momentum transfer ΔP_ω to the top surface is

$$\Delta P_\omega = A_\omega f(\omega,T) + E_\omega f(\omega,T) + 2R_\omega f(\omega,T), \quad (3)$$

where A_ω is the absorptivity, E_ω is the emissivity, R_ω is the reflectivity, and T is the temperature. For a body in thermodynamic equilibrium, $A_\omega = E_\omega$, and by definition, $1 = A_\omega + R_\omega$. By using these restrictions, it follows that $\Delta P_\omega = 2f(\omega,T)$, which is independent of the material properties. Therefore the force on the opposite side of the sail just cancels this force, and there is no net acceleration. This conclusion holds at every frequency.

We assumed that the temperature of the sail is the same on both sides because of the intimate contact. If the radiation spectrum corresponds to that at zero temperature, the zero-point field, then both sides of the sail would be at 0 K. On the other hand, there would be a net force if one side of the sail was pointed at a source of photons (such as our sun), causing a different radiation density on one side of the sail than the other. If one made a powered sail in which a temperature gradient was maintained across the sail, a net force could occur, and it would be a function of the energy required to maintain the temperature difference.

For the most general rigid, unpowered object, consider that the Hamiltonian H of the object depends on the various internal coordinates (q_i, p_i) corresponding to the object's geometry. We assume because of the translational invariance of space that the energy of the object does not depend on the location of the center of mass, for which the corresponding operator is $\mathbf{Q}$, nor does it depend explicitly on time. Then it follows that

$$[H(q_i,p_i,\mathbf{P}_i),\mathbf{P}_j] = i\hbar\frac{\partial H}{\partial Q_j} = 0, \quad (4)$$

where $P_j = -i\hbar\partial/\partial Q_j$ is the operator for the j component of the center-of-mass momentum. The Hamiltonian might depend on the center-of-mass momentum $\mathbf{P}$. Since the Hamiltonian is also the generator of the translations in time, it follows that

$$[H(q_i,p_i,\mathbf{P}_i),\mathbf{P}] = i\hbar\frac{\partial \mathbf{P}}{\partial t} = 0 \quad (5)$$

and the momentum of the center of mass is conserved.

There are two possibilities: (1) The inertial mass remains constant, so the center-of-mass velocity must also be constant and there is no acceleration, or (2) the inertial mass and velocity both vary but in a way that conserves the center-of-mass momentum. However, such a variation would not be consistent with a constant kinetic energy of the center of mass. This would imply that there must be a compensating change in another form of energy within the system. In effect, the mass energy is being converted to kinetic energy. This might be due to the decay of an excited or radioactive atom emitting particles or radiation. However, if we do not have decay or a some similar process converting mass to energy, then we conclude that there is no net acceleration of our object.

This gedanken experiment shows, for example, that neither a passive air foil nor a rigid open cavity (box with no top on it) can accelerate by itself in vacuum.

Although this result might seem obvious or trivial, there are some assumptions and subtleties. We have not explicitly included a Hamiltonian for the vacuum fields and therefore we have not explicitly considered the role of curvature and singularities in the energy-momentum tensor for the vacuum field. Since no real photons are generated by a curved surface, one would not expect this to alter our conclusion. On the other hand, if nuclei with $Z\alpha > 1$ were present, then real photons could be produced from the quantum vacuum, so we are excluding this possibility. Another subtlety is that near a surface, the vacuum force shows small fluctuations (and therefore there will be small variations in the net force) that tend to accelerate the surface, statistically in random directions [41]. From dimensional considerations the root-mean-square value of the fluctuations in the vacuum force scales as $\hbar/c^3T^8$, where T is the time interval between measurements. The effect is negligible except for extraordinarily short times. Calculation shows that the effect may add slightly to the fluctuations based on the time-energy uncertainty relation. In the same spirit, Rueda has suggested that very high-energy particles observed in space may derive their kinetic energy from a long-term acceleration due to the stochastic vacuum field [42].

The assumption that the device is not powered arises because of the possibility that if the device were powered, then it could be accelerated in the vacuum. Consider, for example, an anharmonically vibrating plate which causes the emission of photons by the adiabatic Casimir effect [43]. The radiative reaction will result in a small net acceleration of the plate. In this example of a vibrating plate, the Hamiltonian must include the radiation field of the photons, which is correlated with the moving center of mass.

One wonders whether there are other unrecognized subtleties in the seemingly innocuous assumptions that may modify this gedanken experiment. For example, perhaps there may be presently unknown distortions or excitations in the vacuum field that do not correspond to the emission of photons but that nevertheless carry momentum and energy, like a form of dark energy-momentum.

C. Gedanken experiment 3: Vacuum energy contributes to inertial mass

From general relativity, various conventional forms of energy E are considered to contribute to the inertial mass as given by the equation $E = mc^2$. This gedanken experiment is designed to show that a change in vacuum energy also gives a corresponding change in inertial mass. Imagine an apparatus,

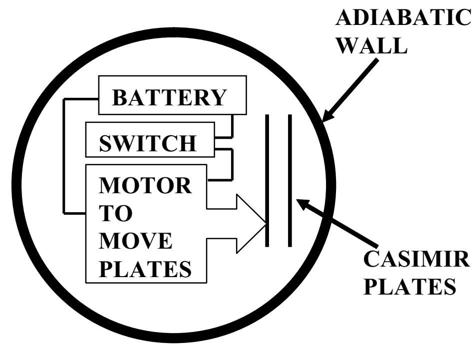

FIG. 3. Schematic showing apparatus described in gedanken experiment 3, consisting of a motor-generator to alter the spacing between two parallel plates, a battery to power the motor, and a timer switch, all surrounded by a wall impervious to thermal energy and matter.

the details of which will be described later, that is contained within a small sphere with uniform, rigid, insulating walls so that the system is closed. The sphere is small enough so that it can serve as an inertial frame. We assume that no heat, thermal radiation, energy, nor mass pass through the walls, which do not vibrate. The sphere is embedded in the quantum vacuum, far from any other objects or gravitational forces. The Casimir force on the sphere will be inward, and therefore it will not tend to accelerate the center of mass of the sphere. As mentioned before, to a higher order in $\hbar$ there are stochastic forces that tend to accelerate the sphere in random directions, as in Brownian motion [44]. As a practical matter these can be neglected over the duration of the experiment. We assume that the sphere is not moving initially, or is in uniform nonrelativistic motion. For simplicity we consider only the vacuum field at 0 K; for fields at higher temperatures there are, as Einstein proved, additional forces present [45]. We assume vacuum fluctuations are present within the sphere.

Within the sphere is a system consisting of parallel Casimir plates, a battery which powers a motor which can change the spacing between the plates, and a timer switch that controls when the motor turns on and off (Fig. 3). When the plates are moved closer together quasistatically, the vacuum energy between them decreases (becomes more negative), and work is done on the motor, charging the battery. If we assume no dissipative forces, the total energy in the sphere is conserved, and the decrease in the vacuum energy between the plates equals the increase of the chemical energy in the ideal battery. This process is reversible. (The same results would be obtained if we used a coiled spring with a mechanical linkage instead of a motor and battery.) The question is whether this transformation of energy alters the motion of the sphere. As in gedanken experiment 2, no external force has acted on the closed system while the spacing between the plates was changed, so the momentum and energy must remain constant. As before, there are two possibilities: (1) The velocity and the inertial mass are constant or (2) the inertial mass decreases and the velocity increases.

If the velocity and inertial mass both changed in a manner that kept the momentum constant, then the kinetic energy would have to change. However, an increase in kinetic energy would violate conservation of energy; therefore a sphere in the quantum vacuum would not accelerate. In other words, it is not possible for a single moving body to conserve momentum and kinetic energy unless the velocity is constant. Since the center-of-mass velocity does not change as the plate separation is decreased, it follows from the conservation of momentum that the inertial mass is constant.

We conclude that vacuum energy contributes to inertial mass in the same way as the chemical energy in the battery.

The same result would follow for other forms of energy. For example, assume we used a spring to drive the motion of the plates instead of a motor-battery combination. The vacuum energy would contribute to the inertial mass just as the potential energy in the spring contributed. Even if dissipative forces were present, and vacuum energy was degraded to heat, the total energy would remain constant as would the inertial mass. All forms of energy, including quantum vacuum energy, contribute to the inertial mass according to general relativity.

The total energy in the sphere is conserved, although the form of the energy may change. Within the container, there may be quasistatic, adiabatic transformations, in which one type of energy increases, while another type decreases, so the total remains constant. This suggests that one can associate an increase in effective inertial mass with one element and a decrease of effective inertial mass with another element. The increase and decrease are with respect to the original state. This suggests that it might be possible to make components that have negative inertial mass. Such objects would tend to rise in a uniform gravitational field. Indeed negative vacuum energy in the stack of parallel plate capacitors considered by Calloni *et al.* theoretically resulted in a force in a gravitational field that was in the opposite direction from that experienced by normal positive matter, but the positive force due to the mass of the silicon wafers, was much larger. Could one make an object that floated in a gravitational field? Since the objects we know of are composed of material with a positive mass, it is not clear that one can make an object whose overall mass is negative. The total energy in a Casimir parallel plate arrangement, even with plates an atom thick, remains positive [46].

To get a sense of the magnitude of forces involved consider two square parallel plates L long separated by a distance a in a gravitational field g. Then the total attractive Casimir force between the plates is $F_C = -L^2K/a^4$. To lowest order in g, the gravitational force F_g on the parallel plates due to the negative vacuum energy $E_C = -aL^2(K/3a^4)$ is [28]

$$F_g = \frac{gE_C}{c^2} = \frac{-gL^2}{c^2}\frac{K}{3a^3}. \tag{6}$$

The ratio of this gravitational force to the Casimir force is

$$\frac{F_g}{F_C} = \frac{ag}{3c^2}. \tag{7}$$

For a typical plate separation $a \sim 100$ nm, the effective mass of the Casimir energy E_C is minus that of about 450 protons per cm^2. The ratio of the gravitational force to the Casimir force is thus 1.1×10^{-23}. Although this number is incredibly small, current gravity wave detectors can monitor the position of a test mass to one part in 10^{21}, so at some point it may be possible to conduct an experiment [47].

D. Gedanken experiment 4 (falling spheres): Vacuum energy contributes to passive gravitational mass

This gedanken experiment is designed to show that a shift in vacuum energy gives a corresponding passive gravitational mass. We imagine two spheres as described in the previous experiment, with Casimir plates, batteries, and motors falling near each other in the quantum vacuum in a weak gravitational field. We assume that the chemical energy of the battery gives a passive gravitational mass that couples to gravity in the normal way. On the other hand, we assume that vacuum energy does not couple to gravity. As one sphere is falling, the plate spacing remains fixed, while in the other sphere the motor alters the spacing between the plates, converting chemical energy from the battery into changes in the quantum vacuum energy between the parallel plates. By our assumption, the acceleration of the second sphere will increase or decrease as energy is transferred from the battery to the Casimir plates.

In this system, the kinetic energy of both spheres is increasing with time. A change in the acceleration of one sphere relative to the other sphere would require an additional external force, which is not present. Hence, the acceleration must not change with changes in vacuum energy. This shows that changes in the quantum vacuum energy give rise to corresponding changes in the passive gravitational mass and inertial mass. Vacuum energy couples to gravity the same way any other form of energy is expected to couple to gravity.

E. Gedanken experiment 5 (explicit calculation in the gravitational field of a mass): Vacuum energy contributes to passive gravitational mass

In this experiment, we consider the coupling of vacuum energy to a gravitational field of a mass M. In the gedanken experiment, we will determine the quantitative consequences if vacuum energy does not couple to the gravitational field. Consider the same apparatus used in previous gedanken experiments consisting of a battery with chemical energy U_B, a motor-generator, and Casimir plates with vacuum energy U_C. The apparatus is initially at a distance R_1 from a gravitational mass M. The initial gravitational potential energy of the chemical energy of the battery is

$$U_i = -GMU_B/(c^2R_1). \tag{8}$$

By assumption, there is no potential energy corresponding to the vacuum energy U_C. Assume we have a device, such as a motor-generator and rope, that can lower the apparatus from R_1 to a distance R_2 from the mass M. When this is done, the lowering device will have net positive work done on it, and the potential energy of the apparatus will decrease, but the sum of both will remain constant since energy is conserved. If we raise the apparatus back to R_1, then this net positive energy of the lowering device is used up, and there is no net change in energy in the system since the gravitational field is conservative. We assume there is no friction or other dissipative force, and we neglect the mass of the rope in the calculation.

Now imagine lowering the apparatus again from R_1 to R_2. Once the apparatus is at R_2, the gravitational potential energy of the chemical energy in the battery is $-GMU_B/(c^2R_2)$. The amount of work done by the lift device to lower the apparatus equals the change in potential energy

$$W_d = GMU_B\big(R_2^{-1} - R_1^{-1}\big)/c^2. \tag{9}$$

Assume the battery now turns on, which sends energy E to the motor, which increases the separation of the Casimir plates, which increases the vacuum energy to $U_C + E$. Conversely, the battery energy is reduced by the same amount to $U_B - E$, so the battery is lighter. Its potential energy at R_2 is reduced to $-GM(U_B - E)/(c^2 R_2)$. We assume that the energy in the quantum vacuum does *not* couple to gravity, so there is no increase in gravitational potential energy corresponding to the change in vacuum energy E.

Imagine now raising the apparatus from R_2 to R_1. Less work will be done to raise the apparatus to R_1 than before since the battery is lighter. At R_1 the potential energy of the battery is

$$U_f = -GM(U_B - E)/(c^2 R_1). \tag{10}$$

The amount of work done by the lift device is

$$W_u = -GM(U_B - E)\big(R_2^{-1} - R_1^{-1}\big)/c^2 \tag{11}$$

and the energy of the Casimir plates remains $U_c + E$.

Once the apparatus is at R_1, we imagine extracting vacuum energy E from the Casimir plates, so the vacuum energy is now U_C, and charging the battery to its original energy state U_B. This conversion will result in an additional gravitational potential energy of

$$U_E = -GME/(c^2 R_1). \tag{12}$$

The system has been returned to its original state, but there is a net increase in energy of the system. The net change in energy of the system equals the total energy of the final state minus the energy of the initial state:

$$\Delta E = W_d + W_u + U_f + U_E - U_i = GME\big(R_2^{-1} - R_1^{-1}\big)/c^2. \tag{13}$$

There is a net increase in energy of the system but no change in the state of the system. This is a clear violation of the conservation of energy. Hence our assumption is not valid and we must conclude that vacuum energy couples to the gravitational field like any conventional form of energy.

F. Gedanken experiment 7: Vacuum energy contributes to active gravitational mass

In order to show that vacuum energy contributes to active gravitational mass, we consider a variation on the experimental arrangement in the preceding gedanken experiment. We have a fixed apparatus, consisting of the motor, battery, and Casimir plates, and we assume its equivalent active gravitational mass is M. Assume we have a test mass m separated from the sphere by a distance R_1. We then move the mass m until it is a distance R_2 from the apparatus. The change in potential energy is $-GMm(R_2^{-1} - R_1^{-1})$. Then we increase the plate separation using energy E from the battery, which reduces the active gravitational mass of the battery by E/c^2 and the active gravitation mass of the apparatus to $M - E/c^2$. We assume the change in vacuum energy does not change the equivalent active gravitational mass. We now move the mass m back to its original location, doing an amount of work $-G(M - E/c^2)m(R_1^{-1} - R_2^{-1})$. We then use the battery to operate the motor and move the plates toward each other until they are at their original separation. An energy E is extracted from the vacuum and is used to charge the battery to its original energy state. This causes a shift in the potential energy of the mass m equal to $-Gm(E/c^2)(R_1^{-1})$. The system has been returned to its original state and there is a net increase in energy equal to $G(E/c^2)(R_2^{-1})$. This violates the conservation of energy. Hence our assumption that vacuum energy does not contribute to active gravitational mass is not true.

G. Gedanken experiment 8: No free energy with gravity shields

This experiment explores gravitational shields, the stuff of science fiction. A few experiments have been done, for example, with rotating superconductors to determine whether there is any evidence of gravitational shielding, with null results [31]. If such shielding devices were possible, how would they operate? What would be their limitations?

We consider a box with special walls that totally shield the interior of the box from any external gravitational field. The box has a door which can be opened and closed to insert a mass M. We assume that the inertial mass of M is not affected by the box. Assume the gravitational potential energy of the mass M is U_1 when we insert it into the box and close the door. (Closing the door can be understood as a euphemism for "turning on" the gravity shield for whatever is inside the box.) For simplicity, we assume that gravity does not exert any force on the box. Now imagine moving the box to a different location. Since there is no external force of gravity on the box and the box is stationary at the beginning and end of the movement, no net work is done. Imagine we now open the door and remove the mass. At this new location the gravitational potential energy of the mass M is U_2. By the conservation of energy, the change in potential energy $U_2 - U_1$ should equal the work done on the system. By our assumptions, no net work was done to move the box, so we conclude that to not violate the conservation of energy we must do an amount of work $U_2 - U_1$ to operate the door of the box. In general, the amount of work necessary to operate the door will equal the difference in energy between the mass M at its final location and its initial location. Assuming U_1, $U_2 < 0$, then positive work $-U_1$ is done to close the door and negative work U_2 must be done to open the door (or turn off the gravity shield).

For example, imagine we put a space capsule into the box. We then accelerate the box to begin an interstellar trip. No energy is used to overcome gravitational fields, only to overcome inertia, reducing fuel needs by several orders of magnitude. At the end of the trip, on some distant planet, the energy to open the box will simply be the change in potential energy. Conceivably, opening and closing the door might be done en route, near gravitational sources, as part of the navigational technology.

If we were to put the space capsule into the box on earth, and shut the door, then the earth's radial gravitational acceleration would suddenly disappear, and the mass would accelerate tangentially to the earth's surface at about

1000 miles/h near the equator, an interesting way to use the earth's rotational velocity to launch a space capsule. If the accelerating mass pushed against the wall of the gravitational box and accelerated the box tangent to the earth's surface, then in a simple geometric model (neglecting air resistance and the effect of gravity on the box itself) the box would be about 100 miles above the surface of the earth after one hour.

This gedanken experiment may be based on a material that is impossible to make. Using the analogy from electrostatics, shielding depends on the existence of positive and negative charges, whose effects can cancel each other. An atom of antimatter could indeed cancel the gravitational energy of an atom of matter, but they do not coexist in any known form, so the existence of a gravity shield might actually violate physical laws.

This observation that gravity shielding may be impossible brings to mind a recent proposal regarding the theoretical expressions of Lifshitz which are used to model Casimir forces for real materials [10,11]. A suggestion was made that the Lifshitz theory needed to be modified to account for screening effects and diffusion currents [48]. In the Lifshitz theory of Casimir forces thermal equilibrium is assumed. On the other hand, diffusion currents and screening effects occur when thermal equilibrium is not present. It appears that including these effects violates thermal equilibrium and hence is not consistent with the basic Lifshitz formulation [49]. This illustrates the subtleties that may lie in seemingly innocuous assumptions about screening. Would a box that shields against vacuum fluctuations be fundamentally impossible?

III. CONCLUSION

Gedanken experiments are used to explore properties of vacuum energy that are currently challenging or impossible to explore experimentally. A constant lateral Casimir force is predicted between two overlapping finite parallel plates; otherwise it would be possible to extract an arbitrary amount of energy from the quantum vacuum. By considering systems in which vacuum energy and other forms of energy are exchanged, we demonstrate that a change ΔE in vacuum energy, whether positive or negative with respect to the free field, corresponds to an equivalent inertial mass and gravitational mass $\Delta M = \Delta E/c^2$.

The first gedanken experiment demonstrated that there is a constant, finite lateral force at 0 K between two parallel, finite plates that overlap. The force tries to maximize the amount of overlap. Other gedanken experiments have shown that changes in vacuum energy formally couple to gravity like ordinary forms of energy. Otherwise, it is possible to design gedanken experiments in which an arbitrary amount of energy can be extracted from a physical system without changing the state, violating our usual form of the law of conservation of energy. Specifically, changes in vacuum energy correspond to equivalent active and passive gravitational masses. Positive shifts in vacuum energy act like ordinary matter, whereas negative shifts in vacuum energy correspond to negative masses, which are repelled by the gravitational force with ordinary matter. This unusual property of negative vacuum energy makes it very interesting, since it might allow, in principle at least, the formation of structures which have zero equivalent mass, and the cancellation of gravitational forces. Unfortunately, in practice, the methods used to generate the negative vacuum energy, for example, Casimir plates, are so limited in the negative energy density they can produce that it does not appear possible, without some new approach, to make an actual object that has net zero or a negative vacuum energy. Perhaps, in astrophysical systems, other boundary conditions pertain, and larger negative vacuum energies are possible.

Within the next decade, experiments may be done to verify some of the conclusions drawn from the gedanken experiments, for example, the lateral Casimir force. Extracting energy from the quantum vacuum is clearly possible if there is a change in the state of the system. It is done when the spacing between the Casimir plates is changed by the motor-battery combination in our gedanken experiments. Experiments on the exchange of energy between the quantum vacuum and ordinary physical systems will help us understand the role of vacuum energy. It is possible that new methods or new boundary conditions will be found that can be used to extract a large amount of energy from the quantum vacuum. Cole has considered this possibility in an astrophysical situation [50].

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[1] U. Mohideen and A. Roy, Phys. Rev. Lett. **81**, 4549 (1998); B. W. Harris, F. Chen, and U. Mohideen, Phys. Rev. A **62**, 052109 (2000).

[2] G. Bressi, G. Carugno, R. Onofrio, and G. Ruoso, Phys. Rev. Lett. **88**, 041804 (2002).

[3] C. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, revised 1993). This is an excellent book that clearly describes the key ideas and experiments in gravitational physics.

[4] E. Fischbach, D. Sudarsky, A. Szafer, C. Talmadge, and S. H. Aronson, Phys. Rev. Lett. **56**, 3 (1986).

[5] R. S. Decca, D. Lopez, H. B. Chan, E. Fischbach, D. E. Krause, and C. R. Jamell, Phys. Rev. Lett. **94**, 240401 (2005).

[6] B. Haisch, A. Rueda, and H. E. Puthoff, Spec. Sci. Technol. **20**, 99 (1997).

[7] V. Weisskopf, Rev. Mod. Phys. **21**, 305 (1949).

[8] G. Barton, Proc. R. Soc. London Ser. A **410**, 175 (1987); **320**, 251 (1970).

[9] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989); also see R. Adler, B. Casey, and O. Jacob, Am. J. Phys. **63**, 620 (1995).

[10] K. Milton, *Physical Manifestations of Zero-Point Energy, The Casimir Effect* (World Scientific, Singapore, 2001).

[11] M. Bordag, G. Klimchitskya, U. Mohideen, and V. Mostepanenko, *Advances in the Casimir Effect* (Oxford University Press, New York, 2009).

[12] P. Milonni, *The Quantum Vacuum. An Introduction to Quantum Electrodynamics* (Academic, San Diego, 1994).

[13] D. Deutsch and P. Candelas, Phys. Rev. D **20**, 3063 (1979); P. Candelas, Ann. Phys. (NY) **143**, 241 (1982); **167**, 257 (1986).

[14] B. DeWitt, in *Physics in the Making*, edited by A. Sarlemijn and M. Sparnaay (Elsevier, Dordrecht, 1989), pp. 247–272.

[15] G. J. Maclay, Phys. Rev. A **61**, 052110 (2000).

[16] G. Barton, J. Phys. A **37**, 1011 (2004); **34**, 4083 (2001).

[17] N. Graham, R. Jaffe, V. Khemani, M. Quandt, O. Schroeder, and H. Weigel, Nucl. Phys. B **677**, 379 (2004).

[18] S. Fulling, *Aspects of Quantum Field Theory in Curved Space Time* (Cambridge University Press, Cambridge, 1989).

[19] N. Birrell and P. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, 1984), pp. 48 and 102.

[20] A. Lambrecht and S. Reynaud, Phys. Rev. Lett. **84**, 5672 (2000).

[21] M. Jaekel and S. Reynaud, Phys. Lett. A **180**, 9 (1993).

[22] T. S. Gendler, *Thought Experiment: On the Powers and Limits of Imaginary Cases* (Garland, New York, 2000); M. Buzzoni, *Thought Experiment in the Natural Sciences* (Koenigshausen and Neumann, Würzburg, 2008).

[23] K. Milton, S. Fulling, P. Parashar, A. Romeo, K. Shajesh, and J. Wagner, J. Phys. A **41**, 164052 (2008).

[24] K. Milton, K. Shajesh, P. Parashar, and J. Wagner, J. Phys. A **41**, 164058 (2008).

[25] K. Scharnhorst, Phys. Lett. B **236**, 354 (1990).

[26] G. Barton, Phys. Lett. B **237**, 559 (1990). This paper shows that the Schorhorst effect is a direct consequence of Euler-Heisenberg interaction that includes radiative corrections to the electromagnetic field equations.

[27] P. Milonni and K. Svozil, Phys. Lett. B **248**, 437 (1990).

[28] E. Calloni, L. DiFiore, G. Esposito, L. Milano, and L. Rosa, Phys. Lett. A **297**, 328 (2002).

[29] S. Reynaud, A. Lambrecht, C. Genet, and M.-T. Jaekel, C. R. Acad. Sci. Paris **2**, 1287 (2001).

[30] H. E. Puthoff, S. R. Little, and M. Ibison, J. Brit. Interplanet. Soc. **55**, 137 (2002).

[31] *Frontiers in Propulsion Science*, edited by M. Millis and E. Davis (AIAA, Weston, VA, 2009).

[32] A. C. Clarke (personal communication). See the acknowledgments in *The Songs of Distant Earth* (Ballantine, New York, 1986). Numerous science fiction writers, including Clarke, Asimov, and Sheffield, have based spacecraft on the quantum vacuum.

[33] K. Ingle, *First Contact: Escape to 55 Cancri* (Books-ForABuck.com, Long Beach, CA, 2009).

[34] M. Morris and K. Thorne, Am. J. Phys. **56**, 395 (1988).

[35] M. Tajmar and O. Bertolami, J. Propulsion Power **21**, 692 (2005).

[36] F. Chen and U. Mohideen, G. L. Klimchitskaya, and V. M. Mostepanenko, Phys. Rev. Lett. **88**, 101801 (2002).

[37] J. Maclay, in *Proceedings of STAIF-2000* (Space Technology and Applications International Forum), edited by M El-Genk (AIP, Melville, NY, 2000).

[38] T. Philbin and U. Leonhardt, New J. Phys. **11**, 033035 (2009).

[39] M. Millis, Acta Astronaut. **44**, 175 (1999); AIAA J. Propulsion Power **13**, 577 (1997); also NASA Report No. TM-2004-213082, 2004.

[40] J. Maclay, in *Frontiers in Propulsion*, edited by M. Millis and E. Davis (AIAA, Weston, VA, 2009).

[41] G. Barton, J. Phys. A **24**, 991 (1991); G. Barton, in *Cavity Quantum Electrodynamics*, edited by P. Berman (Academic, San Diego, CA, 1994).

[42] A. Rueda, Space Sci. **53**, 223 (1990).

[43] J. Maclay and R. Forward, Found. Phys. **34**, 477 (2004).

[44] G. Barton, Comm. Mod. Phys. D **2**, 301 (2000).

[45] J. Maclay, H. Fearn, and P. Milloni, Eur. J. Phys. **22**, 463 (2001).

[46] M. Visser, *Lorentzian Wormholes: From Einstein to Hawking* (AIP, Melville, NY, 1996), pp. 81–87.

[47] E. Hand, Nature (London) **463**, 147 (2010).

[48] D. A. R. Dalvit and S. K. Lamoreaux, Phys. Rev. Lett. **101**, 163203 (2008).

[49] V. Mostepanenko, R. Decca, E. Fishbach, B. Geyer, G. Klimchitskaya, D. Krause, D. Lopez, and U. Mohideen, Int. J. Mod. Phys. A **24**, 1721 (2009).

[50] D. C. Cole, Phys. Rev. E **51**, 1663 (1995).