

## Shot-noise-limited measurement of sub-parts-per-trillion birefringence phase shift in a high-finesse cavity

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We report on a promising approach to high-sensitivity anisotropy measurements using a high-finesse cavity locked by optical feedback to a diode laser. We provide a simple and effective way to decouple the weak anisotropy of interest from the inherent mirror's birefringence whose drift may be identified as the key limiting parameter in cavity-based techniques. We demonstrate a shot-noise-limited phase shift resolution previously inaccessible in an optical cavity, readily achieving the state-of-the-art level of  $3 \times 10^{-13}$  rad.

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Ultimate performance in the definition of the polarization state of light plays a key role in fundamental physical investigations. Detecting changes in a polarization state is widely used to study optical effects in crystals, biological systems, or polymers. It has also demonstrated its ability to provide powerful tests of our understanding of light-matter interaction by the observation of magnetochiral birefringence [1], magnetoelectric Jones and linear birefringence [2,3], or magnetoelectric Jones dichroism [4].

Today, observing the vacuum in a field acting on the polarization of light is one of the all-optical candidates for probing photon-photon scattering, a fundamental feature of QED [5]. The two-photon interaction through the creation of virtual electron-positron pairs yields to a change of vacuum properties such as the Kerr birefringence (induced by an transverse electric field), the Cotton-Mouton birefringence (induced by a transverse magnetic field), or the magnetoelectric birefringence (induced by transverse electric and magnetic fields) [6]. The strength of these nonlinear interactions scales with the QED correction of the Lagrangian  $\frac{2\alpha^2\hbar^3}{45m^4c^5} \simeq 2 \times 10^{-30} \text{ m}^3 \text{ J}^{-1}$  where  $m$  is the electron mass, which yields to an induced phase shift in a vacuum as weak as  $3 \times 10^{-17} \text{ rad/T}^2\text{m}$  for optical wavelengths in the case of an external transverse magnetic field.

At such a low level of interaction, the contribution of new light scalar-pseudoscalar particles that couple to photons, such as axions, should also manifest itself as a sizable deviation from the QED prediction [7]. We may also recall that the PVLAS Collaboration, aiming at the detection of the vacuum magnetic birefringence [8], interpreted a thought-to-be signal as induced by intracavity axion production [9], even though it was finally attributed to an instrumental artefact [10]. The interest in axions, providing the answer to the strong-charge-parity problem, lies beyond particle physics and overlaps with cosmology, since such a particle is considered as a serious dark-matter candidate [11–14].

Probing polarization-state changes using the high sensitivity provided by a high-finesse cavity is expected to be very promising. As noticed by Hall *et al.* [15], the ultimate shot-noise-limited sensitivity, together with milliwatt optical power and state-of-the-art magnetic fields (10 T),

may resolve the QED predicted birefringence in a vacuum, but up to now no cavity-based measurement has been able to approach this fundamental limit. As pointed out by authors of advanced experiments [15,16], this is due to the drift of the inherent residual birefringence of high-reflectivity (HR) cavity mirrors. In this Rapid Communication, we present an approach which is unaffected by this problem and demonstrate a shot-noise-limited phase shift resolution using an optical cavity, down to the state-of-the-art level of  $3 \times 10^{-13}$  rad. We believe that this approach, which works well in the limit of very high cavity finesse, is a major step toward the measurement of effects such as the vacuum birefringence.

Let us begin by considering that the effect of any intracavity anisotropy (mirror coating, Kerr or Cotton-Mouton birefringence, etc.) is to lift the degeneracy of cavity polarization modes. For each transverse cavity mode, two eigenvectors of polarization are then defined by different resonance frequencies. One way to access the intracavity anisotropy is therefore to measure the mode splitting [15,17]. Hall *et al.* achieved a phase sensitivity of  $10^{-9} \text{ rad}/\sqrt{\text{Hz}}$  per pass across a dynamic range of seven orders of magnitude using this method [15]. However, in the limit of weak mode splittings, with partial spectral overlap of the resonance profiles, another scheme is available. As in usual anisotropy measurements, the sample is sandwiched between crossed polarizers. The cavity is illuminated by  $45^\circ$  polarized light relative to its birefringence axes, which excites both polarization modes (approximately linear and orthogonal). The ratio between the crossed and the parallel signals from the analyzer is then proportional to the square of the intracavity polarization change. This idea was first applied to detect small circular dichroisms induced in an atomic vapor by a polarized saturating field [18], and then generalized to different polarization effects (Faraday rotation, Cotton-Mouton effect, Kerr effect, circular dichroism) by the same group [19]. Although limited in dynamic range, this approach appears to provide the best performance in term of sensitivity [16].

In high-finesse cavities, the first source of anisotropy is HR mirror birefringence. Its phase shift per reflection, around  $10^{-6}$  rad, is four orders of magnitude above either the Kerr or the Cotton-Mouton birefringence induced in atomic gases for

typical electric or magnetic field values under STP conditions. Given the small round-trip mirror birefringence, the combined effect of both mirrors is equivalent (to the first order) to that of a single birefringent element. Its phase shift is bounded between the difference and the sum of the phase shifts of the two mirrors following a sinusoidal function of the angle between their neutral-axis. One way to eliminate the combined mirror birefringence would be to use mechanical stress to equalize their phase shifts and then orient the axes orthogonally. Then, by modulating the field inducing the gas birefringence and by adding outside the cavity a much larger ellipticity, the measurement dependency with the weak Kerr or Cotton-Mouton birefringence would be linearized, and the useful signal would thus be enhanced [16]. However, the measurement would be very sensitive to any small drift of the combined mirror birefringence, which may rapidly produce a large signal offset. In particular, thermal stress induced by intracavity optical field buildup has been highlighted [15] as the main limitation of the achievable sensitivity in mode-splitting measurements. When using the crossed polarizers approach, mirror birefringence inhomogeneities combined with mechanical drift was suggested as the main noise source [16]. In this paper, we present a method to profitably exploit the mirror birefringence as a dc bias around which the weak induced ellipticity is modulated. This, as with an externally added ellipticity [16], linearizes the response to the useful signal, but above all it offers the unique advantage of a measurement free of mirror birefringence drift, with ideal averaging properties allowing the measurement to approach the ultimate limit. Indeed we demonstrate here shot-noise performance in an application to the measurement of the Kerr effect in gases.

To fully benefit from the use of a high-finesse cavity, it is necessary to lock the laser frequency to the exact center of a cavity mode. To reach this objective, different servo-locked configurations have been established [20,21], that by Pound-Drever-Hall being the most popular. An attractive alternative is to take advantage of the frequency locking that occurs when a semiconductor laser is coupled to a high-finesse cavity by a weak resonant optical feedback (OF) [22]. The simultaneous and drastic linewidth reduction is so effective that the optically locked diode laser emission may be regarded as monochromatic compared to the sub-kHz cavity resonance. Usually, to separate the OF from the direct reflection of the cavity input mirror, particular cavity or injection configurations (V cavity or confocal cavity) are used [22]. As demonstrated by Morville *et al.* [23], a simpler axisymmetric two-mirror cavity configuration is exploitable: the cavity mirror birefringence allows resonant OF to be selected by its modified polarization. For an external-cavity diode laser (ECDL) in the Littrow configuration, an OF as a fraction of laser power in the range of  $10^{-6}$  to a few  $10^{-4}$  acts in the perturbative regime of the free-running laser and forces optical locking. This corresponds to a mirror birefringence value in the range of  $10^{-7}$  to a few  $10^{-6}$  assuming a finesse of 250 000 and a cavity transmission of 10%. Because of the OF, the ECDL frequency is no longer defined by its free-running value (depending on injection current and grating orientation), but mostly by the cavity resonance frequency and the OF phase. Thus, the laser frequency may be locked to the exact resonance

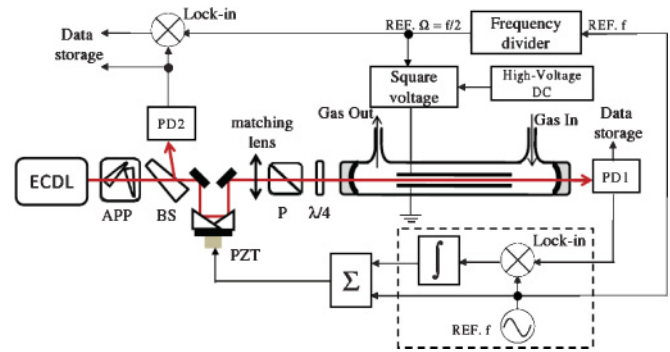


FIG. 1. (Color online) Experimental setup. ECDL, Extended Cavity Diode Laser; APP, anamorphic prism pairs; BS, beam splitter; PZT, piezoelectric transducer; P, polarizer; PD, amplified photodiode;  $\Sigma$ , summation;  $\int$ , integrator stage.

center by servo-controlling the OF phase and the injection current thanks to a modulation-demodulation scheme of the cavity transmission similar to the one proposed by Ohshima *et al.* [24].

A scheme of our experimental setup for a gas-phase Kerr effect measurement is shown in Fig. 1. A 3 mW diode laser beam ( $\lambda = 810$  nm) is first circularized by anamorphic prisms, then focused into a very high finesse cavity (250 000) in order to mode-match its  $TEM_{00}$  mode. The cavity of 52 cm long is formed by two mirrors of 50 cm of curvature radius and by a glass tube in which the gas circulates as a laminar flow under STP conditions. The transverse electric field is generated by applying a voltage between two 47 cm long electrodes separated by 3 mm and placed inside the glass cell on both sides of the optical axis.

The polarizer P followed by the quarter-wave retarder  $\lambda/4$  (a Fresnel rhomb) constitute an optical isolator against the direct cavity reflection which preserves the incident (circular) polarization. On the other hand, the resonant intracavity field is sensitive to the HR coating birefringence, and its polarization is appreciably different from the incident one [19,23]. Light resonating inside the cavity is therefore partially returned through the isolator as the OF. The linearly polarized probe exploited in [19,23] is here replaced by circularly polarized light. This makes the orientation of the mirrors and the electrodes independent of the laboratory frame and should reduce the optical-field-induced mirror birefringence [15].

The computerized servo-control (dashed box in Fig. 1) generates a reference modulation at frequency  $f$  which is applied to the laser frequency. By lock-in phase detection on the cavity transmission (photodiode PD1), an error signal is then generated to correct small resonance frequency drifts by acting on the OF phase through a piezoelectric transducer (PZT) mounted mirror. The modulation frequency is limited by the cavity bandwidth ( $\sim 1$  kHz) and has been fixed to 800 Hz corresponding to the quietest spectral range. The diode laser current source and the mechanical environment are stable enough that correction of the OF phase alone is sufficient to keep the exact resonance for about 1 h.

A flip-flop digital circuit is used to generate an in-phase square modulation at frequency  $\Omega = f/2$  which is applied to the transverse electric field inducing the Kerr gas

birefringence. A fraction ( $\sim 4\%$ ) of the OF power feeding back from the cavity is deflected by a beam splitter (BS) onto a low-noise-amplified photodiode (PD2), providing the birefringence signal. With respect to polarization analysis in transmission, this analysis in reflection allows us to use a single set of high-quality polarizer plus Fresnel rhomb which may be adjusted with high precision. The very critical tilt of the rhomb is optimized by setting the laser in scanning mode while looking at fringes from a thick etalon (not shown in the figure) and minimizing the direct cavity reflection effects. The resulting extinction factor of this analyzer has been evaluated at better than  $10^{-6}$  [25]. The PD2 output is then demodulated at  $\Omega$  by a lock-in amplifier, whose output is digitized together with the dc components of PD1 and PD2 using a 16-bit data acquisition system (NI USB 6251). The  $\Omega$  and dc levels of PD2 are then normalized by dividing out the dc component of the cavity transmission (PD1).

Finally, the total observable cavity anisotropy  $\rho$ , which is the ratio of the birefringence signal over the dc cavity transmission, can be written in a similar way to that in [19] as a sum of its dc and  $\Omega$  components [25]:

$$\rho = \rho_{dc} + \rho_{\Omega} \cos(\Omega t), \quad (1)$$

with

$$\rho_{dc} = \kappa \left( \frac{F}{2\pi} \right)^2 \phi_M^2, \quad (2a)$$

$$\rho_{\Omega} = \frac{\kappa}{\sqrt{2}} \left( \frac{F}{\pi} \right)^2 \phi_M \phi_g, \quad (2b)$$

where  $\phi_g$  is the phase shift induced by the Kerr effect in the gas, while  $\phi_M$  is that associated with the cavity mirrors. The slow axes of these birefringences need to be aligned in order to maximize the useful signal, which is one of the optimization steps of our scheme. From these relations it is clear that measuring  $\rho_{dc}$  and  $\rho_{\Omega}$ , together with the cavity finesse  $F$  and the BS reflection coefficient  $\kappa$  (which would be 1 in the case of direct polarization analysis at the cavity output), immediately yields the Kerr phase shift  $\phi_g = \frac{\pi}{F} \frac{1}{\sqrt{2\kappa}} \frac{\rho_{\Omega}}{\sqrt{\rho_{dc}}}$ .

From Eq. (2b), it seems tempting to increase the dc bias through  $\phi_M$  in order to improve the system resolution. However, each component of the ratio used to experimentally obtain  $\rho_{\Omega}$  depends on the dc bias which leads to non trivial resolution dependency upon the dc bias value. Given that the  $\rho_{\Omega}$  noise is mainly due to the weakest component, which is the small birefringence signal, the theoretical shot-noise-limited resolution is illustrated in Fig. 2 (dashed line) as a function of the dc bias  $\phi_M$ . It is approximately constant below  $\pi/F$ , but rapidly increases above this threshold due to the spectral splitting of the two eigenmodes which becomes larger than their linewidth. A complete analysis, however, needs to account for additional noise sources: photodetector noise (input noise of the current amplifier, thermal noise of the transimpedance gain, and quantization noise at the digitalizer stage) and cavity transmission power fluctuations. The latter has two origins: the intrinsic diode laser power fluctuations and its residual frequency fluctuations. Normalized to the shot noise, the cavity transmission fluctuations are proportional to the birefringence signal, while the photodetector noise is

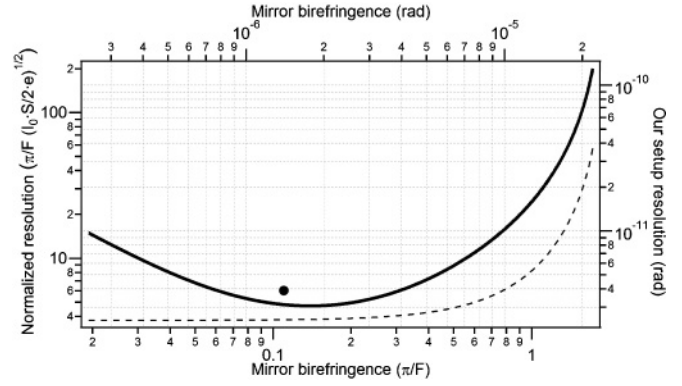


FIG. 2. Kerr phase-shift resolution as a function of the dc bias. Dashed line: photon-noise limited; thick line: technical noise; black dot: our experiment.  $I_0$  is the cavity transmitted power,  $S$  the photodiode responsivity, and  $e$  the electron charge. Our parameters:  $I_0 = 240 \mu\text{W}$ ,  $F = 250\,000$ , and  $S = 0.5 \text{ A/W}$ .

inversely proportional to it. This counterbalancing behavior leads to an optimum of resolution as a function of dc bias as shown in Fig. 2 (thick line). It is noteworthy that theory predicts a shot-noise-limited measurement provided that  $\phi_M$  is around  $0.1(\pi/F)$ . For a finesse of 250 000, this yields an optimal combined mirrors birefringence of  $10^{-6}$  rad. Since the intrinsic coating birefringence of our mirrors was somewhat larger than this value, a simple adjustment of their relative orientation allowed this condition to be achieved.

Figure 3 presents the experimental Kerr phase-shift distributions and their Gaussian fits for four settings of the modulated electric field. These distributions are obtained from measurements of  $\rho_{\Omega}$  and  $\rho_{dc}$  after application of Eqs. (2a) and (2b) in the above optimal experimental conditions. The test gas used is nitrogen at near STP conditions. Integration time for an elementary phase-shift measurement is 1 s, and each distribution is formed by several thousand points. The standard deviation of these distributions is  $4 \times 10^{-12}$  rad and defines the weakest value of Kerr anisotropy which can be resolved in 1 s. The experimental resolution agrees perfectly with the theoretical estimation including the contribution of the relevant noise sources shown in Fig. 2 (black dot). By increasing the integration time, the measured Allan variance of the phase shift shown Fig. 4(a) indicates that the resolution continues to drop with a white-noise dependence to the value of  $3 \times 10^{-13}$  rad at 800 s, the maximal integration time

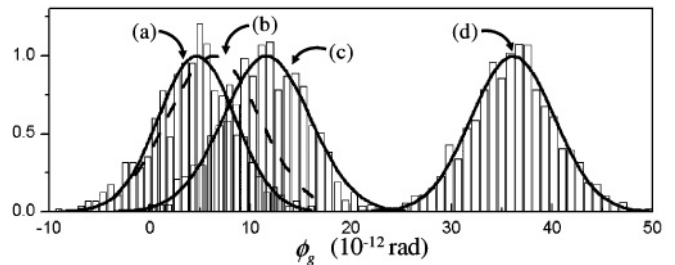


FIG. 3. Nitrogen phase-shift distributions for 1 s measurements at four electric field levels: (a)  $E = 0 \text{ V/m}$ ; (b)  $E = 1.3 \times 10^3 \text{ V/m}$  (only the Gaussian fit is shown); (c)  $E = 2.7 \times 10^3 \text{ V/m}$ ; (d)  $E = 6.4 \times 10^3 \text{ V/m}$ .

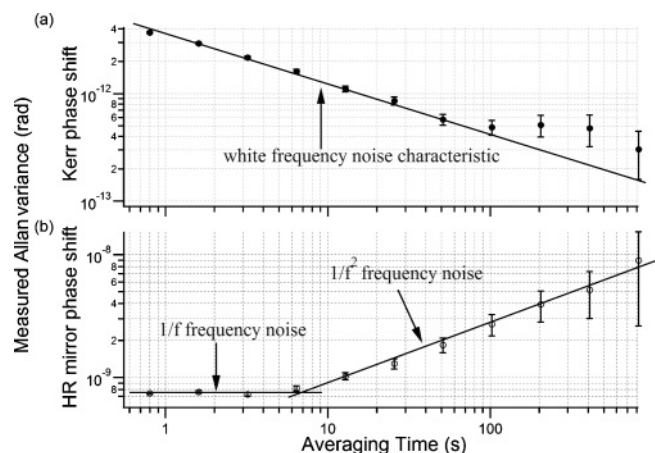


FIG. 4. Simultaneously measured Allan variance for (a) the Kerr phase shift induced in nitrogen by  $E = 9 \times 10^3$  V/m and (b) the phase shift induced by the HR mirror. Error bars are from the statistical errors from the calculation of the Allan variance, which have the usual property of growing at longer integration times.

currently limited by the laser frequency servo loop. This limit could be overcome by also controlling this drift as shown in a similar setup [25] wherein the exact resonance could be maintained for more than 10 h. In Fig. 4(b), the simultaneously measured Allan variance of the HR mirror birefringence is shown, and its evolution clearly illustrates that indeed the mirror birefringence value drifts according to a  $1/f^2$  noise spectrum. This demonstrates the ability to decouple the two measurements.

In conclusion we have shown an effective way to cancel out the drift of the mirror birefringence in very weak anisotropy measurement in a high-finesse cavity. This method has been applied to the detection of the Kerr effect in nitrogen and has demonstrated for the first time a resolution at the shot-noise limited level. In particular, the resolution of  $3 \times 10^{-13}$  rad is three orders of magnitude better than previously reported Kerr birefringence measurements. Finally this technique may constitute a major step toward vacuum birefringence detection using a polarization-state probe.

- [1] M. Vallet *et al.*, *Phys. Rev. Lett.* **87**, 183003 (2001).  
 [2] T. Roth and G. L. J. A. Rikken, *Phys. Rev. Lett.* **85**, 4478 (2000).  
 [3] T. Roth and G. L. J. A. Rikken, *Phys. Rev. Lett.* **88**, 063001 (2002).  
 [4] D. Budker and J. E. Stalnaker, *Phys. Rev. Lett.* **91**, 263901 (2003).  
 [5] E. Iacopini and E. Zavattini, *Phys. Lett. B* **85**, 151 (1979).  
 [6] G. L. J. A. Rikken and C. Rizzo, *Phys. Rev. A* **63**, 012107 (2000).  
 [7] L. Maiani, R. Petronzio, and E. Zavattini, *Phys. Lett. B* **175**, 359 (1986).  
 [8] D. Bakalov *et al.*, *Quantum Semiclass. Opt.* **10**, 239 (1998).  
 [9] E. Zavattini *et al.*, *Phys. Rev. Lett.* **96**, 110406 (2006).  
 [10] E. Zavattini *et al.*, *Phys. Rev. Lett.* **99**, 129901(E) (2007).  
 [11] S. Lamoreaux, *Nature* **441**, 31 (2006).  
 [12] A. C. Melissinos, *Phys. Rev. Lett.* **102**, 202001 (2009).  
 [13] S. Asztalos *et al.*, *Phys. Rev. Lett.* **104**, 041301 (2010).  
 [14] J. E. Kim and G. Carosi, *Rev. Mod. Phys.* **82**, 557 (2010).  
 [15] J. L. Hall, J. Ye, and L.-S. Ma, *Phys. Rev. A* **62**, 013815 (2000).  
 [16] M. Bregant *et al.*, *Phys. Rev. D* **78**, 032006 (2008).  
 [17] G. Bailly, R. Thon, and C. Robilliard, *Rev. Sci. Instrum.* **81**, 033105 (2010).  
 [18] A. Schenzle *et al.*, *Appl. Phys. B* **28**, 297 (1982).  
 [19] M. Vallet *et al.*, *Opt. Commun.* **168**, 423 (1999).  
 [20] R. W. P. Drever *et al.*, *Appl. Phys. B* **31**, 97 (1983).  
 [21] T. W. Hänsch and B. Couillaud, *Opt. Commun.* **35**, 441 (1980).  
 [22] B. Dahmani, L. Hollberg, and R. Drullinger, *Opt. Lett.* **12**, 876 (1987).  
 [23] J. Morville and D. Romanini, *Appl. Phys. B* **74**, 495 (2002).  
 [24] S. Ohshima and H. Schnatz, *J. Appl. Phys.* **71**, 3114 (1992).  
 [25] M. Durand, Ph.D. thesis, Université Claude Bernard Lyon 1, 2009, available in french at [<http://tel.archives-ouvertes.fr/tel-00432201/fr>].