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$\mathcal{PT}$-symmetric laser absorber

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In a recent work, Y. D. Chong *et al.* [Phys. Rev. Lett. **105**, 053901 (2010)] proposed the idea of a coherent perfect absorber (CPA) as the time-reversed counterpart of a laser, in which a purely incoming radiation pattern is completely absorbed by a lossy medium. The optical medium that realizes CPA is obtained by reversing the gain with absorption, and thus it generally differs from the lasing medium. Here it is shown that a laser with an optical medium that satisfies the parity-time ($\mathcal{PT}$) symmetry condition $\epsilon(-\mathbf{r}) = \epsilon^*(\mathbf{r})$ for the dielectric constant behaves simultaneously as a laser oscillator (i.e., it can emit outgoing coherent waves) and as a CPA (i.e., it can fully absorb incoming coherent waves with appropriate amplitudes and phases). Such a device can thus be referred to as a $\mathcal{PT}$-symmetric CPA laser. The general amplification or absorption features of the $\mathcal{PT}$ CPA laser below lasing threshold driven by two fields are determined.

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Introduction. It is well known that a gain medium embedded in an optical cavity, that is, a laser oscillator, emits a coherent electromagnetic radiation that escapes from the cavity in the form of outgoing monochromatic waves when the amplification of photons in the medium reaches a threshold value that balances light leakage out of the cavity [1]. In a recent work [2], Y. D. Chong and collaborators have introduced the concept of a coherent perfect absorber (CPA) as the time-reversed counterpart of a laser, in which by reversing the gain with absorption the same optical system supports a purely incoming radiation pattern—the time reverse of the lasing mode—with complete absorption and zero reflection. Generally, the laser medium and its time-reversed medium are different media because gain must be replaced with loss.

In this Rapid Communication it is shown that an optical medium that satisfies the parity-time ($\mathcal{PT}$) symmetry condition $\epsilon(-\mathbf{r}) = \epsilon^*(\mathbf{r})$ for the dielectric constant can behave *simultaneously* as a laser oscillator (i.e., it can emit outgoing coherent waves) and as a CPA, fully absorbing incoming coherent waves with the right amplitudes and phases. Owing to this rather special property, we refer to such an optical device as a $\mathcal{PT}$ CPA laser. Optical structures with a complex refractive index with even (odd) symmetry for the real (imaginary) part of the refractive index belong to the so-called $\mathcal{PT}$ optical structures [3,4]. Such structures have received increasing attention in the optical and physical communities in the past few years [3,4], mainly because they provide an experimentally accessible laboratory system for simulation with optics of certain features of $\mathcal{PT}$-symmetric and pseudo-Hermitian quantum mechanics [5,6]. A $\mathcal{PT}$ CPA laser can be realized using a $\mathcal{PT}$-invariant distributed feedback optical structure, which was recently proposed in the framework of $\mathcal{PT}$ relativistic quantum mechanics [7].

The idea of the $\mathcal{PT}$-symmetric CPA laser. The main idea underlying the $\mathcal{PT}$ CPA laser can be captured by considering monochromatic wave propagation in a dielectric structure with a spatially dependent complex dielectric constant ϵ, which realizes the laser oscillator, in the plane-wave and scalar approximations. Such approximations hold, for example, for a laser oscillator realized in a fiber or in a waveguide. By writing the electric field in the structure as $\mathcal{E}(x,t) = E(x)\exp(-i\omega t) + \text{c.c.}$, where ω is the frequency of the field, the spatial field envelope $E(x)$ satisfies the Helmholtz equation

$$\frac{d^2E}{dx^2} + \left(\frac{\omega}{c_0}\right)^2 \epsilon(x)E = 0, \tag{1}$$

where c_0 is the speed of light in vacuum. In general, the frequency ω is permitted to assume complex values; in this case the imaginary part of ω corresponds to the growth rate in time of the spatial mode $E(x)$. The lasing medium is assumed to be embedded in the spatial region $|x| < L/2$, where $\epsilon(x)$ is complex-valued. The imaginary part of ϵ describes the local gain or loss of the medium. Outside the laser oscillator, that is, for $|x| > L/2$, $\epsilon(x)$ is assumed to be real-valued and equal to $\epsilon(x) = n_0^2$, where n_0 is the (modal) refractive index of the waveguide or fiber [see Fig. 1(a)]. For a laser at threshold, Eq. (1) admits a solution $E = E_0(x)$ describing outgoing waves at some real frequency $\omega = \omega_0$ (the frequency of the most unstable mode), that is, with the behavior $E_0(x) = \exp(ikn_0x)$ for $x > L/2$ and $E_0(x) = q\exp(-ikn_0x)$ for $x < -L/2$, where $k = \omega/c_0$ and q is a constant describing the unbalance of the outgoing waves from the two sides of the cavity. If we take the complex conjugate on both sides of Eq. (1), use the relation $\epsilon(-x) = \epsilon^*(x)$, and make the spatial inversion $x \to -x$, it readily follows that $E_1(x) = E_0^*(-x)$ is also a solution to Eq. (1) with the same frequency $\omega = \omega_0$ and dielectric constant $\epsilon(x)$. Such a new solution has the asymptotic behavior $E_1(x) = q^*\exp(-ikn_0x)$ for $x > L/2$ and $E_1(x) = \exp(ikn_0x)$ for $x < -L/2$; that is, it corresponds to two incoming waves that are fully absorbed in the medium, without being reflected. Thus a laser at threshold for which the dielectric constant satisfies the $\mathcal{PT}$ symmetry condition can *simultaneously* generate outgoing monochromatic waves and perfectly absorb incoming waves with appropriate amplitude and phase. Further properties of the $\mathcal{PT}$ CPA laser can be gained by a more detailed analysis of the scattering properties of the optical structure.

Scattering analysis and $\mathcal{PT}$CPA laser. The most general solution to Eq. (1) has the asymptotic behavior $E(x) = a\exp(ikn_0x) + b\exp(-ikn_0x)$ for $x < -L/2$, and $E(x) = c\exp(ikn_0x) + d\exp(-ikn_0x)$ for $x > L/2$. The

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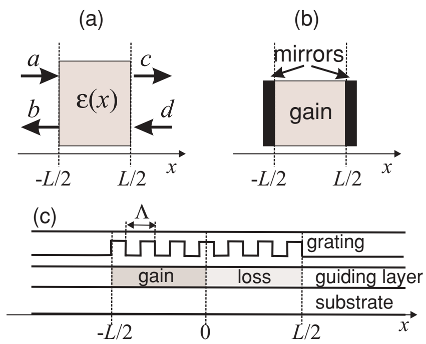

FIG. 1. (a) Schematic of wave scattering in a one-dimensional optical structure with complex dielectric constant $\epsilon(x)$. (b) Schematic of a laser oscillator comprising two equal lossless mirrors filled with a gain medium. (c) Distributed-feedback structure, consisting of a uniform index grating with two homogeneous and symmetric gain and loss regions, that realizes a $\mathcal{PT}$ CPA laser.

amplitudes of forward- and backward-propagating waves out of the cavity are related by the algebraic relation

$$\begin{pmatrix} c \\ d \end{pmatrix} = \mathcal{M}(\omega) \begin{pmatrix} a \\ b \end{pmatrix}, \tag{2}$$

where $\mathcal{M}(\omega)$, with $\det\mathcal{M} = 1$, is the 2×2 transfer matrix of the optical structure from $x = -L/2$ to $x = L/2$ [see Fig. 1(a)]. The spectral transmission and reflection coefficients for left (l) and right (r) incidence, that is, the coefficients of the scattering matrix, can be expressed in terms of the transfer matrix elements as

$$t^{(l)} = t^{(r)} \equiv t = \frac{1}{\mathcal{M}_{22}}, \quad r^{(l)} = -\frac{\mathcal{M}_{21}}{\mathcal{M}_{22}}, \quad r^{(r)} = \frac{\mathcal{M}_{12}}{\mathcal{M}_{22}}. \tag{3}$$

For a laser oscillator without injected signal, the boundary conditions $a = d = 0$ apply, which imply [from Eq. (2)] that $\mathcal{M}_{22}(\omega) = 0$. If all the zeros of $\mathcal{M}_{22}(\omega)$ (i.e., the poles of the transmission t) lie in the lower half of the imaginary plane, that is, $\mathrm{Im}(\omega) < 0$, the laser is below threshold. As the gain is increased, the poles of t move toward the real axis, and the laser threshold is reached when the most unstable pole, say $\omega = \omega_0$, becomes real. In contrast, for a perfect absorber the boundary conditions $b = c = 0$, that is, the absence of reflected waves, hold. From Eq. (2) this implies that $\mathcal{M}_{11}(\omega) = 0$, and the amplitudes of incident waves must satisfy the condition $d = \mathcal{M}_{21}(\omega)a$. For a laser at threshold, for which $\mathcal{M}_{22}(\omega_0) = 0$ at some real frequency ω_0, the condition $\mathcal{M}_{11}(\omega) = 0$ is generally never satisfied at any frequency ω; that is, a laser *does not generally behave* like a perfect absorber. For example, let us consider a standard laser model comprising two equal lossless end mirrors, at $x = \pm L/2$, filled by a gain medium with gain coefficient g [Fig. 1(b)]. The transfer matrix $\mathcal{M}$ of this simple laser oscillator is given by the product of the transfer matrices of the mirrors and gain medium, that is, $\mathcal{M} = \mathcal{M}_M \mathcal{M}_g \mathcal{M}_M$, where [1,8]

$$\mathcal{M}_M = \frac{1}{t_M} \begin{pmatrix} t_M^2 - r_M^2 & r_M \\ -r_M & 1 \end{pmatrix}, \qquad \mathcal{M}_g = \begin{pmatrix} \exp(gL + i\delta) & 0 \\ 0 & \exp(-gL - i\delta) \end{pmatrix}, \tag{4}$$

$\delta = (\omega/c_0) n_g(\omega) L$, $n_g(\omega)$ is the real part of the refractive index in the gain medium, and r_M and t_M are the field reflection and transmission coefficients of the mirrors. For a lossless passive mirror, one has $|t_M|^2 + |r_M|^2 = 1$ and the phases of r_M and t_M differ by $\pm\pi/2$ (see, e.g., [8]). For the sake of definiteness, we assume r_M to be real-valued, so that t_M is purely imaginary. After straightforward calculations, the condition $\mathcal{M}_{22}(\omega) = 0$ for a real frequency ω yields $\exp(-2gL - 2i\delta) = r_M^2$, that is, $\exp(-2gL) = r_M^2$ and $\delta = l\pi$ (l is an integer number). The former equation corresponds to the usual threshold condition for laser oscillation, stating the balance of coupling losses with gain in the medium; the latter equation states that the lasing frequency should correspond to one among the longitudinal resonance frequencies of the resonator [1]. In contrast, the condition for a perfect absorber, $\mathcal{M}_{11} = 0$, reads explicitly $r_M^2 \exp(-2gL - 2i\delta) = 1$, which can never be satisfied for any real frequency ω. Hence an ordinary laser oscillator does not behave as a perfect absorber.

Let us now consider a $\mathcal{PT}$-symmetric laser oscillator; the possible realization of such an oscillator is discussed in the following sections. In this case, taking the complex conjugate of Eq. (1), using the relation $\epsilon(-x) = \epsilon^*(x)$ and after setting $x \to -x$, from Eq. (2) it can be readily shown that $\mathcal{M}^{-1}(\omega) = \mathcal{M}^*(\omega^*)$, that is,

$$\mathcal{M}_{22}(\omega) = \mathcal{M}_{11}^*(\omega^*), \tag{5}$$

$$\mathcal{M}_{12}(\omega) = -\mathcal{M}_{12}^*(\omega^*), \quad \mathcal{M}_{21}(\omega) = -\mathcal{M}_{21}^*(\omega^*). \tag{6}$$

Therefore, if the medium is at threshold for lasing, there exists a real frequency ω_0 such that $\mathcal{M}_{22}(\omega_0) = 0$. From Eq. (5), it follows that at the same frequency one has $\mathcal{M}_{11}(\omega_0) = 0$, that is, the lasing medium also behaves as a CPA, according to the simple argument presented in the previous section.

Amplification and absorption properties of the $\mathcal{PT}$ CPA laser below threshold. It is interesting to investigate the amplification and absorption properties of the $\mathcal{PT}$-symmetric laser just below threshold for oscillation [9] when two signals, of amplitudes a and d [see Fig. 1(a)], are injected from the two sides of the cavity, that is, when the laser is operated as a cavity amplifier with two input ports (see, e.g., [10]). Near threshold and for frequencies ω of the injected signals close to the laser frequency ω_0, the most general form of the coefficients of the transfer matrix $\mathcal{M}(\omega)$, satisfying the constraints, Eqs. (5) and (6), imposed by the $\mathcal{PT}$ invariance, reads

$$\mathcal{M}_{11} = \kappa(\omega - \omega_0 - i\epsilon), \tag{7}$$

$$\mathcal{M}_{22} = \kappa^*(\omega - \omega_0 + i\epsilon), \tag{8}$$

$$\mathcal{M}_{12} = i\alpha + i\beta(\omega - \omega_0), \tag{9}$$

$$\mathcal{M}_{21} = \frac{\mathcal{M}_{11}\mathcal{M}_{22} - 1}{\mathcal{M}_{12}} \simeq \frac{i}{\alpha} - i\frac{\beta}{\alpha^2}(\omega - \omega_0) + o(\epsilon), \tag{10}$$

where $\epsilon > 0$ is a small parameter that measures the distance from laser threshold, α and β are real-valued parameters, and κ is a complex parameter. In writing Eqs. (7)–(10), we assumed that $\mathcal{M}_{22}$ has a simple 0 at $\omega = \omega_0 - i\epsilon$ (i.e., $t = 1/\mathcal{M}_{22}$ has a simple pole), and $\omega - \omega_0 \sim \epsilon$. For a single injected signal at frequency ω, for example, for $d = 0$ (left incident signal), the $\mathcal{PT}$ laser behaves like an ordinary cavity amplifier

[9], and the spectral intensity transmission $T = |t(\omega)|^2 = 1/|\kappa^2|[(\omega-\omega_0)^2+\epsilon^2]$ has a typical Lorentzian profile. For the following discussion, it is worth introducing an overall reflection or transmission coefficient Θ, defined as the ratio of the total intensity of outgoing (reflected or transmitted) waves to the total intensity of the incoming (injected) waves [2]; that is,

$$\Theta = \frac{|b|^2+|c|^2}{|a|^2+|d|^2}. \tag{11}$$

Note that the vanishing of Θ is the signature of perfect absorption, whereas $\Theta > 1$ indicates that an overall amplification has been realized in the medium [11]. For a single-port injection ($d = 0$), one has $\Theta = (|b|^2+|c|^2)/|a|^2 = T + R^{(l)}$, where $R^{(l)} = |r^{(l)}|^2$ is the spectral intensity reflection for left incidence. A typical behavior of $\Theta(\omega)$ in this case is shown by the dotted curve in Fig. 2(a). Note that, because $\Theta > T$, near the resonance, $\omega = \omega_0$, $\Theta(\omega)$ takes large values, diverging as the lasing threshold is approached (i.e., $\epsilon \to 0$). It is now interesting to investigate how the second signal of amplitude d, injected into the other side of the cavity, can fully change the behavior of $\Theta(\omega)$ and, in particular, how perfect absorption can be attained in a narrow spectral range at about $\omega = \omega_0$ for appropriate amplitude and phase of d. To this aim, let us first consider a coherent signal and set $d/a = \sigma\exp(i\phi)$; in this case one obtains

$$\Theta(\omega) = \frac{|1+\sigma\mathcal{M}_{12}\exp(i\phi)|^2 + |\sigma\exp(i\phi)-\mathcal{M}_{21}|^2}{(1+\sigma^2)|\mathcal{M}_{22}|^2}. \tag{12}$$

To achieve a CPA, according to the previous scattering analysis, let us assume a second coherent signal with amplitude and phase defined by $d/a = \sigma\exp(i\phi) = \mathcal{M}_{21}(\omega_0)$. In this case, from Eq. (12) one obtains $\Theta(\omega_0) \sim \epsilon^2$, that is, perfect absorption is attained at $\omega = \omega_0$ as $\epsilon \to 0$. A typical behavior of $\Theta(\omega)$ in this case, depicted in Fig. 2(a) (solid curve), shows a marked dip near $\omega = \omega_0$. Thus the second injected signal, of appropriate amplitude and phase, makes the amplifier a (near)

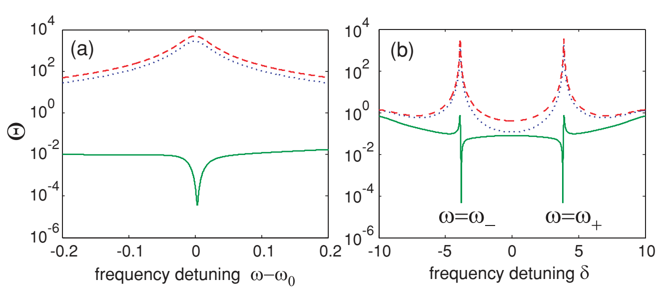

FIG. 2. (Color online) (a) Behavior of the overall reflection or transmission coefficient Θ (on a logarithmic scale) versus frequency detuning $\omega - \omega_0$ for the transfer matrix defined by Eqs. (7)–(10) and for parameter values $\alpha = 3$, $\beta = 0.3$, $\epsilon = 0.02$, and $\kappa = 1$. Solid curve, two-port coherent input excitation with $d/a = \mathcal{M}_{21}(\omega_0)$; dashed curve, two-port incoherent input excitation with $d/a = |\mathcal{M}_{21}(\omega_0)|\exp(i\phi)$, averaged over the random phase ϕ; dotted curve, single-port excitation ($d = 0$). (b) Same as (a), but for the distributed-feedback structure in Fig. 1(c) for parameter values $q_0L = 1$ and $gL = 4.43$. The normalized frequency detuning in the horizontal axis is defined as $\delta = (\omega-\omega_B)n_0L/c_0$, where ω_B is the Bragg reference frequency.

perfect absorber in a narrow spectral range around ω_0. It should be noted that such perfect absorption requires that the two injected fields be coherent. For example, let us assume that the two fields have a definite amplitude ratio $\sigma = |d/a|$, however their relative phase ϕ fluctuates. Averaging the expression of $\Theta(\omega)$ over the fluctuating phase ϕ yields

$$\Theta(\omega) = \frac{1+\sigma^2+\sigma^2|\mathcal{M}_{12}|^2+|\mathcal{M}_{21}|^2}{(1+\sigma^2)|\mathcal{M}_{22}|^2}. \tag{13}$$

As in the CPA case, let us assume that $\sigma = |\mathcal{M}_{21}(\omega_0)|$. A typical behavior of $\Theta(\omega)$ for incoherent excitation is shown by the dashed curve in Fig. 2(a). Note that, similarly to the single injected signal, in this case Θ diverges (rather than vanishing) at $\omega = \omega_0$ as the lasing threshold is attained. This indicates that, as in [2], a CPA requires both amplitude and phase control of the two incoming waves. Therefore, depending on the amplitude and phase of the two injected signals, the $\mathcal{PT}$ CPA laser below threshold can either amplify or absorb the input fields. At exact threshold, the $\mathcal{PT}$ CPA laser behaves like an ordinary laser; that is, outgoing waves are spontaneously emitted starting from spontaneous emission noise in the medium. However, compared to an ordinary laser, the $\mathcal{PT}$ CPA laser has the additional property to fully absorb incoming radiation from the two ports with the appropriate amplitude and phase relationship.

Distributed-feedback $\mathcal{PT}$ CPA laser. How can one realize a $\mathcal{PT}$-symmetric CPA laser? At first sight one could naively think that, because the overall (spatially averaged) gain coefficient in a $\mathcal{PT}$-invariant structure always vanishes, a $\mathcal{PT}$ laser would never reach the threshold for oscillation. However, this is not the case, and the threshold for laser oscillation can generally be reached at the so-called $\mathcal{PT}$ symmetry-breaking point [7]. At this point, a pair of resonance and antiresonance frequencies crosses the real frequency axis [7], defining the lasing and CPA modes of the CPA-laser system. The photon in the former (lasing) mode lives more in the gain part of the structure, while the photon in the latter (CPA) mode lives more in the lossy part. A possible way to realize a $\mathcal{PT}$ CPA laser with a reasonably low threshold is to consider a distributed-feedback structure, composed of a uniform index grating with refractive index $n(x) = n_0 + \Delta n\cos(2\pi x/\Lambda)$, with two homogeneous gain and lossy regions at $-L/2 < x < 0$ and $0 < x < L/2$, respectively, with a gain or loss coefficient per unit length equal to g [Fig. 1(c)]. Such a structure was recently introduced in Ref. [7] as an optical system to test the onset of spectral singularities and $\mathcal{PT}$ symmetry breaking in an optical analog of the non-Hermitian relativistic Dirac equation. As g is increased from 0, the threshold for laser oscillation is attained at two frequencies, $\omega_\pm = \omega_B \pm \Delta\omega$, symmetrically placed at around the Bragg frequency $\omega_B = \pi c_0/\Lambda$ (see, e.g., Fig. 2(c) in Ref. [7]). The transfer matrix of the distributed-feedback structure is given by [7] $\mathcal{M} = \mathcal{M}^+\mathcal{M}^-$, where

$$\mathcal{M}_{11}^{\pm} = \cosh(\lambda_\pm L) - i\frac{\rho_\pm}{\lambda_\pm}\sinh(\lambda_\pm L), \tag{14}$$

$$\mathcal{M}_{12}^{\pm} = -i\frac{q_0}{\lambda_\pm}\sinh(\lambda_\pm L), \quad \mathcal{M}_{21}^{\pm} = i\frac{q_0}{\lambda_\pm}\sinh(\lambda_\pm L), \tag{15}$$

$$\mathcal{M}_{22}^{\pm} = \cosh(\lambda_\pm L) + i\frac{\rho_\pm}{\lambda_\pm}\sinh(\lambda_\pm L). \tag{16}$$

In Eqs. (14)–(16), we have set $q_0 = \omega_B \Delta n/(2c_0)$, $\rho_\pm = n_0(\omega - \omega_0)/c_0 \pm ig$, and $\lambda_\pm = \sqrt{q_0^2 - \rho_\pm^2}$. The threshold gain value $g_{th}L$ for laser oscillation can be found numerically by searching for the zeros in $\mathcal{M}_{22}$; in particular, $g_{th}L$ turns out to be a function of $q_0 L$ solely. For example, for $q_0 L = 1$, one has $g_{th}L \simeq 4.46$. When the structure is kept below the lasing threshold and is illuminated with a single field, or with two fields—either coherent or incoherent—with amplitude ratio $|d/a| = |\mathcal{M}_{21}(\omega_\pm)|$, the numerically computed behavior of Θ versus the normalized frequency detuning $\delta = (\omega - \omega_B)n_0 L/c_0$ is shown in Fig. 2(b) for $gL = 4.43$. Note that, for coherent signal injection, a CPA is observed near the lasing frequencies $\omega_\pm$, according to the general analysis developed in the previous section. It is worth observing the dips in the CPA near $\omega_\pm$ are much narrower than the peaks in the amplifier when the $\mathcal{PT}$ system is operated with a single input signal or with two incoherent signals [see Fig. 2(b)].

Conclusions. In Ref. [2], the idea that backward lasing yields a CPA was proposed. The optical media that realize the laser oscillator and the CPA are generally different from each other. Here we have introduced the idea of a $\mathcal{PT}$-symmetric CPA laser and shown that an optical medium that satisfies the $\mathcal{PT}$ symmetry condition $\epsilon(-\mathbf{r}) = \epsilon^*(\mathbf{r})$ can behave simultaneously as a laser oscillator, emitting outgoing coherent waves, and as a CPA, fully absorbing incoming coherent waves with appropriate amplitudes and phases. The amplification and absorption properties of the $\mathcal{PT}$ CPA laser below the lasing threshold have also been discussed.

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