

Quantum logical gates with four-level superconducting quantum interference devices coupled to a superconducting resonator

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We propose a way for realizing a two-qubit controlled phase gate with superconducting quantum interference devices (SQUIDs) coupled to a superconducting resonator. In this proposal, the two lowest levels of each SQUID serve as the logical states and two intermediate levels of each SQUID are used for the gate realization. We show that neither adjustment of SQUID level spacings during the gate operation nor uniformity in SQUID parameters is required by this proposal. In addition, this proposal does not require the adiabatic passage or a second-order detuning and thus the gate is much faster.

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Introduction. Superconducting devices, including Cooper-pair boxes, Josephson junctions, and superconducting quantum interference devices (SQUIDs), have appeared to be among the most promising candidates for scalable quantum computing, due to design flexibility, large-scale integration, and compatibility to conventional electronics [1–3]. In the past few years, for SQUID systems, many theoretical methods for realizing a single-qubit gate and a two-qubit controlled-phase (CP) or controlled-NOT (CNOT) gate have been presented [4–15].

For realizing a two-qubit CP gate with SQUIDs, several methods have been proposed based on a cavity QED technique [8–15]. These methods are of importance in building quantum logic gates and open a new avenue for the physical realization of quantum information processing with SQUIDs in cavity QED. However, we note that these methods have some disadvantages. For instance, (i) the methods presented in Refs. [8,9] require adjustment of the level spacings of SQUIDs during the gate operation, thus decoherence caused due to the adjustment of level spacings may pose a severe problem; (ii) the methods proposed in Refs. [10,11] require slowly changing the Rabi frequencies to satisfy the adiabatic passage and the approaches introduced in Refs. [12–14] require a second-order detuning to achieve an off-resonant Raman coupling between two relevant levels; note that when the adiabatic passage or a second-order detuning is applied, the gate becomes slow (the gate time is on the order of 1 ms to a few microseconds [11,13]); and (iii) the proposal reported in Ref. [15] employs a two-mode resonator or cavity as well as a second-order detuning between the two cavity modes; technically speaking, the requirement for a SQUID interacting with more than one cavity or resonator modes is difficult to meet. In addition, it is noted that although two-qubit CNOT, CP, or iSWAP gates have been experimentally demonstrated in superconducting charge qubits, flux qubits, and phase qubits [16–18], to the best of our knowledge, no experimental demonstration of a two-qubit gate with SQUID qubits in cavity QED has been reported.

In this Brief Report, we present an alternative method for implementing a two-qubit CP gate with two SQUIDs coupled to a superconducting resonator. As shown below, this proposal

has the following advantages: (a) there is no need for adjusting the level spacings of SQUIDs during the gate operation, thus decoherence caused by tuning the SQUID level spacings is avoided; (b) neither slowly changing the Rabi frequency nor the use of second-order detuning is required, thus the gate is significantly faster (as shown below, the operation time of the gate is on the order of 10 ns); and (c) only one mode of the resonator is employed. In addition, this proposal does not require identical coupling constants of each SQUID with the resonator and thus is tolerable to inevitable nonuniformity in device parameters. We believe that this work is of interest because it avoids most of the problems existing in the previous proposals.

Basic theory. The SQUIDs considered throughout this Brief Report are rf SQUIDs each consisting of a Josephson tunnel junction enclosed by a superconducting loop. The Hamiltonian for an rf SQUID, with junction capacitance C and loop inductance L , can be written in the usual form [19]

$$H_s = \frac{Q^2}{2C} + \frac{(\Phi - \Phi_x)^2}{2L} - E_J \cos\left(2\pi \frac{\Phi}{\Phi_0}\right), \quad (1)$$

where Φ , the magnetic flux threading the ring, and Q , the total charge on the capacitor, are the conjugate variables of the system, Φ_x is the static (or quasistatic) external magnetic flux applied to the ring, and $E_J \equiv I_c \Phi_0 / 2\pi$ is the Josephson coupling energy, where I_c is the critical current of the junction and $\Phi_0 = h/2e$ is the flux quantum.

SQUID-resonator resonant interaction. Consider a SQUID (say SQUID a) coupled to a single-mode resonator and driven by a classical microwave pulse. The SQUID is biased properly to have four lowest levels, which are denoted by $|0\rangle$, $|1\rangle$, $|2\rangle$, and $|3\rangle$, respectively [Fig. 1(a)]. The resonator mode is resonant with the $|2\rangle \leftrightarrow |3\rangle$ transition but decoupled (highly detuned) from the transition between any other two levels, which can be readily achieved by adjusting level spacings of the SQUID [8,20]. In the interaction picture, the interaction Hamiltonian for the SQUID and the resonator mode, after making the rotating-wave approximation, can be written as [8]

$$H_I = \hbar(g_a c^\dagger |2\rangle_a \langle 3| + \text{H.c.}). \quad (2)$$

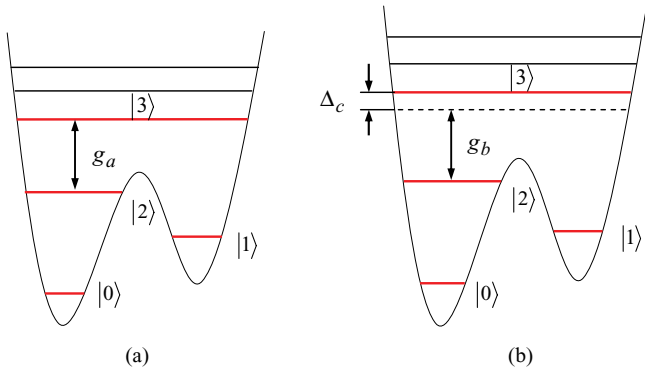


FIG. 1. (Color online) (a) SQUID-resonator resonant interaction. (b) SQUID-resonator off-resonant interaction. The difference between the level spacings in (a) and the level spacings in (b) can be achieved by choosing different device parameters for SQUIDs.

Here, the subscript a represents SQUID a ; c^+ and c are the photon creation and annihilation operators of the resonator mode with frequency ω_c ; g_a is the coupling constant between the resonator mode and the $|2\rangle \leftrightarrow |3\rangle$ transition of SQUID a . The initial states $|3\rangle_a|0\rangle_c$ and $|2\rangle_a|1\rangle_c$ of the system, under the Hamiltonian (2), evolve as follows

$$\begin{aligned} |3\rangle_a|0\rangle_c &\rightarrow \cos g_a t |3\rangle_a|0\rangle_c - i \sin g_a t |2\rangle_a|1\rangle_c, \\ |2\rangle_a|1\rangle_c &\rightarrow \cos g_a t |2\rangle_a|1\rangle_c - i \sin g_a t |3\rangle_a|0\rangle_c, \end{aligned} \quad (3)$$

where $|0\rangle_c$ and $|1\rangle_c$ are the vacuum state and the single-photon state of the resonator mode, respectively.

SQUID-resonator off-resonant interaction. Consider a system composed of a SQUID (say SQUID b) and a single-mode resonator. Suppose that the resonator mode is off-resonant with the $|2\rangle \leftrightarrow |3\rangle$ transition (i.e., $\Delta_c = \omega_{32} - \omega_c \gg g_b$) while decoupled from the transition between any other two levels of SQUID b [Fig. 1(b)]. Here, Δ_c is the detuning between the $|2\rangle \leftrightarrow |3\rangle$ transition frequency ω_{32} of SQUID b and the resonator mode frequency ω_c , and g_b is the coupling constant between the resonator mode and the $|2\rangle \leftrightarrow |3\rangle$ transition. The effective interaction Hamiltonian in the interaction picture can be written as [21,22]

$$H_e = \hbar \frac{g_b^2}{\Delta_c} (|3\rangle_b \langle 3| - |2\rangle_b \langle 2|) c^+ c, \quad (4)$$

where the subscript b represents SQUID b .

From the Hamiltonian (4), it is straightforward to see that if the resonator mode is initially in a single-photon state $|1\rangle_c$, the time evolution of the states of the system is then given by

$$\begin{aligned} |2\rangle_b|1\rangle_c &\rightarrow e^{ig_b^2 t/\Delta_c} |2\rangle_b|1\rangle_c, \\ |3\rangle_b|1\rangle_c &\rightarrow e^{-ig_b^2 t/\Delta_c} |3\rangle_b|1\rangle_c, \end{aligned} \quad (5)$$

which introduces a phase shift $e^{ig_b^2 t/\Delta_c}$ to the state $|2\rangle$ while $e^{-ig_b^2 t/\Delta_c}$ to the state $|3\rangle$ of the SQUID, when the resonator mode is in the state $|1\rangle_c$. Note that the states $|2\rangle_b|0\rangle_c$ and $|3\rangle_b|0\rangle_c$ remain unchanged under the Hamiltonian (4).

In the following gate operations, we will need this resonant interaction between the pulse and SQUIDs. Note that the resonant interaction between the pulse and the SQUIDs can be completed within a very short time by increasing the pulse

Rabi frequency (i.e., by increasing the intensity or amplitude of the pulse).

Two-qubit CP gate. Let us consider two SQUIDs, a and b . By choosing different device parameters for each SQUID, SQUIDs a and b can have the four-level configurations as depicted in Fig. 1(a) and Fig. 1(b), respectively. The two logic states of a SQUID qubit are represented by the two lowest levels $|0\rangle$ and $|1\rangle$, while the two intermediate levels $|2\rangle$ and $|3\rangle$ of each SQUID are utilized for the gate realization. For the notation convenience, we here denote the ground state (the first excited state) as level $|1\rangle$ ($|0\rangle$) for SQUID b [Figs. 2(a') and 2(b')]. We suppose that the resonator mode is resonant with the $|2\rangle \leftrightarrow |3\rangle$ transition of SQUID a while off-resonant with the $|2\rangle \leftrightarrow |3\rangle$ transition of SQUID b , which can be reached by prior adjustment of the level spacings of SQUIDs a and b . In addition, we assume that the resonator mode is initially in the vacuum state $|0\rangle_c$. The notations $\omega_{31}^{(i)}$, $\omega_{20}^{(i)}$, and $\omega_{21}^{(i)}$ involved in the following gate operations are the $|1\rangle \leftrightarrow |3\rangle$ transition frequency, the $|0\rangle \leftrightarrow |2\rangle$ transition frequency, and the $|1\rangle \leftrightarrow |2\rangle$ transition frequency of SQUID i ($i = a, b$).

The operations for realizing a two-qubit CP gate are listed as follows:

Step (i): Apply a microwave pulse (with a frequency $\omega_{\mu w} = \omega_{31}^{(a)}$ and a phase $\phi = \pi$) to SQUID a for a time interval $t_1 = \pi/(2\Omega_{13})$ [Fig. 2(a)] to transform the state $|1\rangle_a$ to $i|3\rangle_a$. Then wait for a time interval $t'_1 = \pi/(2g_a)$ during which the $|2\rangle \leftrightarrow |3\rangle$ transition of SQUID a resonantly interacts with the resonator mode [Fig. 2(b)] to transform the state $|3\rangle_a|0\rangle_c$ to $-i|2\rangle_a|1\rangle_c$, as shown in Eq. (3).

It can be found that after this step, the following transformation is obtained:

$$|1\rangle_a|0\rangle_c \xrightarrow{\text{after } t_1} i|3\rangle_a|0\rangle_c \xrightarrow{\text{after } t'_1} |2\rangle_a|1\rangle_c. \quad (6)$$

On the other hand, the state $|0\rangle_a|0\rangle_c$ remains unchanged.

Step (ii): Apply a microwave pulse (with a frequency $\omega_{\mu w} = \omega_{20}^{(a)}$ and a phase $\phi = \pi/2$) to SQUID a [Fig. 2(c)] while a microwave pulse (with a frequency $\omega_{\mu w} = \omega_{21}^{(b)}$ and a phase $\phi = -\pi/2$) to SQUID b [Fig. 2(b')]. The Rabi frequency of the pulse applied to SQUID a is Ω_{02} while the Rabi frequency of the pulse applied to SQUID b is Ω_{12} . We set $\Omega_{02} = \Omega_{12}$, which can be achieved by adjusting the intensities of the two pulses. After the pulse duration $t_2 = \pi/(2\Omega_{02}) = \pi/(2\Omega_{12})$, the state $|2\rangle$ ($|0\rangle$) of SQUID a is transformed to the state $|0\rangle$ ($-|2\rangle$) while the state $|1\rangle$ of SQUID b is transformed to the state $|2\rangle$.

Step (iii): Wait for a time t_3 . Note that in the case when the resonator mode is in the photon state $|1\rangle_c$, the levels $|2\rangle$ and $|3\rangle$ of SQUID a are not populated after the above operations. Therefore, there is no coupling between the resonator mode and SQUID a . The resonator mode is off-resonant with the $|2\rangle \leftrightarrow |3\rangle$ transition of SQUID b [Fig. 2(a')]. It can be seen from Eq. (5) that for $t_3 = \pi \Delta_c / g_b^2$, the state $|2\rangle_b|1\rangle_c$ changes to $-|2\rangle_b|1\rangle_c$. On the other hand, the state $|0\rangle_b|0\rangle_c, |0\rangle_b|1\rangle_c$, and $|2\rangle_b|0\rangle_c$ remain unchanged.

Step (iv): Apply a microwave pulse (with a frequency $\omega_{\mu w} = \omega_{20}^{(a)}$ and a phase $\phi = -\pi/2$) to SQUID a [Fig. 2(c)] while applying another microwave pulse (with a frequency $\omega_{\mu w} = \omega_{21}^{(b)}$ and a phase $\phi = \pi/2$) to SQUID b [Fig. 2(b')].

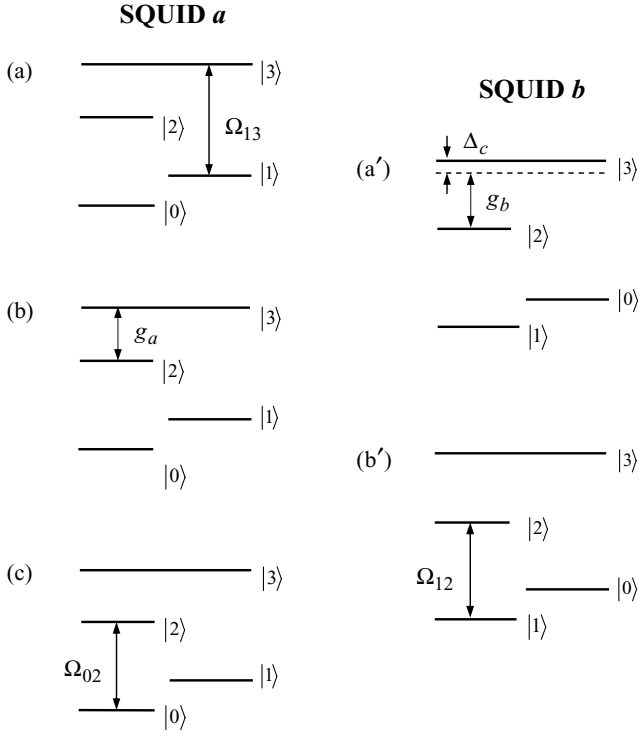


FIG. 2. Illustration of SQUIDs interacting with the resonator mode and/or the microwave pulses during the gate performance. The figures on the left (right) side correspond to SQUID a (b).

Like step (ii), we set $\Omega_{02} = \Omega_{12}$. After the pulse duration t_2 given in step (ii), the state $|0\rangle$ ($|2\rangle$) of SQUID a is transformed to the state $|2\rangle$ ($-|0\rangle$) while the state $|2\rangle$ of SQUID b is transformed back to the state $|1\rangle$.

Step (v): Perform an inverse operation of step (i) [Figs. 2(a) and 2(b)]. That is, wait for a time interval t'_1 given in step (i), during which the $|2\rangle \leftrightarrow |3\rangle$ transition of SQUID a resonantly interacts with the resonator mode and then apply a microwave pulse (with a frequency $\omega_{\mu w} = \omega_{31}^{(a)}$ and a phase $\phi = \pi$) to SQUID a for a time interval t_1 given in step (i). It can be verified that after this step, the following transformation is achieved:

$$|2\rangle_a |1\rangle_c \xrightarrow{\text{after } t'_1} -i |3\rangle_a |0\rangle_c \xrightarrow{\text{after } t_1} |1\rangle_a |0\rangle_c. \quad (7)$$

On the other hand, the state $|0\rangle_a |0\rangle_c$ remains unchanged.

The states of the whole system after each step of the above operations are summarized in the following table:

$$\begin{array}{cccc}
 |00\rangle|0\rangle_c & |00\rangle|0\rangle_c & |20\rangle|0\rangle_c & |20\rangle|0\rangle_c \\
 |01\rangle|0\rangle_c & \xrightarrow{\text{Step(i)}} |01\rangle|0\rangle_c & \xrightarrow{\text{Step(ii)}} |22\rangle|0\rangle_c & \xrightarrow{\text{Step(iii)}} |22\rangle|0\rangle_c \\
 |10\rangle|0\rangle_c & \xrightarrow{\text{Step(i)}} |20\rangle|1\rangle_c & \xrightarrow{\text{Step(ii)}} |00\rangle|1\rangle_c & \xrightarrow{\text{Step(iii)}} |00\rangle|1\rangle_c \\
 |11\rangle|0\rangle_c & \xrightarrow{\text{Step(i)}} |21\rangle|1\rangle_c & \xrightarrow{\text{Step(ii)}} |02\rangle|1\rangle_c & \xrightarrow{\text{Step(iii)}} -|02\rangle|1\rangle_c \\
 & & & \\
 & \xrightarrow{\text{Step(iv)}} & \xrightarrow{\text{Step(v)}} & \\
 & |00\rangle|0\rangle_c & |00\rangle|0\rangle_c & \\
 & |01\rangle|0\rangle_c & |01\rangle|0\rangle_c & \\
 & |20\rangle|1\rangle_c & |10\rangle|0\rangle_c & \\
 & -|21\rangle|1\rangle_c & -|11\rangle|0\rangle_c &
 \end{array}, \quad (8)$$

where $|kl\rangle$ is abbreviation of the state $|k\rangle_a |l\rangle_b$ of SQUIDs (a, b) with $k, l \in \{0, 1, 2\}$. It can be concluded from Eq. (8) that a two-qubit CP gate was achieved with two SQUIDs (i.e., the control SQUID a and the target SQUID b) after the above process.

From the description above, it can be found that: (i) in contrast to the previous proposals [8,9], the method presented above does not require adjustment of the level spacings of the SQUIDs during the gate operation; (ii) this method does not require slow variation of the Rabi frequency in contrast to Refs. [10,11]; (iii) compared with the previous approaches [12–14], this method does not require a finite second-order detuning $\delta = \Delta_c - \Delta_{\mu w}$ and thus the gate speed is improved by one order (here $\Delta_{\mu w}$ is the detuning of the pulse frequency with the transition frequency between the two associated levels of SQUIDs; for the details, see Refs. [12–14]); and (iv) this method employs only one mode of the resonator, which differs from the previous proposal [15].

Discussion. Let us give a brief estimate on the gate time. As shown above, the total operation time is

$$\begin{aligned}
 \tau &= 2t_1 + 2t'_1 + 2t_2 + t_3 \\
 &= \pi/g_a + \pi \Delta_c/g_b^2 + \pi/\Omega_{13} + \pi/\Omega_{02}, \quad (9)
 \end{aligned}$$

where Ω_{02} is equal to Ω_{12} [see steps (ii) and (iv) above]. Without loss of generality, let us consider $g_a \sim g_b \sim 3.0 \times 10^9 \text{ s}^{-1}$, which is available at present [9]. By choosing $\Delta_c = 10g_b$, $\Omega_{13} \sim \Omega_{02} \sim 10g_a$, we have $\tau \sim 12 \text{ ns}$.

Several issues related to the gate operations above need to be addressed as follows:

(i) The level $|3\rangle$ of SQUID a is occupied in steps (i) and (v). Since only SQUID-pulse resonant interaction and SQUID-resonator resonant interaction are used in steps (i) and (v), the operation time $t_1 + t'_1$ in step (i) or (v), equal to $\pi/(2\Omega_{13}) + \pi/(2g_a)$, can be significantly shortened by increasing the pulse Rabi frequency Ω_{13} and the coupling constant g_a . Alternatively, one can design the SQUID a to have a sufficiently long energy relaxation time γ_3^{-1} for the level $|3\rangle$. By doing these, we can have $\gamma_3^{-1} \gg t_1 + t'_1$, such that decoherence caused by the energy relaxation of the level $|3\rangle$ of SQUID a is negligibly small.

(ii) The occupation probability p_3 of the level $|3\rangle$ for SQUID b during step (iii) is given by [12]

$$p_3 \simeq \frac{4g_b^2}{4g_b^2 + \Delta_c^2}, \quad (10)$$

which need to be negligibly small in order to reduce the gate error. For the choice of $\Delta_c = 10g_b$, we have $p_3 \sim 0.04$, which can be further reduced by increasing the ratio of Δ_c/g_b .

(iii) For steps (i), (ii), (iv), and (v), the resonant interaction between the resonator mode and the $|2\rangle \leftrightarrow |3\rangle$ transition of SQUID a , involved during the application of the pulse, is unwanted. To minimize the effect of this unwanted interaction on the gate, the Rabi frequencies Ω_{13} and Ω_{02} require to be much larger than the coupling constant g_a , i.e., $\Omega_{13}, \Omega_{02} \gg g_a$. Note that this condition can be achieved by increasing Ω_{13} and Ω_{02} (i.e., via increasing the pulse intensity).

(iv) For either step (ii) or step (iv), when the SQUID b is in the state $|2\rangle$ and the resonator mode is in the single-photon state $|1\rangle_c$, the unwanted off-resonant interaction between the resonator mode and the $|2\rangle \leftrightarrow |3\rangle$ transition of SQUID b induces a phase shift $e^{ig_b^2 t_2/\Delta_c}$ to the state $|2\rangle$ of SQUID b , which will affect the desired gate performance. The effect of this unwanted SQUID-resonator off-resonant interaction on the gate can be made negligibly small as long as the

condition $\Omega_{12} \gg g_b^2/\Delta_c$ is met. In the following, we will give a discussion on the effect of this unwanted interaction on the fidelity of the gate.

Suppose that the two SQUID qubits are initially in a generic state described by $|\psi(0)\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \theta|11\rangle$, where the coefficients satisfy the normalization. In the ideal case, it can be seen from Eq. (10) that after the five-step operations described above, the state $|\psi(0)\rangle$ becomes $|\psi_{id}(\tau)\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle - \theta|11\rangle$. On the other hand, when the effect of the off-resonant interaction between the resonator mode and SQUID b is included during steps (ii) and (iv), one can easily work out the expression for the final state $|\psi(\tau)\rangle$ after performing the same operations above. To simplify our presentation, we will not give a complete expression for $|\psi(\tau)\rangle$ due to its complexity.

The fidelity is given by

$$F = |\langle\psi_{id}(\tau)|\psi(\tau)\rangle|^2 = 1 - 2x(1 + p^2 - q^2 - r^2) + x^2[(1 - q^2 - r^2)^2 + 2p^2(1 + q^2 - r^2) + p^4], \quad (11)$$

where

$$x = |\theta|^2, \quad p = \cos \varphi, \quad q = \frac{s}{2\sqrt{\Omega_{12}^2 + s^2/4}} \sin \varphi, \quad (12)$$

$$r = \frac{\Omega_{12}}{\sqrt{\Omega_{12}^2 + s^2/4}} \sin \varphi,$$

with $s = g_b^2/\Delta_c$ and $\varphi = \pi\sqrt{\Omega_{12}^2 + s^2/4}/(2\Omega_{12})$.

Equation (11) shows that the fidelity F is a function of $x \in [0, 1]$. Thus, the average fidelity over all possible two-qubit initial states is given by

$$\bar{F} = \int_0^1 F(x) dx = \frac{1}{3}[1 + p^4 + q^4 + r^2 + r^4 + p^2(-1 + 2q^2 - 2r^2) + q^2(1 + 2r^2)]. \quad (13)$$

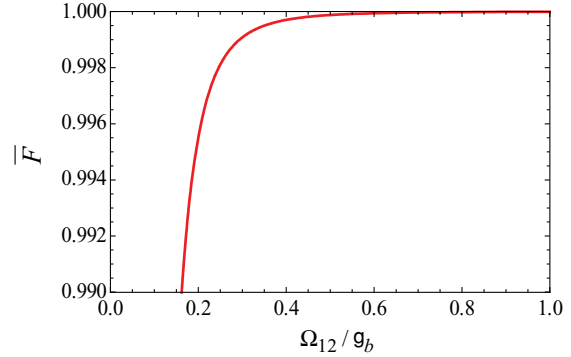


FIG. 3. (Color online) Average fidelity \bar{F} as a function of the Rabi frequency Ω_{12} (in unit of g_b) for $\Delta_c = 10g_b$.

It can be verified that when the unwanted ‘‘SQUID b ’’-resonator off-resonant interaction in steps (ii) and (iv) is not considered (i.e., the case for $g_b^2/\Delta_c = 0$ or $s = 0$), we have $p = q = 0$ and $r = 1$, leading to $F = 1$ and $\bar{F} = 1$. We have plotted the average fidelity \bar{F} for the case $\Delta_c = 10g_b$ (Fig. 3). One can see from Fig. 3 that the average fidelity \bar{F} increases as the Rabi frequency Ω_{12} of the pulse applied to SQUID b becomes larger, and the \bar{F} is ~ 1 when $\Omega_{12} = 0.6g_b$.

Conclusion. We have presented a way to realize a two-qubit controlled phase gate with two SQUIDs, by the use of a microwave superconducting resonator. As shown above, in this proposal, (a) SQUIDs, which often have considerable parameter nonuniformity, can be used; (b) the adjustment of the level spacings, which is undesirable in experiment, is avoided; and (c) neither the adiabatic passage nor a second-order detuning is needed and thus the gate can be performed much faster.

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