Reanalysis of Casimir force measurements in the 0.6-to-6-μm range

S. K. Lamoreaux

Yale University, Department of Physics, P.O. Box 208120, New Haven, Connecticut 06520, USA

A systematic correction for Casimir force measurements is proposed and applied to the results of an experiment that was performed more than a decade ago. This correction brings the experimental results into good agreement with the Drude model of the metallic plates' permittivity. The systematic is due to time-dependent fluctuations in the distance between the plates caused by mechanical vibrations, tilt, or position measurement uncertainty and is similar to the correction for plate roughness.

Introduction. This is a brief report of a correction to the Casimir force [1] and the results of its application to my experiment that was performed some 15 years ago [2]. One might question whether it is worth reanalyzing an experiment that is so very dated; however, this work stands, together with work with germanium [3], as the experiments with the largest plate separations and is particularly sensitive to a number of fundamental, thermal, and systematic effects. Indeed [2], led to a resurgence in interest in the Casimir measurement field [4] and has been discussed in various works, for example [5,6].

Although my experiment [2] was intended as a demonstration, the deviation between its results and the theory presented by Böstrom and Sernelius [5] remains as a puzzle. Despite years of theoretical work and years of questioning, there has been no satisfactory explanation of the discrepancy. In particular, it would appear that the Drude model must be the correct one for describing the plates because the so-called plasma model implies a superconducting boundary condition at zero frequency.

Unfortunately, the raw data that led to [2] are no longer available; however, there is enough information in that article to make a good estimate of the correction. As the correction depends on external factors that were not measured, it is not clear that the raw data would help much in any case. The work reported here was inspired by our recent and ongoing remeasurement of the Casimir force between Au plates, which support the Drude model better than the plasma model. In the course of our recent work, it occurred to us that time-dependent fluctuations between the plates can lead to a correction, much like the surface-roughness correction that has been studied by Mostepanenko and collaborators [6].

The interesting features of [2] are as follows: The Drude model appears to better describe the long-distance (greater that 1.5 μm) data. The short-distance data appear to agree with the plasma model prediction. The roughness usually associated with optical surfaces is too small to account for the deviation between the Drude theory and experiment and seems to have the wrong form, as in this case, the discrepancy should fall, as a fraction of the force, as $1/d^2$, where d is the separation between the plates. Thus, for the Casimir force alone, the effect should be very short range. In [2], a background potential existed, creating an electrostatic force that was greater than the Casimir force over the measurement range. Thus, the effects of both forces must be considered together.

Corrections due to vibration and distance calibration uncertainties. The force between two plates, for small variations $\delta(t)$ of the distance d, is

$$F\big(d + \delta(t)\big) = F(d) + F'(d)\delta(t) + \tfrac{1}{2}F''(d)\delta^2(t). \quad (1)$$

If we assume that $\delta(t)$ represents a stationary random process with zero mean, there are two cases to consider. First, if the correlation time of $\delta(t)$ implies frequencies higher than the measurement bandwidth, the term linear in $\delta(t)$ does not contribute to anything, while the second-order term results in a change in the apparent force,

$$F_a(d) = F(d) + \tfrac{1}{2}F''(d)\langle\delta^2\rangle, \quad (2)$$

where $\langle\delta^2(t)\rangle = \delta_{\rm rms}^2$ is the mean-square fluctuation. It should be noted that $\delta_{\rm rms}$ can have contributions from multiple sources, which, if uncorrelated, can be added in quadrature. In addition, a finite surface roughness can be included here; the form of Eq. (2) does not distinguish between spatial and temporal roughness.

Second, if the fluctuations frequency is within the measurement bandwidth, which is the case for uncertainties in the distance determination, there will be an excess scatter associated with the force measurement,

$$\sigma_{F_a}^2 = \sigma_F^2 + [F'(d)\delta_{\rm rms}]^2, \quad (3)$$

while the apparent average force is given by Eq. (2), as before.

Application to the experiment. In [2], there is a large background electrostatic force that is used for determining the absolute distance, obtained by fitting to $\beta/(d - d_0)$, using points at distances greater than 2 μm. For the approximations here, let us assume that there are no significant corrections at these long distances. Based on Fig. 3 of [2], the background electrostatic force is

$$F_e(d) = \frac{\beta}{d}; \quad \beta = (215 \pm 7)\ \mu{\rm dyn}\ \mu{\rm m}. \quad (4)$$

The total force is the electric plus Casimir force,

$$F(d) = F_e(d) + F_c(d), \quad (5)$$

and the first derivative at $d = 0.62$ μm is approximately 1000 μdyn/μm. Comparing the size of the error bars in Fig. 4 of [2], where the 0.1-μm bins have 1/10 the data of the 1-μm bins and should be $\sqrt{10}$ times larger. It is observed that they are $1.32\sqrt{10}$ larger, so the contribution to the error due to fluctuations is 4 μdyn. This implies that the rms fluctuations

in position on a time scale of the measurement of each point (50 s) is 40 nm. This is to be compared to 14 nm in our present experiment, where the intrinsic signal to noise is similar, as is the applied calibration voltage. The excess noise in [2] is likely due to the faster rate of drift in position. It is stated in [2] that the fluctuations in absolute position measurement is less than 100 nm, which is consistent with the result here. On the other hand, the quality of the data in Fig. 3 of [2] suggests that perhaps 40 nm is optimistic; unfortunately, the original data set is not available to further investigate this point. For fluctuations at this level, the apparent change in force is less than 5% so these fluctuations do not contribute to the discrepancy.

Our recent work shows that, due to vibrations and tilt, there is an rms position fluctuation of 20 nm in a 0.01-to-5-Hz bandwidth for our torsion pendulum supported by a tungsten wire of a few cm length. The variations at low frequency are dominated by the tilt of the pendulum and correspond to angular fluctuations of the order of 1×10^{-7} rad. Given that the pendulum in the present experiment has a length of 4 cm, compared to an effective length of nearly 80 cm for [2], we might expect position rms fluctuations of order 400 nm, as the change in position is roughly the pendulum length times the tilt angle, which is not to be confused with the torsional motion angle. However, the bandwidth of the swinging mode of the longer pendulum is lower (it is relatively smaller by a approximately a factor of $1/\sqrt{40} = 1/6.3$, so the effective noise should be a factor of $1/\sqrt{6.3} = 1/2.5$), implying an rms noise of 400 nm/2.5 = 160 nm. We can therefore take $\delta_{\rm rms} = 100$ nm as a lower limit. In operating the experiment [2], the problems with tilt noise were about an order of magnitude worse than those in our present experiment, consistent with relative size of the rms position fluctuations as discussed here.

Only angular fluctuations at frequencies below the swinging mode frequency will contribute significantly to the relative separation fluctuation between the plates because the magnetic damper tends to stabilize the relative position of the plates. On the other hand, vibrations that cause a net translational motion of the system couple in a different way, and frequencies above the mode frequency can contribute. The angular noise dominates so I neglect translational vibrations in this discussion.

Let us now consider the combined effect of $F_e(d)$ and $F_c(d)$ for either the plasma or Drude models of permittivity. The correction to the force is given by Eq. (2) as

$$F_a(d) - F(d) = \tfrac{1}{2}[F_e''(d) + F_c''(d)]\delta_{\rm rms}^2. \tag{6}$$

It is easy to calculate $F_e''(d)$, while $F_c''(d)$ can be numerically evaluated. The plasma model and Drude model forces were calculated using tabulated Au properties, with the interesting result that the second derivative of the Drude model is about twice as large, in the 0.5-to-1-μm range, as that for the plasma model, which is not too surprising as the Drude model force falls off more rapidly in this region. It should also be noted that $F_e'' \propto 1/d^3$, so there is a constant offset for large d, when the correction is multiplied by d^3, as shown in the graph.

The results are shown in Fig. 1, where it is clear that the Drude model has much better agreement. Furthermore, both the large and the small distance data agree with the theory, unlike the case of any other model. As an aside, a remeasurement of the radius of curvature of the spherical plate
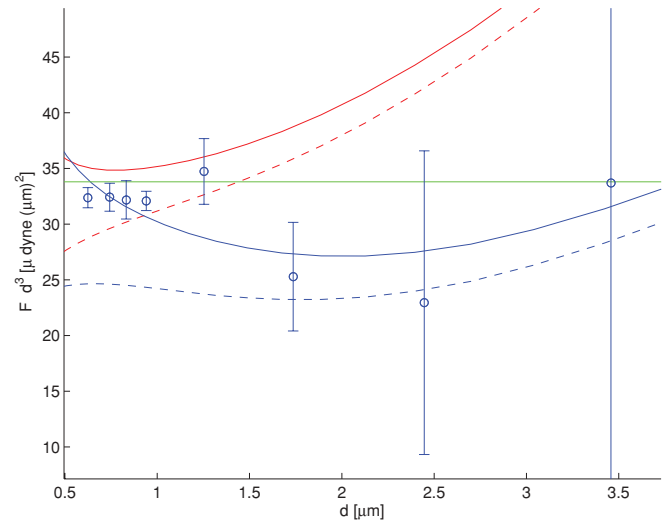

FIG. 1. (Color online) Results taking $\delta_{\rm rms} = 100$ nm. Horizontal green line, perfect conductor, zero temperature; solid red (first line from the top), plasma model with distance fluctuations; dashed red (second line from the top), plasma model without distance fluctuation correction; solid blue (third line from the top), Drude model with distance fluctuations; dashed blue (fourth line from the top), Drude model without distance fluctuation correction. Reduced $\chi^2 = 6.17$ (probability $<10^{-5}$, 6 degrees of freedom) for the plasma model with distance correction, while $\chi^2 = 1.75$ for the Drude model with distance correction (probability 10%, 6 degrees of freedom).

used in [2] shows $R = 12.4 \pm 0.1$ cm, which has lower error than the number reported in the erratum [2].

The agreement can be made better by allowing $\delta_{\rm rms}$ to vary with distance. It is reasonable to assume that the runs which attained the lowest separation were obtained when the system and environment was particularly quiet. For example, if we let $\delta_{\rm rms} = \sqrt{\frac{d}{3\mu m}}$ μm, then $\chi^2 = 0.79$ for the Drude model, while $\chi^2 = 6.9$ for the plasma model.

Discussion and conclusion. By assuming that the relative separation between the plates of a Casimir experiment is fluctuating on a short time scale, the results given in [2] can be brought into agreement with theory. The required rms fluctuations appear as large, of the order of 100 nm, but such a level is not unreasonable. Given that the total pendulum length is nearly 1 m, a tilt of 10^{-7} rad is all that is required to generate the required fluctuations. Such level of tilt is easily generated by air currents moving past the apparatus and by unavoidable oscillations of the floor. It should be noted that the rms position fluctuations are due mainly to angle fluctuations with a frequency of less than about 0.5 Hz, the natural frequency of the pendulum's swinging mode. Measurements with our new apparatus as Yale shows less than 50 ± 20-nm rms fluctuations; however, the pendulum is only a few cm in length. A full analysis of the pendulum and how various tilts and vibrations affect the relative positions of the plates is a tedious but elementary exercise. I do note, however, that within the feedback bandwidth (which is greater than the measurement bandwidth), the system compensates for a tilt by adjusting the torsion pendulum angle in order to keep the differential capacitor balanced. Thus, for frequencies below about 0.1 Hz, a tilt position offset is approximately

doubled for the interplate separation. Indeed, the extreme sensitivity of the apparatus to tilt and vibration was readily apparent; in order to take sensible data, the experiment could only be operated between 11 p.m. and about 5 a.m., and the air-conditioning ducts into the room had to be blocked. The required rms position fluctuation of 100 nm is below the minimum plate separation of 600 nm and falls into what can be considered a reasonable range. The angular fluctuations appeared as a very slow drift, causing a change in the distance offset between the plate, which was of order 1 μm/h. On top of this slow drift, according to the calculations here, were rapid fluctuations with periods up to 2 s (the frequency of the pendulum swinging mode) with rms deviations of order 10^{-7} rad in the 0.01-to-0.5-Hz bandwidth.

It appears that $\delta_{rms} = 100$ nm gives close to a best fit and thus should be considered as an adjustable parameter, the value of which is verified by other means described in this brief report. Also to be noted is that other contributions to δ_{rms} can be included by adding all contributions in quadrature. Of course, other systematic effects can contribute, such as a contact potential that varies with distance. The contact potential was not measured as a function of distance in [2]; however, our recent work with Au suggests that the contact potential is nearly constant. This does not preclude the possibility that there was such a variation in [2].

If $\delta_{rms}(d)$ depends on distance, which is a very distinct possibility, it is easy to see that the the data can be brought into better agreement with the Drude model in particular. Certainly, δ_{rms} depends on time, and the data runs that attained the closest separation were likely obtained when the system and the environment were the quietest. This level of fine tuning is beyond the scope of the brief analysis presented here and beyond the scope of credibility.

Effects of vibration are important for all Casimir experiments. Even in the absence of external perturbations, atomic force microscope (AFM) cantilevers, for example, exhibit Brownian motion and the effects of such need to be taken into account. The implication is that very stiff springs should be used. In the case of the torsion pendulum experiment, feedback is used to keep the torsion angle fixed; this reduces position fluctuations due to Brownian motion, but can make the system more sensitive to vibrations and tilts, as discussed earlier.

The apparent force due to the large $1/d$ electrostatic force varies as $1/d^3$, with magnitude relative to the Casimir force (perfect conductors) of $\beta\delta_{rms}^2/2 = 1.2$ μdyn μm^3, about 3% of the perfect conducting force.

Again, the work described in [2] was presented as a demonstration; the analysis here shows that there is a possible systematic effect that can lead to a substantial increase in the apparent Casimir force. In this case, the large electrostatic force, present for calibration, contributes substantially, particularly at large separations. Because its contribution scales as $1/d^3$, it appears as a scale factor for the Casimir force for distances around 0.5 μm. Recent work at Yale led to the consideration of these effects and also appears to support the Drude model for the permittivity. We hope to complete these studies in the very near future.

Finally, it must be emphasized that [2] was intended as a demonstration; the results presented here should not be considered as a verification of the Boström-Sernelius theory [5] or as evidence against the plasma model, but the discovery of a systematic effect that brings the experimental results into agreement with the theory described in [5]. It is unclear whether additional systematic effects exist; however, it seems likely that there are additional, possibly large systematics [7].

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