Deterministic source of a train of indistinguishable single-photon pulses with a single-atom-cavity system

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We present a mechanism to produce indistinguishable single-photon pulses on demand from an optical cavity. The sequences of two laser pulses generate, at the two Raman transitions of a four-level atom, the same cavity-mode photons without repumping of the atom between photon generations. Photons are emitted from the cavity with near-unit efficiency in well-defined temporal modes of identical shapes controlled by the laser fields. The second-order correlation function reveals the single-photon nature of the proposed source. A realistic setup for the experimental implementation is presented.

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I. INTRODUCTION

Deterministic sources of high-quality single-photon (SP) states are of great importance for quantum-information processing [1]. A basic requirement for many quantum optics applications, including quantum computing with linear optics [2,3], quantum cryptography [4], and entanglement swapping [5], is to have single photon pulses with well-defined identical shapes, frequency, and polarization, as these schemes based on photon-interference effects are very sensitive to the parameters of SP pulses and their repetition rate. A good source has to ensure a pure SP state without mixture from both the multiphoton and zero-photon states, as well as prevent the entanglement between the photons, which degrades the purity of the SP state. Since the individual photons are usually emitted during the spontaneous decay of atomic systems, the SP sources must be immune to the environmental effects that induce the dephasing of atomic transitions. Most of the schemes proposed earlier to produce single photons on demand from solid-state single emitters [6], organic molecules [7,8], and quantum dots [9,10] are confronted with this difficulty. Besides, they do not offer a high efficiency because of the isotropic nature of fluorescence, which prevents photon collect, not to mention the spectral dephasing and inhomogeneity of solid-state emitters. Deterministic sources of single photons are realized also in cold atomic ensembles with a feedback circuit [11,12]. But these schemes are not suitable for generating SP train with an arbitrary repetition rate because of strong temporary bounds caused by the feedback and write-read processes.

At present, all these requirements can be achieved together with a Λ-type atom trapped in high-finesse optical cavities [13–16], where the single photons are generated via vacuum-stimulated Raman scattering of a classical laser field into a cavity mode. These systems not only provide a strong interaction between a photon and an atom but also support very high collection efficiency because the photons leave the cavity through one mirror with a transmissivity incomparably larger than that of the opposite one. By carefully adjusting the parameters of the laser pulse one can also easily control the wave form of output single photons. However, the main disadvantage of these schemes is the necessity to use a repumping field to transfer the population of the atom to its initial state after the generation of a cavity photon and only then to generate the next one. In this paper we propose a scheme featuring a double-Raman atomic configuration, which is able to deterministically generate a stream of identical SP pulses without using the repumping field, while maintaining the high generation efficiency, as well as providing simple control of the output photon wave forms. The removal of the repumping field is a principal task, because its usage strongly restricts the process: The repetition rate of emitting photons is limited by the acting time of the repumping field. Unlike this, our mechanism allows us to freely change the repetition rate of the single-photon pulses up to zero, because even in this case nonentangled single photons are generated. One of the most important applications of this property is to generate Fock states with a programmable number of photons. Moreover, in the good-cavity limit (atom-cavity mode coupling is larger than the cavity decay rate) our scheme can serve as an one-atom laser with a controllable statistics of generated photons that allows the quantitative study of the quantum-to-classical transition in our system raised with gradual change of parameters. It is worth noting that a similar scheme without repumping field was employed in [17] for generating a sequence of single photons of alternating polarization.

This paper is organized as follows. In the next section we present the generic atomic system and derive the basic equations for the time evolution of the atomic state amplitudes and the cavity field. Here we find the analytic solutions for the flux and numbers of output photons and discuss the main features of the model. Then, in Sec. III we consider the generation of cavity photons under more realistic conditions when all relevant atomic levels and the spontaneous decays are taken into account. In Sec. IV the results for the correlation between cavity photons are presented, demonstrating the single-photon nature of the source. Our conclusions are summarized in Sec. V.

II. THE GENERIC FOUR-LEVEL ATOM

Our scheme, illustrated in Fig. 1, involves a four-level atom trapped in a one-mode high-finesse optical cavity. The two

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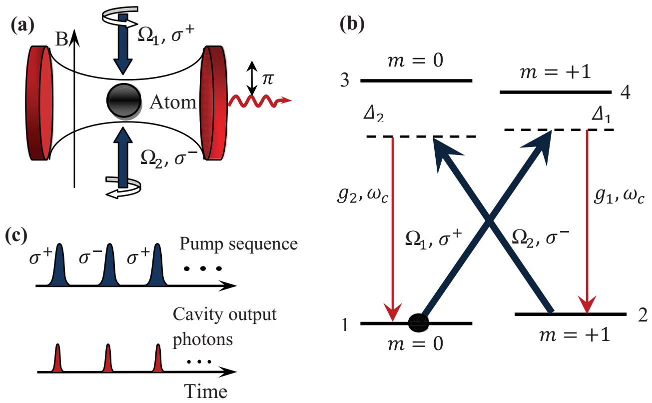

FIG. 1. (Color online) Schematic setup. (a) A single atom trapped in a high-Q cavity is driven by two laser pulses. (b) Relevant atomic level structure in an external magnetic field. Shown is the case when the Landé g_L factors of the ground and excited states have opposite signs. (c) Sequence of laser pulses and generated cavity output single photons.

ground states 1 and 2 and two upper states 3 and 4 of the atom are Zeeman sublevels [Fig. 1(b)], which are split by a magnetic field acting perpendicular to the cavity axis. The atom is initially prepared in one of the ground states, for instance, in state 1 of magnetic quantum number $m = 0$, and interacts in turns with a sequence of two pumping fields as shown in Fig. 1(a). At first, a coherent σ^+ polarized field of Rabi frequency Ω_1 applied between ground state 1 and excited state 4 transfers the atom to ground state 2 while creating a cavity-mode Stokes photon. Then, after a programmable delay time τ_d, which is larger as compared to the pump pulse duration T, the σ^- polarized pump pulse Ω_2 generates the anti-Stokes photon at the $3 \rightarrow 1$ transition and transfers the atom back to ground state 1. The Stokes and anti-Stokes photons have identical frequencies, so that the cavity is coherently coupled to the atom on both transitions $4 \rightarrow 2$ and $3 \rightarrow 1$ with the coupling constants g_1 and g_2, respectively, resulting in the generation of linearly polarized cavity photons in both cases. The laser fields are tuned to the two-photon resonance, while the one-photon detunings are very large compared to the cavity damping rate k and the Larmor and Rabi frequencies: $\Delta_i \gg k$, $\Delta_B^{(g,e)}, g_i, \Omega_i$, $i = 1,2$, where $\Delta_B^{(g,e)} = g_L^{(g,e)} \mu_B B$ is Zeeman splitting of atomic levels in the magnetic field B with $g_L^{(g,e)}$ the Landé factor of the ground and excited states and μ_B the Bohr magneton. This condition makes the system robust against the spontaneous loss from upper levels and dephasing effects induced by other excited states. More importantly, in the off-resonant case the Raman process with effective atom-photon coupling $G_i = g_i \Omega_i / \Delta_i, i = 1,2$, can be made much slower than the cavity field decay: $G_i \ll k$. This ensures that a generated photon leaves the cavity long before the next cavity photon is emitted and, hence, no entanglement between the photons will be created, if we also take into account that the coherence between atomic ground states is always zero. Therefore, we can construct identical wave packets for outgoing photons independently from each other, as they are entirely determined by the temporal shape of the corresponding pump pulse. A remarkable feature of our scheme is that, despite the smallness of $G_{1,2}$, it is able to produce cavity photons with near-unit efficiency, as discussed in more detail in the following.

Despite of its simplicity, the four-level scheme is realized, for example, in the atom of the lithium isotope ^{6}Li (or ^{40}Ca$^+$) with the ground states $2S_{1/2}(F = 1/2, m_F = \mp 1/2)$ and exited states $2P_{1/2}(F' = 1/2, m_{F'} = \mp 1/2)$ as the states (1,2) and (3,4) in our scheme, respectively. The central drawback of this scheme is that the spontaneous decay of upper states into the ground state $2S_{1/2}(F = 3/2)$ constitutes a loss channel that moves the system outside the considered level configuration. However, we show that even in this case, for reasonable values of parameters, the atom can generate about 70 identical SP pulses before falling into the ground $F = 3/2$ state. To restore the generation a repumping field must be applied to transfer the atom into the initial state. For continuous generation of single photons, a closed system can be used by employing cycling transitions, for example, of the D_2 line in the ^{87}Rb atom with $5S_{1/2}(F = 2)$ and $5P_{3/2}(F' = 3)$ as the ground and excited states. In this case, the only limitation is the atom lifetime in cavities, which amounts to at most one minute [16]. When analyzing this system all Zeeman sublevels participating in the interaction with the laser pulses must be taken into account. However, we show in the next section that owing to the fact that no coherence is created between the Zeeman sublevels the results of four-level scheme is easily generalized to this complicated case. In this sense the four-level atom serves as a generic scheme for multilevel atoms.

Here we derive the equations for atomic and cavity field operators in the case of a four-level atom. Using analytical solutions of these equations, we calculate the number and flux of cavity photons, which determine the SP detection time distribution or the shape of SP wave packets.

The laser fields propagating perpendicular to the cavity axis are given by

$$E_j(t) = \mathcal{E}_j f_j^{1/2}(t) \exp(-i\omega_j t), \quad j = 1,2. \tag{1}$$

where $f_1(t) = \sum_{l=1}^N f_1^l(t)$ and $f_2(t) = \sum_{l=1}^N f_2^l(t - \tau_d)$ represent the sum of N well-separated temporal modes with profiles $f_1^l(t)$ and $f_2^l(t - \tau_d)$ for the lth temporal mode in the pump series 1 and 2, respectively. $\mathcal{E}_j$ is the peak amplitude of the field j.

In the far off-resonant case, we can adiabatically eliminate the upper atomic states 3 and 4 and write the effective Hamiltonian in the rotating-wave approximation (RWA) as

$$H = \hbar\left[G_1 f_1^{1/2}(t)\sigma_{21} + G_2 f_2^{1/2}(t)\sigma_{12}\right]a^\dagger + \text{H.c.} \tag{2}$$

with σ_{ij} and $a(a^\dagger)$ the atomic and cavity mode operators, respectively. The peak Rabi frequencies of the laser fields are given by $\Omega_1 = \mu_{41}\mathcal{E}_1/\hbar, \Omega_2 = \mu_{32}\mathcal{E}_2/\hbar$ with μ_{ij} the dipole matrix element of the $i \rightarrow j$ transition.

For Ω_i and g_i of the same order, the Stark shifts of the atomic ground levels, of the form $\Omega_i^2 f_i(t)/\Delta_i$ and g_i^2/Δ_i, are negligibly small with respect to the cavity linewidth k in the bad-cavity limit, $G_i \ll k$, as considered here.

The system evolution is described by the master equation for the whole density matrix ρ for the atom and cavity mode [18]:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \left.\frac{d\rho}{dt}\right|_{\rm rel}, \tag{3}$$

where the second term in the right-hand side (r.h.s.) accounts for all relaxations in the system. With the use of the Lindblad operator $L[\hat{O}]\rho = \hat{O}\rho\hat{O}^\dagger - (\hat{O}^\dagger\hat{O}\rho + \rho\hat{O}^\dagger\hat{O})/2$ it is written in the form

$$\left.\frac{d\rho}{dt}\right|_{\rm rel} = kL[a]\rho + \Gamma_1(t)L[\sigma_{21}]\rho + \Gamma_2(t)L[\sigma_{12}]\rho - (\Gamma_{1,\rm out}\sigma_{11} + \Gamma_{2,\rm out}\sigma_{22})\rho. \tag{4}$$

The first term on the r.h.s. of this equation represents the cavity output coupling, while the second and third terms describe the optical pumping (OP) to ground states 2 and 1 from the states 1 and 2, respectively, and give rise to noise of corresponding rates

$$\Gamma_1(t) = \frac{\Omega_1^2}{\Delta^2} f_1(t)\gamma_{42}, \quad \Gamma_2(t) = \frac{\Omega_2^2}{\Delta^2} f_2(t)\gamma_{31}. \tag{5}$$

The last term of Eq. (4) introduces the losses of atomic population due to the decay of upper atomic states 3 and 4 into states outside of the system with rates $\gamma_{3,\rm out}$ and $\gamma_{4,\rm out}$, respectively:

$$\Gamma_{1,\rm out}(t) = \frac{\Omega_1^2}{\Delta_1^2} f_1(t)\gamma_{4,\rm out} = \Gamma_{1,\rm out} f_1(t), \tag{6a}$$

$$\Gamma_{2,\rm out}(t) = \frac{\Omega_2^2}{\Delta_2^2} f_2(t)\gamma_{3,\rm out} = \Gamma_{2,\rm out} f_2(t). \tag{6b}$$

The spontaneous emission channel corresponding to the cycling transition returning the atom back to the starting state should be in principle included. However, in contrast to the decays of Eqs. (5) and (6), this channel neither changes the population of the system nor leads to any noises, and therefore it can be ignored.

Our aim is to find the flux of the output photons,

$$\frac{dn_{\rm out}(t)}{dt} = \langle a_{\rm out}^\dagger(t) a_{\rm out}(t)\rangle, \tag{7}$$

which describes the sequence of outgoing SP wave packets. Here $n_{\rm out}(t)$ is the mean photon number of the output field $a_{\rm out}(t)$ from the cavity which is connected with the input $a_{\rm in}(t)$ and intracavity $a(t)$ fields by the input-output formulation [18]

$$a_{\rm out}(t) - a_{\rm in}(t) = \sqrt{k}a(t) \tag{8}$$

and satisfies the commutation relation $[a_{\rm out}(t), a_{\rm out}^\dagger(t')] = [a_{\rm in}(t), a_{\rm in}^\dagger(t')] = \delta(t-t')$. With the Hamiltonian (1), the Heisenberg-Langevin equation for $a(t)$ is given by [18]

$$\dot{a} = -iG_1 f_1^{1/2}(t)\sigma_{21} - iG_2 f_2^{1/2}(t)\sigma_{12} - (k/2)a - \sqrt{k}a_{\rm in}(t). \tag{9}$$

In the bad-cavity limit, $k \gg G_{1,2}$, we adiabatically eliminate the cavity mode $a(t)$, yielding

$$a = -\frac{2i}{k}\left[G_1 f_1^{1/2}(t)\sigma_{21} + G_2 f_2^{1/2}(t)\sigma_{12}\right] - \frac{2}{\sqrt{k}}a_{\rm in}(t). \tag{10}$$

Upon substituting this solution into the Hamiltonian, for the case of a vacuum input $\langle a_{\rm in}^\dagger(t)a_{\rm in}(t)\rangle = 0$, we eventually obtain from Eq. (3) the following equations for the photon flux:

$$\frac{dn_{\rm out}(t)}{dt} = \alpha_1 f_1(t)\langle\sigma_{11}(t)\rangle + \alpha_2 f_2(t)\langle\sigma_{22}(t)\rangle \tag{11}$$

and for atomic variables $i,j = 1,2;\ j \neq i$,

$$\langle\dot{\sigma}_{ii}(t)\rangle = -[\alpha_i f_i(t) + \Gamma_i(t) + \Gamma_{i,\rm out}(t)]\langle\sigma_{ii}(t)\rangle + [\alpha_j f_j(t) + \Gamma_j(t)]\langle\sigma_{jj}(t)\rangle, \tag{12}$$

$$\langle\dot{\sigma}_{21}(t)\rangle = -\frac{1}{2}\sum_{i=1}^{2}[\alpha_i f_i(t) + \Gamma_i(t)]\langle\sigma_{21}(t)\rangle, \tag{13}$$

which are subjected to initial conditions $\langle\sigma_{11}(-\infty)\rangle = 1$, $\langle\sigma_{22}(-\infty)\rangle = \langle\sigma_{21}(-\infty)\rangle = 0$. Here

$$\alpha_i = 4G_i^2/k, i = 1,2. \tag{14}$$

Thus, in the bad-cavity limit the problem is reduced to the solution of the dynamical equations for the atom. Equations (12) are easily solved analytically. However, the final solutions are lengthy and will be given here only graphically. We first discuss some properties of Eqs. (11)–(13). It is seen that state 1 (and similarly state 2) is populated in two ways: (i) via cavity-photon generation with the rate $\alpha_{1(2)}(t)$ and (ii) by optical pumping of rate $\Gamma_{1(2)}(t)$, which gives for the signal-to-noise ratio $R_{\rm sn} = 4g_{1(2)}^2/(k\gamma_{42(31)})$, which must be quite large: $R_{\rm sn} \gg 1$. The second observation is that the overall population of the atom after a total of n pulses of two laser sequences for $\alpha_i T \gg 1$ decreases as

$$\langle\sigma_{11}(t) + \sigma_{22}(t)\rangle|_{t\geqslant(n-1)\tau_d+T} = (1 - n\Gamma_{\rm out}/\alpha). \tag{15}$$

Here it is assumed that $\Gamma_{1\rm out} = \Gamma_{2\rm out} = \Gamma_{\rm out}$, $\alpha_1 = \alpha_2 = \alpha$. Thus, the population leakage is negligibly small until $n\alpha/\Gamma_{\rm out} < 1$. Further, the ground-state coherence is always zero: $\langle\sigma_{21}(t)\rangle = 0$, as expected due to the spontaneous nature of Raman transitions. Finally, from the explicit expression of the flux calculated from Eq. (7) for one pump pulse and considering that for $\Gamma_{\rm out} = 0$

$$\frac{dn_{\rm out}(t)}{dt} = \alpha_1 f_1(t) e^{-\alpha_1\int_{-\infty}^{t} f_1(t')dt'}, \tag{16}$$

we conclude that the wave form of the emitted single photon is simply related to the shape of the pump pulse and, thereby, is much easier to control compared to schemes proposed so far in literature, where this dependence is in integral form [14,19,20]. From this equation one finds $n_{\rm out}(t) = 1 - \exp[-\alpha_1\int_{-\infty}^{t} f_1(t')dt']$, showing that our system is able to produce photons with near-unit efficiency, if

$$\alpha_i T \gg 1. \tag{17}$$

In Fig. 2 the calculated flux and total number of output photons, as well as the populations of atomic ground states, are shown for the case when each laser sequence contains four Gaussian-shaped subpulses with duration $T = 1\ \mu$s corresponding to a linewidth of 1 MHz. The atom is initially in the ground state 1. We present the results for cavity-photon generation with wavelength $\lambda \sim 671$ nm on the D1 line transition $2P_{1/2}(F' = 1/2) \rightarrow 2S_{1/2}(F = 1/2)$ in the ^{6}Li

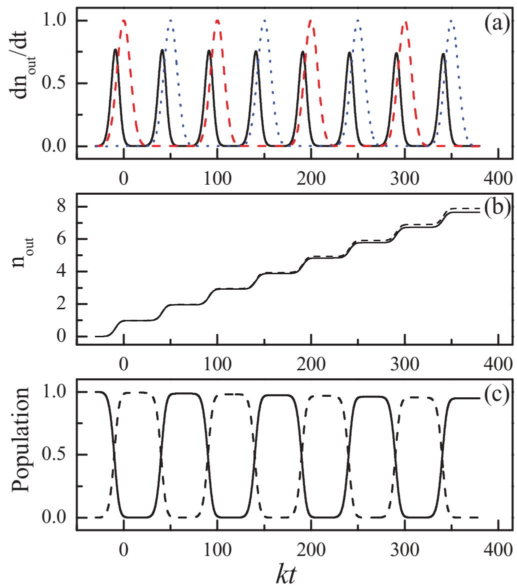

FIG. 2. (Color online) Flux in the units of k (a), total number of output field photons (b), and population of atomic ground states 1 (solid) and 2 (dashed) (c) as a function of time (in units of k^{-1}), in the case when four identical pulses of each laser beam [red-dashed and blue-dotted in (a)] are applied with delay time $\tau_d = 3\ \mu$s. For the rest of the parameters see the text. The scale of output pulses in (a) is increased by a factor of 3.

atom obtained with the following parameters: $\Omega_{1,2} = 2\pi \times 10$ MHz $= 0.1\Delta(\Delta_1 \simeq \Delta_2 = \Delta)$, $(g_{1,2},k,\gamma_{sp})/2\pi = (10,3,5.87)$ MHz, and $\gamma_{3,out} = \gamma_{4,out} \sim \gamma_{sp}$, γ_{sp} being the natural linewidth of Li atom. A magnetic field of 10 G produces the Zeeman splitting $\Delta_B/2\pi = 7$ MHz. These parameters are within experimental reach and ensure the fulfillment of all the necessary conditions indicated here. The results demonstrate two important features of the scheme. The photons are generated deterministically at the leading edge of each pump pulse with identical duration $T_{cav} \sim T/2$ and time-symmetric wave packets. The efficiency of one-photon generation by each pump pulse is close to 100% [see Fig. 2(b)]. As one can see in Figs. 2(a) and 2(c), the peak values of the generated SP pulses and of the atomic populations display a slight decrease in time caused by population losses through the channel $2P_{1/2}(F' = 1/2) \to 2S_{1/2}(F = 3/2)$. Because of this fact the total number of emitted photons does not reach its maximum value of 8 [Fig. 2(b), solid line]. Nevertheless, from Eq. (15) it follows that about $n_{out} \sim \alpha/\Gamma_{out} \simeq R_{sn} \simeq 70$ cavity photons are generated before the losses become significant. For comparison, n_{out} is shown also for the lossless case of $\Gamma_{out} = 0$ [Fig. 2(b), dashed line].

III. CONTINUOUS GENERATION OF SP PULSES FROM ALKALI ATOMS

The continuous generation of SP pulses of identical polarization is possible on the cycling transitions of alkali atoms with $F',F \neq 0$. As an example we consider the transition $5S_{1/2}(F = 2) \to 5P_{3/2}(F' = 3)$ in the ^{87}Rb atom where the state $5P_{3/2}(F' = 3)$ is well isolated from other hyperfine levels. Using the fact that Zeeman coherence is always 0, the results of the previous section are easily generalized to this case, leading to an expression for the photon flux of

$$\frac{dn_{out}(t)}{dt} = f_1(t)\sum_{m_F=-F}^{F-1}\alpha_{1,m_F}\langle\sigma_{m_F}(t)\rangle + f_2(t)\sum_{m_F=-F+1}^{F}\alpha_{2,m_F}\langle\sigma_{m_F}(t)\rangle, \tag{18}$$

where $\sigma_{m_F}(t)$ is the population of the ground Zeeman sublevel $|F,m_F\rangle$, while α_{1,m_F} and α_{2,m_F} are the probabilities of two-photon transitions $|F,m_F\rangle \to |F,m_F+1\rangle$ and $|F,m_F\rangle \to |F,m_F-1\rangle$ with absorbing one laser photon from the σ^+ and σ^- pump pulses and emitting one cavity photon, respectively. They have the same form as the probabilities $\alpha_{1,2}$ obtained in Eq. (14) for the four-level system.

The equation for the Zeeman sublevel populations takes the form

$$\begin{aligned}\langle\dot\sigma_{m_F}(t)\rangle &= A^{(1)}_{m_F-1}(t)\langle\sigma_{m_F-1}(t)\rangle + A^{(2)}_{m_F+1}(t)\langle\sigma_{m_F+1}(t)\rangle\\ &\quad - \big[A^{(1)}_{m_F}(t) + A^{(2)}_{m_F}(t) + \Gamma^{(1)}_{m_F,m_F+2}(t)\\ &\quad + \Gamma^{(2)}_{m_F,m_F-2}(t)\big]\langle\sigma_{m_F}(t)\rangle + \Gamma^{(1)}_{m_F-2,m_F}(t)\langle\sigma_{m_F-2}(t)\rangle\\ &\quad + \Gamma^{(2)}_{m_F+2,m_F}(t)\langle\sigma_{m_F+2}(t)\rangle,\end{aligned} \tag{19}$$

where

$$A^{(1)}_{m_F}(t) = f_1(t)\alpha_{1,m_F} + \Gamma^{(1)}_{m_F,m_F+1}(t), \tag{20}$$

$$A^{(2)}_{m_F}(t) = f_2(t)\alpha_{2,m_F} + \Gamma^{(2)}_{m_F,m_F-1}(t). \tag{21}$$

The expression (19) includes all OP rates, where $\Gamma^{(1)}_{m_F,m_F+1}(t)$ and $\Gamma^{(1)}_{m_F,m_F+2}(t)$ describe the OP from $|F,m_F\rangle$ into the states $|F,m_F+1\rangle$ and $|F,m_F+2\rangle$, respectively, induced by the Ω_1 pump pulse and similarly $\Gamma^{(2)}_{m_F,m_F-1}(t)$ and $\Gamma^{(2)}_{m_F,m_F-2}(t)$ represent the OP caused by the Ω_2 pulse. The first term on the r.h.s. of Eq. (18) describes the generation of cavity photons by the Ω_1 field initialized from the Zeeman sublevels $-F \leqslant m_F \leqslant F-1$. Similarly, the flux of output photons generated by Ω_2 from corresponding Zeeman levels is determined by the second term in Eq. (18). A remarkable feature of multilevel Zeeman systems is that more than one cavity photon per laser pulse can be emitted. This is achieved for sufficiently high laser power, when the atomic population prepared initially, for example, in the state $|F,-F\rangle$ is completely transferred to the extreme right state $|F,F\rangle$, ensuring the maximal number of output photons, $n_{out} = 2F$. This property can be used to produce Fock states with a given number of photons. Thus, to guarantee single-photon emission in our scheme, the laser power has to be appropriately chosen, in contrast to the case of the four-level atom, where the condition (17) simply imposes a minimum laser intensity.

Figure 3 presents the time dependence of the flux and the total number of output photons generated on the transition $5S_{1/2}(F = 2) \to 5P_{3/2}(F' = 3)$ in the ^{87}Rb atom in the case of four Gaussian-shaped laser subpulses. To provide the single-photon generation, the duration and intensity of laser subpulses are taken as half the values of those used in Sec. II. Two important features are readily seen. First, Fig. 3(a) shows that the first few SP pulses are different from the next ones. This is due to the asymmetry of the initial conditions. As a

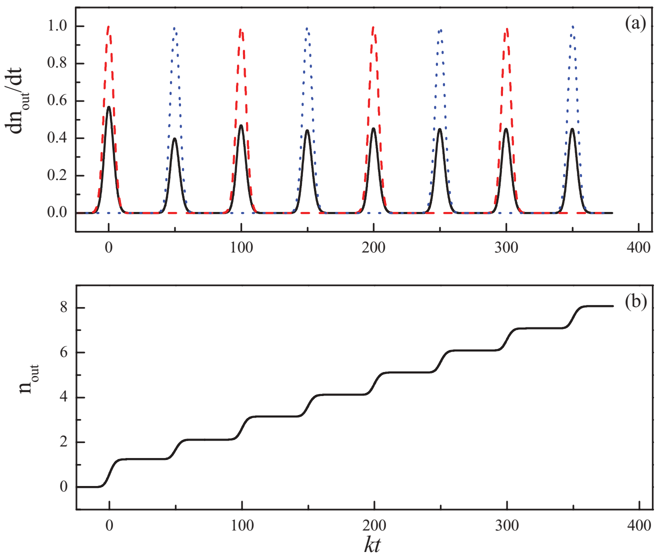

FIG. 3. (Color online) Flux (a) and total number (b) of output cavity photons versus the time (in units of k^{-1}) generated on the cycling transition $5S_{1/2}(F=2) \rightarrow 5P_{3/2}(F'=3)$ in the ^{87}Rb atom. The atom is initially in the state $|F,-F\rangle$. The laser subpulses have a duration $T = 0.5\ \mu$s and the same delay time as in Fig. 2.

consequence, the number of initially generated output photons [Fig. 3(b)] per pump pulse is not constant. However, after the first few photons, a stable generation of identical SP pulses occurs (from $kt \geqslant 100$). Second, in contrast to Fig. 2, the SP pulses are generated at the center of each pump pulse with identical duration $T_{\rm cav} \sim T$ and time-symmetric wave packets. The efficiency of one-photon generation by each pump pulse is again close to 100%, as seen from Fig. 3(b).

IV. PHOTON CORRELATIONS

The probability of a joint detection of two photons produced in the train is given by the intensity correlation function

$$G^{(2)}(t,\tau) = \langle a_{\rm out}^\dagger(t) a_{\rm out}^\dagger(t+\tau) a_{\rm out}(t+\tau) a_{\rm out}(t)\rangle, \qquad (22)$$

where τ is the time delay between the two photon detections. By applying the quantum regression theorem [18,21] and using the input-output relation (8), the second-order temporal correlation function $G^{(2)}(t,\tau)$ is reduced to

$$G^{(2)}(t,\tau) = k[\alpha_1(t+\tau)Z_1(t,\tau) + \alpha_2(t+\tau)Z_2(t,\tau)], \qquad (23)$$

where $Z_i(t,\tau) = \langle a^\dagger(t)\sigma_{ii}(t+\tau)a(t)\rangle, i = 1,2$, as a function of τ, obey equations similar to Eq. (11) with initial values $Z_1(t,0) = \alpha_2(t)\langle\sigma_{22}(t)\rangle/k$ and $Z_2(t,0) = \alpha_1(t)\langle\sigma_{11}(t)\rangle/k$. Since we are interested in the total probability of a joint detection as a function of the time delay τ, we have to integrate Eq. (23) over t. The results of numerical calculations for $G^{(2)}(\tau) = \int_{-\infty}^{\infty} G^{(2)}(t,\tau)dt$ are shown in Fig. 4. The temporal structure of $G^{2)}(\tau)$ reveals the characteristics of a pulsed source of light: The absence of a peak at delay time $\tau = 0$ is evidence of the single-photon nature of the source, and the individual peaks are separated by the pump pulses' delay. The decrease in the peak amplitude of the probability of joint detection for increasing delay time results from having a finite train of emitted photons.

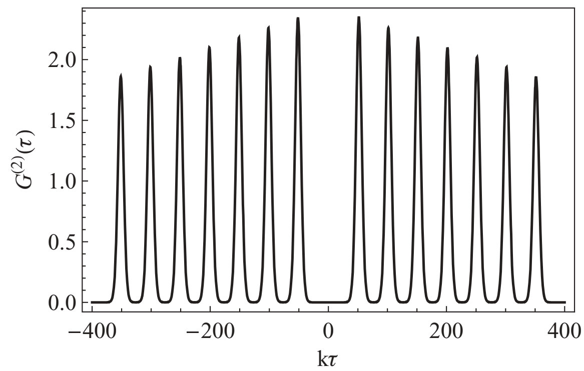

FIG. 4. Intensity correlation integrated over the single-photon train as a function of time delay τ between the two photon detections. The parameters are the same as in Fig. 2.

V. CONCLUSIONS

In conclusion, we have proposed a robust and realistic source of indistinguishable single photons with identical frequency and polarization generated on demand in a well-defined spatio-temporal mode from a coupled double-Raman atom-cavity system. We have considered two cases of four-level systems (^{6}Li or ^{40}Ca$^+$) and atoms with many Zeeman sublevels and have shown that in the first case the number of generated SP pulses is limited by the decay of atomic states outside of the system, while in the second one the continuous SP generation is achievable (e.g., in ^{87}Rb atoms). The high efficiency and simplicity of the scheme, free from such complications as a repumping process and environmental dephasing, makes the generation of many SP identical pulses feasible. We have shown the ability of our scheme to produce a sequence of narrow-band SP pulses with a delay determined only by the pump repetition rate. Such a controlled scheme may pave the way to single-photon-based quantum-information applications, such as deterministic all-optical quantum computation and quantum communication.

Our scheme allows us, as well, to generate Fock states by two different mechanisms. As is mentioned in Sec. III, Fock states containing a fixed number of photons can be generated from each pump subpulse if multilevel atoms with ground state $F \geqslant 1$ are used and laser fields are sufficiently strong. In the second mechanism, the generation of Fock states with a programmable number of photons is possible with overlapped weak pump pulses even in the case of four-level systems such as ^{6}Li atoms or ^{40}Ca$^+$ ions. These questions will be addressed in future publications.

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