Symmetric eikonal model for projectile-electron excitation and loss in relativistic ion-atom collisions

A. B. Voitkiv,[1,*] B. Najjari,[1] and V. P. Shevelko[2]
[1]*Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, D-69117 Heidelberg, Germany*
[2]*P.N. Lebedev Physical Institute of the Russian Academy of Sciences, Leninsky Avenue 53, 117924 Moscow, Russia*
(Received 5 April 2010; published 13 August 2010)

At impact energies $\gtrsim$1 GeV/u the projectile-electron excitation and loss occurring in collisions between highly charged ions and neutral atoms is already strongly influenced by the presence of atomic electrons. To treat these processes in collisions with heavy atoms we generalize the symmetric eikonal model, used earlier for considerations of electron transitions in ion-atom collisions within the scope of a three-body Coulomb problem. We show that at asymptotically high collision energies this model leads to an exact transition amplitude and is very well suited to describe the projectile-electron excitation and loss at energies above a few GeV/u. In particular, by considering a number of examples we demonstrate advantages of this model over the first Born approximation at impact energies of ∼1–30 GeV/u, which are of special interest for atomic physics experiments at the future GSI facilities.

DOI: 10.1103/PhysRevA.82.022707 PACS number(s): 34.10.+x, 34.50.Fa

I. INTRODUCTION

During the last two decades a large number of experimental investigations of projectile-electron excitation and loss in collisions between relativistic highly charged ions and solid and gaseous targets has been performed. In particular, a variety of very heavy projectiles with a net charge of 52–91 a.u. was used in the experiments. Experiments have also covered a very large interval of impact energies, ranging from comparatively low relativistic energies of ∼100–200 MeV/u [1–5] to the extreme relativistic energy of 160 GeV/u [6–8] where the projectile velocity already only fractionally differs from the speed of light $c \approx 137$ a.u.

Most of the data, however, have been collected for impact energies not exceeding a few hundreds of MeV/u. For impact energies above 1 GeV/u just a few experimental results exist. They include the data on the electron loss from 10.8 GeV/u $Au^{78+}(1s)$ ions penetrating solid targets [9,10] and on the electron loss from 160 GeV/u $Pb^{81+}(1s)$ ions colliding with solid [6] and gaseous [7] targets.

Besides, one should also note that the experimental data on the elementary cross sections in collisions at very high energies collected using solid targets are not very accurate. The loss cross sections reported for 160 GeV/u $Pb^{81+}(1s)$ projectiles penetrating solid and gas targets differ between themselves roughly by a factor of 2 (see [6] and [7]). The reason for this difference, as recently explained in [11], lies in the multiple collisions suffered by the projectiles when they move in solids, which does not allow accurate experimental determination of the values for elementary ion-atom cross sections. In the case of 10.8 GeV/u $Au^{78+}(1s)$ ions the data in [9] and [10], which were collected for collisions with solid-state targets, also differ by about of a factor of 2 (but the reason for this is not completely clear).

Concerning difficulties in the theoretical description of the ion-atom collisions, two main points that complicate the treatment of the projectile-electron excitation and loss should be mentioned: the presence of atomic electrons and the fact that, in the case of collisions with a heavy atom, the atomic field may be too strong, making it necessary to go beyond the first Born approximation.

The latter point is known to be of extreme importance when considering the projectile-electron excitation and loss processes occurring in collisions of very highly charged ions with heavy atoms at relatively low impact energies, 0.1–0.2 GeV/u, where the difference between experimental data and first Born calculations reaches an order of magnitude. At such energies, however, atomic electrons play only a minor role in the projectile-electron transitions and their presence can simply be neglected [13].

With increases in the impact energy the role of the higher-order effects in the projectile-target interaction diminishes. Nevertheless, even at energies well above 1 GeV/u the first Born approximation tends to substantially overestimate transition probabilities and the accuracy of first Born results for cross sections remains unclear, especially in the case when the collision causes more than one electron of the projectile to undergo transitions. Besides, at such impact energies, even for the most highly charged projectiles (e.g., hydrogen-like uranium ions), the influence of the electrons of the atom on the projectile-electron transitions can no longer be ignored [13].

In the present paper we demonstrate that the so-called symmetric eikonal (SE) model, extended to account for the presence of atomic electrons, can be used for treating the projectile-electron excitation and loss in collisions with heavy atoms at impact energies of $\gtrsim$1 GeV/u. In particular, we show that in the limit of asymptotically high impact energies, the transition amplitude, derived in the SE model, coincides with the amplitude obtained in the light-cone approximation, which means that the SE model provides essentially an exact solution for the problem of projectile-electron excitation and loss in extreme relativistic ion-atom collisions.

The forthcoming upgrade of the heavy-ion facilities at the GSI (Darmstadt, Germany) will allow one to perform extensive experimental explorations of the different aspects of heavy ion-atom collisions at impact energies in the range

*alexander.voitkiv@mpi-hd.mpg.de

1–30 GeV/u. It is shown here that in this range of impact energies the SE model represents a very valuable tool for describing projectile-electron excitation and loss in collisions with very heavy atoms.

Atomic units are used throughout unless stated otherwise.

II. THEORY

Depending on whether or not the electrons of the atom are "active" in the collision, one can normally distinguish two atomic modes (see, e.g., [12] and [13]) which can contribute to the projectile-electron transitions. In one of them, which is often called *screening*, the electrons of the atom act coherently, with the atomic nucleus screening (partially or fully) the field of the latter. The other mode, in which the atomic electrons actively participate, undergoing transitions, is termed *antiscreening*. In this mode the behavior of the projectile electron is influenced mainly by the electrons of the atom, while the nucleus of the atom plays only a minor role [14].

In collisions with heavy atoms, which are of special interest for the present article, the antiscreening mode is much weaker than the screening one and is not considered here. In the screening mode the field of the atom can be regarded as external, which enables one to reduce a many-electron problem of the ion-atom collision to a problem of the motion of the electron of the projectile in two external fields: the field of the nucleus of the ion and the field of the atom. The latter is taken as a superposition of the field of the atomic nucleus and the field of the atomic electrons, whose space distribution is assumed to be "frozen" during the very short collision time [13].

In the rest frame of the ion the electron is described by the Dirac equation

$$i\frac{\partial \Psi(\mathbf{r},t)}{\partial t} = \hat{H}\Psi(\mathbf{r},t). \tag{1}$$

The Hamiltonian $\hat{H}$ reads

$$\hat{H} = \hat{H}_0 + \hat{W}, \tag{2}$$

where

$$\hat{H}_0 = c\boldsymbol{\alpha}\cdot\mathbf{p} + \beta c^2 - \frac{Z_I}{r} \tag{3}$$

is the Hamiltonian for the electron motion in the field of the ionic nucleus and

$$\hat{W} = \boldsymbol{\alpha}\cdot\mathbf{A}(\mathbf{r},t) - \Phi(\mathbf{r},t) \tag{4}$$

is the interaction between the electron and the field of the atom. In the preceding expressions $\boldsymbol{\alpha}$ and β are the Dirac's matrices, $\mathbf{r}$'s are coordinates of the electron with respect to the ionic nucleus, and Φ and $\mathbf{A}$ are, respectively, the scalar and vector potentials of the electromagnetic field of the incident atom.

The field of the atom in its rest frame is described by the scalar potential, which, using results of [15] and [16], can be taken as

$$\Phi' = \frac{Z_A\phi(r')}{r'}, \tag{5}$$

where

$$\phi(r') = \sum_j A_j \exp(-\kappa_j r'), \tag{6}$$

with the screening parameters A_j ($\sum_j A_j = 1$) and κ_j given in [15] and [16]. We assume that in the rest frame of the ion the atom moves along a classical straight-line trajectory $\mathbf{R} = \mathbf{b} + \mathbf{v}t$, where $\mathbf{b} = (b_x, b_y)$ is the impact parameter and $\mathbf{v} = (0,0,v)$ the atomic velocity. Using Eqs. (5) and (6) and the Lorentz transformation for the potentials, we obtain that in the rest frame of the ion the potentials of the atomic field are given by

$$\Phi(\mathbf{r},t) = \frac{\gamma Z_A}{\sqrt{\gamma^2(z-vt)^2 + (\mathbf{r}_\perp - \mathbf{b})^2}} \times \sum_j A_j \exp[-\kappa_j\sqrt{\gamma^2(z-vt)^2 + (\mathbf{r}_\perp - \mathbf{b})^2}],$$
$$\mathbf{A}(\mathbf{r},t) = \left(0,0,\frac{v}{c}\Phi\right), \tag{7}$$

where γ is the collisional Lorentz factor and $\mathbf{r} = (\mathbf{r}_\perp, z)$ with $\mathbf{r}_\perp\cdot\mathbf{v} = 0$.

Within the SE model the transition amplitude is approximated by

$$a_{fi}(\mathbf{b}) = -i\int_{-\infty}^{+\infty} dt \langle \chi_f(t) \mid [\hat{H} - i\partial/\partial t]\chi_i(t)\rangle, \tag{8}$$

where the initial and final states of the electron, whose motion in the field of the ionic nucleus is affected by the field of the atom, are chosen according to

$$\chi_i(t) = \psi_0 \exp(-i\varepsilon_0 t)\exp\left[i\int_{-\infty}^{t} dt'\Phi(t')\right],$$
$$\chi_f(t) = \psi_n \exp(-i\varepsilon_n t)\exp\left[i\int_{+\infty}^{t} dt'\Phi(t')\right]. \tag{9}$$

Here, ψ_0 and ψ_n are the initial and final undistorted states of the electron in the ion.

Making use of the fact that the dependence of the scalar potential Φ on the electron coordinates and time is of the form $\Phi = \gamma Z_A f(s_\perp, \gamma|z - vt|)$, where $\mathbf{s}_\perp = \mathbf{r}_\perp - \mathbf{b}$, the transition amplitude (8) can be transformed into

$$a_{fi}(\mathbf{b}) = i\frac{c}{v}\int_{-\infty}^{+\infty} dt \exp(i\omega_{n0}t)\langle\psi_n|\exp\left[i\int_{-\infty}^{+\infty} dt'\Phi(t')\right] \times \left[\frac{\Phi(t)}{\gamma^2}\alpha_z - v\left(\boldsymbol{\nabla}_\perp\int_{-\infty}^{t} dt'\Phi(t')\right)\cdot\boldsymbol{\alpha}_\perp\right]|\psi_0\rangle, \tag{10}$$

where $\omega_{n0} = \varepsilon_n - \varepsilon_0$ is the electron transition frequency and $\boldsymbol{\nabla}_\perp$ denotes the two-dimensional [in the (x,y) plane] gradient operator.

The amplitude (10) can be simplified by employing the relation

$$\lim_{\lambda\to+0}\int_{-\infty}^{+\infty} dt \exp(i\omega_{n0}t)\exp(-\lambda|t|)\int_{-\infty}^{t} dt'\Phi(t') = \frac{i}{\omega_{n0}}\int_{-\infty}^{+\infty} dt \exp(i\omega_{n0}t)\Phi(t), \quad (\omega_{n0}\neq 0), \tag{11}$$

which yields

$$a_{fi}(\mathbf{b}) = i\frac{c}{v}\int_{-\infty}^{+\infty} dt\,\exp(i\omega_{n0}t)\langle\psi_n|\exp\left(i\int_{-\infty}^{+\infty} dt'\Phi(t')\right) \times \left(\frac{\Phi(t)}{\gamma^2}\alpha_z - i\frac{v}{\omega_{n0}}\left[\nabla_\perp\Phi(t)\right]\cdot\boldsymbol{\alpha}_\perp\right)|\psi_0\rangle. \quad (12)$$

A. The limit of weak interaction

If the interaction between the electron of the ion and the atom is sufficiently weak, one can replace $\exp[i\int_{-\infty}^{+\infty} dt'\Phi(t')]$ in (12) with 1. After such a replacement we integrate in (12) the term proportional to $[(\nabla_\perp\Phi(t)]\cdot\boldsymbol{\alpha}_\perp$ by parts over the (x,y) plane and use the continuity equation

$$\frac{\partial\rho_{n0}}{\partial t} + \nabla\cdot\mathbf{j}_{n0} = 0 \quad (13)$$

for the transition charge and current densities,

$$\rho_{n0} = -\psi_n^\dagger\,\psi_0\,\exp(i\omega_{n0}t), \quad \mathbf{j}_{n0} = -\psi_n^\dagger\,c\boldsymbol{\alpha}\,\psi_0\,\exp(i\omega_{n0}t). \quad (14)$$

Then we integrate the term proportional to $\partial/\partial z\psi_n^\dagger\alpha_z\psi_0$ by parts over the z coordinate and obtain

$$a_{fi}(\mathbf{b}) = i\frac{c}{v}\int_{-\infty}^{+\infty} dt\,\exp(i\omega_{n0}t)\langle\psi_n|\frac{v}{c}\Phi + \alpha_z\left(\frac{\Phi}{\gamma^2} - i\frac{v}{\omega_{n0}}\frac{\partial\Phi}{\partial z}\right)|\psi_0\rangle. \quad (15)$$

Taking into account that $\partial\Phi/\partial z = -1/v\partial\Phi/\partial t$ and

$$\int_{-\infty}^{+\infty} dt\,\exp(i\omega_{n0}t)\frac{\partial\Phi}{\partial t} = -i\omega_{n0}\int_{-\infty}^{+\infty} dt\,\exp(i\omega_{n0}t)\,\Phi, \quad (16)$$

we arrive at the transition amplitude

$$a_{fi}(\mathbf{b}) = i\int_{-\infty}^{+\infty} dt\,\exp(i\omega_{n0}t)\langle\psi_n|\Phi\left(1-\frac{v}{c}\alpha_z\right)|\psi_0\rangle, \quad (17)$$

which coincides with the expression for the amplitude obtained in the first Born approximation.

B. The high-energy limit

An important question, which is addressed in this subsection, concerns the high-energy limit ($\gamma\to\infty$) of the SE model. Keeping in mind that $\Phi = \gamma Z_A f(s_\perp,\gamma|z-vt|)$, one can show that

$$\int_{-\infty}^{+\infty} dt\,\exp(i\omega_{n0}t)\Phi(t) = \frac{2Z_A}{v}\exp(i\omega_{n0}z/v)\int_{-\infty}^{+\infty} d\xi f(s_\perp,\xi) = \exp(i\omega_{n0}z/v)\,G\left(s_\perp,\frac{\omega_{n0}}{\gamma v}\right), \quad (18)$$

where, for the moment, the explicit form of the function G is not important. Correspondingly,

$$\int_{-\infty}^{+\infty} dt\;\Phi(t) = G(s_\perp,0) \equiv G_0(s_\perp). \quad (19)$$

At sufficiently high impact energies, where the difference between $G = G(s_\perp,\omega_{n0}/\gamma v)$ and G_0 essentially vanishes, we can replace the amplitude (12) with the following expression:

$$a_{fi}(\mathbf{b}) = i\frac{c}{v\gamma^2}\langle\psi_n|\exp(iG)\exp(i\omega_{n0}z/v)\,G\,\alpha_z|\psi_0\rangle - i\frac{c}{\omega_{n0}}\langle\psi_n|\exp(iG)\exp(i\omega_{n0}z/v)(\nabla_\perp G)\cdot\boldsymbol{\alpha}_\perp|\psi_0\rangle. \quad (20)$$

We now take the second line of (20), integrate there by parts, use the continuity Eq. (13), and then again integrate by parts. As a result of these manipulations, expression (20) transforms into

$$a_{fi}(\mathbf{b}) = \langle\psi_n|\exp(i\omega_{n0}z/v)\exp(iG)\left(1-\frac{v}{c}\alpha_z\right)|\psi_0\rangle - \frac{c}{v\gamma^2}\langle\psi_n|\exp(i\omega_{n0}z/v)\exp(iG)(1-iG)\alpha_z|\psi_0\rangle. \quad (21)$$

The limit $\gamma\to\infty$ of the amplitude (21) is given by

$$a_{fi}(\mathbf{b}) = \langle\psi_n|\exp(i\omega_{n0}z/v)\exp(iG_0)\left(1-\frac{v}{c}\alpha_z\right)|\psi_0\rangle, \quad (22)$$

and it coincides with the transition amplitude derived in the so-called light-cone approach (see [13] and [17]).

The light-cone approach is strictly valid at $\gamma\to\infty$ and, in this limit, enables one to solve the problem of electron loss (ionization) and excitation exactly. One should also mention that in the case of strong ion-atom interaction, this exact solution does not coincide with the first Born results, no matter how high the impact energy is [13]. Taking the preceding two points into account, we can make the following conclusions. First, in the high-energy limit the SE model yields an exact solution for the transition amplitude of the projectile-electron excitation and loss. Second, even at $\gamma\to\infty$ the results of the SE model still, in general, differ from those of the first Born approximation.

It is rather obvious that such conclusions would also hold if the electron transitions in the ion are caused by the collision with a charged particle (e.g., a stripped atomic nucleus) and, for instance, can be applied to K-shell ionization of atoms by high-energy bare nuclei. In this respect, one should note that in the literature on relativistic ion-atom collisions, there have been attempts to consider the high-energy limit of distorted-wave models for the case of atomic ionization or excitation by the impact of a nucleus, in which these processes are treated as a three-body Coulomb problem (the incident and atomic nuclei and atomic electron). In particular, starting with the work in [18–20], it has been assumed that the high-energy limit of such distorted-wave models for ionization and excitation processes, like the continuum distorted-wave eikonal initial state (CDWEIS) (see, e.g., [21] and [22]) and the SE, is simply that of the first Born approximation.

We have just seen, however, that for the SE model this is not true and that, in the high-energy limit, the transition amplitude obtained in this model goes over into the amplitude derived in the light-cone approach. Since at $\gamma\gg 1$ the CDWEIS model becomes essentially identical to the SE, the high-energy limit of the CDWEIS coincides with the light-cone approach but differs from the first Born approximation.

C. The explicit form of the amplitude

Using the explicit form (7) of the scalar potential we obtain that

$$\int_{-\infty}^{+\infty} dt \exp(i\omega_{n0}t)\Phi(t) = \frac{2Z_A}{v}\exp(i\omega_{n0}z/v)\sum_j A_j K_0(s_\perp \Lambda_j), \quad (23)$$

where K_0 is the modified Bessel function [23], $s_\perp = |\mathbf{s}_\perp| = |\mathbf{r}_\perp - \mathbf{b}|$, and

$$\Lambda_j = \sqrt{\kappa_j^2 + \omega_{n0}^2/(\gamma^2 v^2)}. \quad (24)$$

Besides, it also follows from (23) that

$$\int_{-\infty}^{+\infty} dt\,\Phi(t) = \frac{2Z_A}{v}\sum_j A_j K_0(\kappa_j s_\perp). \quad (25)$$

Taking Eqs. (23) and (25) into account the transition amplitude becomes

$$a_{fi}(\mathbf{b}) = i\frac{2Z_A c}{v^2}\sum_j A_j \langle\psi_n|\exp\left(i\frac{\omega_{n0}z}{v}\right) \times \exp\left[i\frac{2Z_A}{v}\sum_{j'} A_{j'}K_0(\kappa_{j'}s_\perp)\right] \times\left[\frac{\alpha_z}{\gamma^2}K_0(s_\perp\Lambda_j) - i\frac{v\Lambda_j}{\omega_{n0}s_\perp}K_1(s_\perp\Lambda_j)\,\mathbf{s}_\perp\cdot\boldsymbol{\alpha}_\perp\right]|\psi_0\rangle, \quad (26)$$

where K_1 is the modified Bessel function [23].

D. The limit of vanishing screening

The results already obtained can also be applied to treatment of ionization and excitation of neutral atoms in relativistic collisions with bare nuclei. This can be done by setting $\kappa_j = 0$ in (26), replacing Z_A with Z_N there, where Z_N is the charge of the nucleus incident on the atom, and regarding ψ_0 and ψ_n as the initial and final states of the "active" electron of the atom. Keeping in mind that $\sum_j A_j = 1$, taking into account that $K_0(x) \approx -\ln(x/2) - \Gamma$ for $|x| \ll 1$ (see, e.g., [23]), where Γ is Euler's constant, and disregarding an inessential phase factor, we obtain

$$a_{fi}(\mathbf{b}) = i\frac{2Z_N c}{v^2}\langle\psi_n|\exp\left(i\frac{\omega_{n0}z}{v}\right)\exp\left(-i\frac{2Z_A}{v}\ln s_\perp\right) \times\left[\frac{\alpha_z}{\gamma^2}K_0\left(\frac{\omega_{n0}}{\gamma v}s_\perp\right) - \frac{i}{\gamma s_\perp}K_1\left(\frac{\omega_{n0}}{\gamma v}s_\perp\right)\mathbf{s}_\perp\cdot\boldsymbol{\alpha}_\perp\right]|\psi_0\rangle. \quad (27)$$

III. SOME APPLICATIONS

The SE model is normally regarded as a tool for describing collision-induced transitions between bound states. In our case it means that this model should first be applied to the treatment of the projectile-electron excitation. It is known [24] that at comparatively low relativistic impact energies, the model has a problem with describing transitions involving electron spin flip. However, this problem diminishes when the energy increases and practically disappears at energies $\gtrsim$1 GeV/u.

The SE model can also be used for considering projectile-electron loss (in particular, the total cross section). In this case the model also becomes more accurate when the impact energy increases. In particular, provided that, in the rest frame of the atom, the magnitude of the velocity of the electron emitted by the projectile is of the order of the collision velocity, the SE model can be applied for calculating not only the total but also the differential loss cross sections. In practical terms this condition holds starting already at $\gamma \simeq 2$–3, that is, at impact energies ~1–2 GeV/u.

As shown in the previous section, at asymptotically high impact energies the SE model yields an exact solution. In the case of projectile-electron excitation and loss in collisions with neutral atoms, even for the most highly charged projectiles, the region of such energies is actually already reached at $\gamma \sim 30$–50. Thus, the model should work excellently starting with a magnitude of γ of a few tens.

Taking all this into account one would expect that the SE model performs quite well at impact energies ~1–30 GeV/u, which are of special interest for the present study. Below projectile-electron excitation and loss are considered in this energy range for collisions between hydrogen- and helium-like highly charged ions and heavy atoms by using the SE model. Results of this model are also compared with those of the first Born approximation.

A. Single-electron loss

In Fig. 1 we show results for the electron loss from incident $Au^{78+}(1s)$ projectiles in collisions with neutral Au atoms at impact energies of 1–30 GeV/u. These results include our first Born and eikonal calculations as well as experimental data from [9] and [10] on the electron loss from 10.8 GeV/u $Au^{78+}(1s)$ projectiles.

In the case considered our eikonal and first-order results differ by 15%–35%. As expected, when the impact energy increases, the difference between them decreases. Overall, the difference is not large but, nevertheless, should be taken into account if precise cross section values are needed.

Both the first Born and the eikonal cross sections agree neither with the experimental data in [9] nor with those in [10]. The data in [9] are substantially smaller (by about 30%–50%), while the data reported in [10] are considerably larger (by a factor of about 1.3–1.4) than our results [25].

As mentioned in Sec. I, there is a difference of roughly a factor of 2 in the experimental cross sections reported for the loss from 160 GeV/u Pb^{81+} ions in collisions with solid and gas targets, with the solid-state cross sections being larger. The origin of this difference, which was explained in [11], lies in multiple collisions between the projectile and the atoms inside solids, which effectively enhance the electron loss process. Since the magnitude of this difference depends on the impact energy and decreases when the energy decreases, similar reasons are probably responsible for the observed

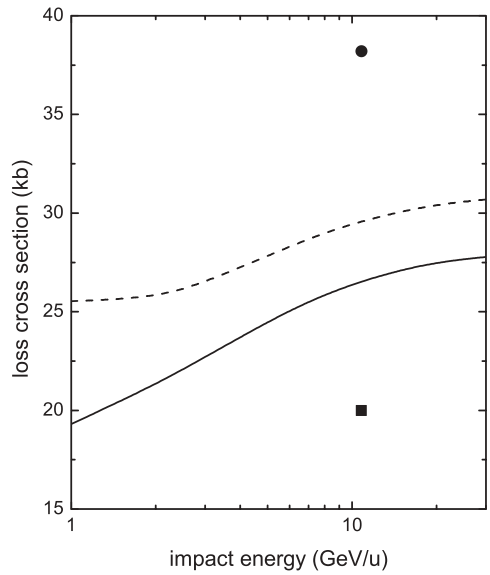

FIG. 1. Cross section for the electron loss from $Au^{78+}(1s)$ ions in collisions with neutral Au atoms. Solid curve, eikonal results; dashed curve, first Born results. The circle displays experimental data from [9], while the square shows the result from [10] scaled to the gold target.

disagreement between our results for atomic targets and the experimental data in [10].

B. Two-electron transitions

If a projectile-ion initially carries several electrons, then more than one electron of the ion can be simultaneously excited and/or lost in a single collision with a neutral atom. Helium-like ions are simplest projectiles, for which simultaneous transitions of more than one electron are possible, and here we consider double-electron loss and loss-excitation for the case of such projectiles.

It is known (see, e.g., [13]) that, provided the condition $Z_I Z_A/v > 0.4$ is fulfilled, two-electron transitions in a heavy helium-like ion occurring in collisions with an atom are governed almost solely by the independent interactions between the atom and each of the electrons of the ion. To describe such transitions one can apply the independent electron model. According to this model the cross section for the double-electron loss from a helium-like ion is given by

$$\sigma_{l,l} = 2\pi \int_0^\infty db\, b\, P_{l,l}(b), \tag{28}$$

where the probability $P_{l,l}(b)$ for the two-electron loss is given by

$$P_{l,l}(b) = P_{\mathrm{loss}}^2(b), \tag{29}$$

where $P_{\mathrm{loss}}(b)$ is the single-electron loss probability in a collision with a given value of the impact parameter b.

The cross section for the simultaneous loss-excitation in the case of a helium-like ion is evaluated as

$$\sigma_{e,l} = 2\pi \int_0^\infty db\, b\, P_{e,l}(b), \tag{30}$$

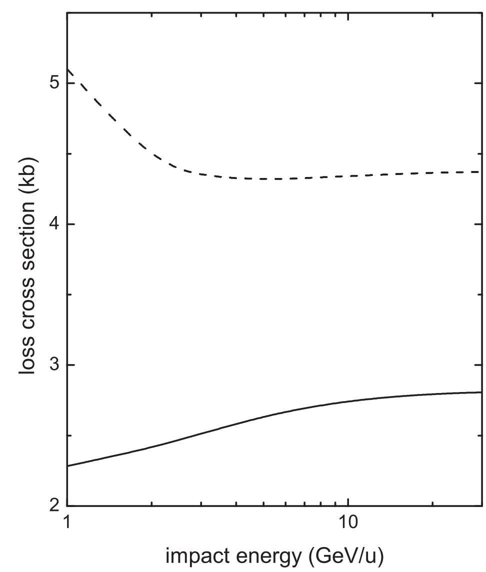

FIG. 2. Cross sections for double-electron loss from $Au^{77+}(1s^2)$ ions in collisions with neutral Au atoms. Solid curve, eikonal results; dashed curve, first Born results.

where the probability $P(b)$ for the two-electron process is given by

$$P_{e,l}(b) = 2\, P_{\mathrm{exc}}(b)\, P_{\mathrm{loss}}(b), \tag{31}$$

where $P_{\mathrm{exc}}(b)$ is the single-electron excitation probability.

1. Double-electron loss

Figure 2 shows calculated cross sections for double-electron loss from $Au^{77+}(1s^2)$ projectiles incident on neutral Au atoms at impact energies of 1–30 GeV/u. Compared to the single loss, now the difference between the eikonal and the first Born results is much more pronounced. For impact energies $\sim$1–5 GeV/u these calculations predict even qualitatively different dependencies of the cross sections on the collision energy. The absolute difference between the first Born and the eikonal cross sections ranges between $\simeq$2.5 at 1 GeV/u to $\simeq$1.5 at 30 GeV/u. Moreover, as additional calculations show, for impact energies above 30 GeV/u, the ratio $\simeq$1.5 remains almost a constant, and thus, even at asymptotically high collision energies the first Born calculation still substantially overestimates the cross-section values.

2. Simultaneous loss-excitation

In Fig. 3 we present results of calculations for the simultaneous projectile-electron excitation and loss, $U^{90+}(1s^2) \to U^{91+}(n=2, j) + e^-$, where n and j are, respectively, the principal quantum number and the total angular momentum of the excited state of the hydrogen-like ion. The projectile is assumed to collide with Kr, Xe, and Au atomic targets at an impact energy of 20 GeV/u. Similarly to the case of double-electron loss, already considered, we observe that the differences between the cross sections, calculated with the first Born and eikonal probabilities, can be quite substantial if the atom is sufficiently heavy. For collisions leading to the population of the states with $j = 1/2$ the first Born results are larger by a factor of $\simeq$1.16 (Kr), $\simeq$1.38 (Xe), and

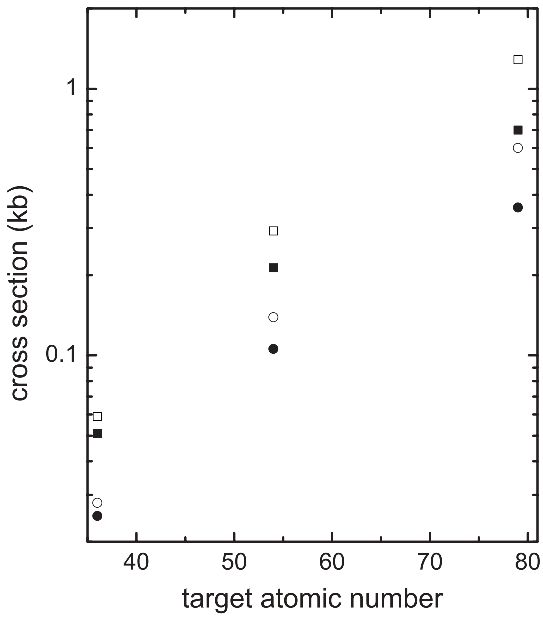

FIG. 3. Cross sections for the simultaneous electron loss-excitation, $U^{90+}(1s^2) \rightarrow U^{91+}(n=2,j) + e^-$, occurring in collisions with Kr, Xe, and Au atomic targets at 20 GeV/u. $j = 1/2$ and $j = 3/2$ are the angular momentum of the states of the hydrogen-like uranium ion. Squares and circles show results for $j = 1/2$ and $j = 3/2$, respectively. Open and filled symbols denote the cross sections obtained using the first Born approximation and the SE model, respectively.

$\simeq$1.84 Au. When the states with $j = 3/2$ are populated this ratio is $\simeq$1.12, $\simeq$1.31, and $\simeq$1.67, respectively.

We see that the difference between the first Born and the eikonal cross sections turns out to be somewhat smaller for transitions involving the states with $j = 3/2$. This can be

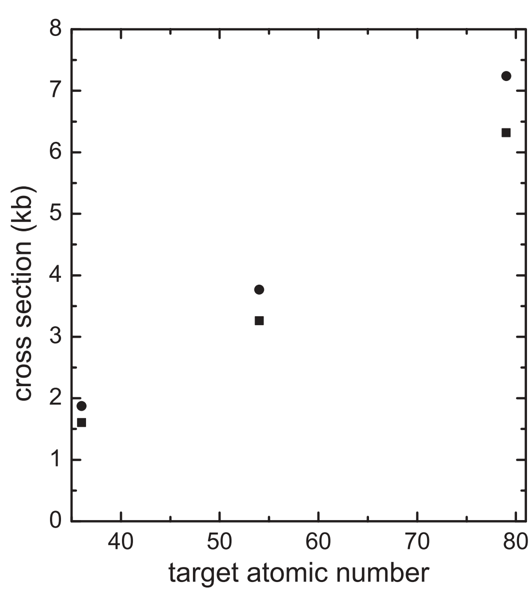

FIG. 4. Cross sections for the excitation of 20 GeV/u $U^{91+}(1s)$ projectiles into states with $n = 2$, $j = 1/2$ (squares) and $n = 2$, $j = 3/2$ (circles) in collisions with Kr, Xe, and Au atomic targets. Cross sections were calculated using the SE model.

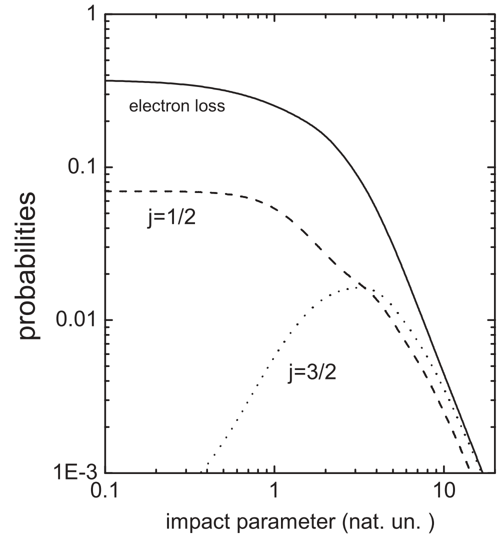

FIG. 5. Probabilities of projectile-electron loss and excitation in collisions of 20 GeV/u $U^{91+}(1s)$ ions with Au atoms given as a function of the impact parameter. Probabilities were calculated using the SE. Solid curve, probability of electron loss; dashed curve, probability of excitation into states with $n = 2$, $j = 1/2$; dotted curve, probability of excitation into states with $n = 2$, $j = 3/2$.

attributed to the fact that, compared to the $j = 1/2$ case, these transitions are characterized on average by larger impact parameters. As a result, the field of the atom acting on the electrons of the ion is weaker, and therefore, the first Born treatment becomes less inaccurate.

As additional calculations show, the differences in the results between the eikonal and the first Born calculation does not substantially change when the impact energy increases further. Thus, as for double-electron loss, even at asymptotically high impact energies the first Born calculation may considerably overestimate the cross section for simultaneous projectile-electron excitation and loss.

At these rather high energies, where the electric dipole transitions already become dominant, the pure excitation $U^{90+}(1s^2) \rightarrow U^{90+}(1s;\ n = 2, j)$ [or the excitation $U^{91+}(1s) \rightarrow U^{91+}(n = 2, j)$; see Fig. 4] proceeds more efficiently into the states with $j = 3/2$ [due to the most "powerful" dipole transition, $1s_{1/2}(+1/2) \rightarrow [2p_{3/2}(+3/2)]$. However, according to both the first Born and the eikonal results, the cross sections for simultaneous loss-excitation are larger for transitions into states with $j = 1/2$. The origin of this interesting peculiarity can be understood by considering the single-electron transition probabilities. At this impact energy the electron loss is also already dominated by the electric dipole transitions. However, compared to the excitation, this process involves larger energy-momentum transfer and, consequently, effectively occurs at smaller impact parameters. As a result, it turns out that the probability of electron loss has a larger overlap with the probability of excitation into states with $j = 1/2$ (for illustration see Fig. 5).

IV. CONCLUSIONS

We have considered the symmetric eikonal model for treating projectile-electron transitions in relativistic collisions with heavy atoms. We have shown that at asymptotically high impact energies, this model yields the same results as the light-cone approach and, thus, offers an exact solution for the transition amplitude. In the case of strong interaction between the projectile-electron and the atom, this solution differs from that given by the first Born approximation. Consequently, even at asymptotically high impact energies the SE and the first Born approximation in general do not coincide. This conclusion contradicts the previous claims (see, e.g., [18]) that in the limit $\gamma \to \infty$, the SE goes over into the first Born approximation.

In the limit $\gamma \gg 1$ another popular distorted-wave model—the CDWEIS approximation—becomes essentially identical to the symmetric eikonal model. Therefore, at asymptotically high impact energies the CDWEIS also becomes exact and, contrary to what was stated earlier [18], does not coincide with the first Born approximation.

Using the SE model and the first Born approximation we have calculated cross sections for the projectile-electron excitation and loss. We have shown that at impact energies of $\sim$1–30 GeV/u, which are relevant for the future GSI facility, there are noticeable deviations between the eikonal and the first Born results for single-electron transitions and very substantial differences between such results for transitions involving two projectile electrons. Moreover, these very substantial differences "survive" even in the asymptotic limit $\gamma \to \infty$.

Our results clearly show a great advantage of the SE model over the first Born approximation.

ACKNOWLEDGMENT

B.N. acknowledges the support from the Deutsche Forschungsgemeinschaft under Project No. VO 1278/2-1.

[1] R. Anholt, W. E. Meyerhof, X.-Y. Xu, H. Gould, B. Feinberg, R. J. McDonald, H. E. Wegner, and P. Thieberger, Phys. Rev. A **36**, 1586 (1987); W. E. Meyerhof, R. Anholt, X.-Y. Xu, H. Gould, B. Feinberg, R. J. McDonald, H. E. Wegner, and P. Thieberger, Nucl. Instrum. Methods A **262**, 10 (1987).
[2] Th. Stöhlker *et al.*, Nucl. Instrum. Methods B **124**, 160 (1997).
[3] C. Scheidenberger and H. Geissel, Nucl. Instrum. Methods B **135**, 25 (1998).
[4] C. Scheidenberger, Th. Stölker, W. E. Meyerhof, H. Geissel, P. H. Mokler, and B. Blank, Nucl. Instrum. Methods B **142**, 441 (1998).
[5] T. Ludziejewski *et al.*, Phys. Rev. A **61**, 052706 (2000).
[6] H. F. Krause, C. R. Vane, S. Datz, P. Grafström, H. Knudsen, C. Scheidenberger, and R. H. Schuch, Phys. Rev. Lett. **80**, 1190 (1998).
[7] H. F. Krause, C. R. Vane, S. Datz, P. Grafström, H. Knudsen, U. Mikkelsen, C. Scheidenberger, R. H. Schuch, and Z. Vilakazi, Phys. Rev. A **63**, 032711 (2001).
[8] C. R. Vane and H. F. Krause, Nucl. Instrum. Methods B **261**, 244 (2007).
[9] A. Westphal and Y. D. He, Phys. Rev. Lett. **71**, 1160 (1993).
[10] N. Claytor, A. Belkacem, T. Dinneen, B. Feinberg, and H. Gould, Phys. Rev. A **55**, R842 (1997).
[11] A. B. Voitkiv, B. Najjari, and A. Surzhykov, J. Phys. B **41**, 111001 (2008).
[12] E. C. Montenegro, W. E. Meyerhof, and J. H. McGuire, Adv. At. Mol. Opt. Phys. **34**, 249 (1994).
[13] A. B. Voitkiv and J. Ullrich, *Relativistic Collisions of Structured Atomic Particles* (Springer-Verlag, Berlin, 2008).
[14] According to first Born considerations, in the antiscreening mode the nucleus of the atom does not interact at all with the electron of the projectile.
[15] G. Moliere, Naturforsch. A **2**, 133 (1947).
[16] F. Salvat, J. D. Martinez, R. Mayol, and J. Parellada, Phys. Rev. A **36**, 467 (1987).
[17] A. J. Baltz, Phys. Rev. Lett. **78**, 1231 (1997).
[18] G. R. Deco and N. Grün, J. Phys. B **22**, 1357 (1989).
[19] G. R. Deco, K. Momberger, and N. Grün, J. Phys. B **23**, 1990 (1989).
[20] G. R. Deco and N. Grün, J. Phys. B **22**, 3709 (1989).
[21] D. S. F. Crothers, *Relativistic Heavy-Particle Collision Theory* (Kluwer Academic/Plenum, London, 2000).
[22] A. B. Voitkiv and B. Najjari, J. Phys. B **40**, 3295 (2007).
[23] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1965).
[24] A. B. Voitkiv, B. Najjari, and J. Ullrich, Phys. Rev. A **75**, 062716 (2007).
[25] Note that the experimental data reported in [10] are in very good agreement with results of theoretical considerations of R. Anholt and U. Becker [Phys. Rev. A **36**, 4628 (1987)] and of A. H. Sørensen [Phys. Rev. A **58**, 2895 (1998)]. However, these theoretical models, which were developed for collisions between individual ions and atoms (and, therefore, should be tested in collisions with gas targets first of all), overestimate experimental data obtained in collisions with rarefied gas targets [7] by a factor of 2–3 (while our results agree very nicely with this atomic target experiment). Critical discussion of the models of Anholt and Becker and of Sørensen is given in [13].