Efficient generation of universal two-dimensional cluster states with hybrid systems

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We present a scheme to generate any two-dimensional cluster state efficiently. The number of the basic gate (entangler) operations is on the order of the entanglement bonds of a cluster state. This deterministic scheme, which demands few ancilla resources and no quantum memory, is suitable for large-scale quantum computation.

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I. INTRODUCTION

Measurement-based quantum computation (MBQC) or one-way quantum computation, which was first introduced by Briegel and Raussendorf [1] and Raussendorf *et al.* [2], has recently been a hot topic in quantum information science [3–12]. Different from the traditional circuit-based quantum computation implemented by single-qubit and multiqubit gates, the necessary operation in MBQC only is single-qubit measurements. However, the efficient generation of the universal resources for MBQC, which are entangled states of large numbers of qubits (conventionally called cluster states or graph states), remains an obstacle for the realization of MBQC. Many proposals have been put forward to create cluster states with various physical systems. They include the optical systems of discrete [3–6] and continuous variable [7] photonic states, the condensed matter systems, such as charge qubits [8], flux qubit [9], quantum dot [10], and atomic ensembles [11], etc.

Here, we focus on the optical approaches to generate cluster states. In 2004, Nielsen proposed the method of adding photons one by one with controlled-Z (CZ) gates to generate a cluster state [3]. This scheme only uses linear optical elements, so it is probabilistic, and the cost for creating a cluster state of a large number of qubits could be very high. Later, many works were developed to generate cluster states more efficiently. One of them is the Browne-Rudolph protocol [4]. Two types of fusion gates are introduced in their protocol. The type-I fusion gate is used to connect two cluster state strings with a success probability of 1/2. After the operation of this gate, an undetected photon will be connected to the photon adjacent to a detected photon. The success of the gate is heralded at the cost of one photon to be detected. For a type-II fusion gate, on the other hand, two photons should be detected to create an L-shape cluster. At least three photons should be consumed (one photon for the σ_x operation and two photons for the type-II fusion gate) in a complete operation. An improved scheme without a type-II fusion gate was later proposed by Gilbert *et al.* [5]. The Browne-Rudolph protocol and its improvements only apply linear optics but are nondeterministic and demand a large quantity of single-photon sources. Therefore, these schemes are not appropriate for large-scale quantum computation.

More recently, another approach for generating cluster states with weak nonlinearities was developed by Louis *et al.* [13]. Its cluster state generation could be deterministic with the $\hat{X}$-quadrature measurements, but the amplitude $|\alpha|$ of the quantum bus (qubus) or communication beams in coherent state $|\alpha\rangle$ should satisfy $\alpha\theta^2 \gg 1$ (where θ is the cross-phase shift). This would be possible with a very strong coherent beam given the small cross-phase-modulation (XPM) phase θ or with giant nonlinearity for a moderate beam amplitude $|\alpha|$. If one chooses the $\hat{P}$-quadrature measurements instead, the amplitude-XPM phase scaling will be improved to $\alpha\theta \gg 1$, but the operation will be nondeterministic with a success probability of 1/2. Moreover, their schemes require a minus XPM phase shift $-\theta$, which is impractical to realize [14].

So far, MBQC has been experimentally demonstrated with optical systems [15,16], but it is impossible to follow these proof-of-principle experiments to perform the realistic MBQC, which involves a large number of qubits. The main reason is that quantum memories will be necessary in generating such cluster states if given probabilistic gates (e.g., probabilistic controlled-phase flip gate [17]). It would take a long time for a repeat-until-success procedure with the probabilistic gates to create a target cluster state, so the already generated part of the cluster state should be stored in quantum memories. Obviously, if the efficiency of the gate operation is not high enough, a large number of photonic qubits in the cluster state should be stored in quantum memory for a long time. Unfortunately, efficient and high-quality quantum memories for photonic qubits are still under development, thus far. Therefore, it is interesting to study how to quickly create photonic cluster states without quantum memory.

In this paper, we propose a scheme to generate a two-dimensional (2D) cluster state with hybrid systems, which involve both discrete qubits and continuous variable ancilla states. This is a deterministic approach to generate a photonic cluster state of large size. Compared with the former works (e.g., Refs. [13]), the deterministic approach is more feasible for experimental realization. Moreover, with the high efficiency of the scheme, only temporary storage such as delay lines would be necessary for the already generated part in a cluster state.

The rest of the paper is organized as follows. First, we describe a hybrid system called an entangler as the tool for

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creating the entanglement links in a cluster state. Then, in Secs. III and IV, we outline the procedures to generate a string or chain cluster state and two types of box cluster states, respectively. Next, we present the main results about the generation of a 2D cluster state in Sec. V. Finally, we conclude the paper with some discussions.

II. BASIC TOOL: ENTANGLER

Before we present the scheme for generating a cluster state, we describe the tool to create the entanglement links in a cluster state. Such a basic gate is called an entangler in short. It was first introduced by Pittman *et al.,* and is used to construct a controlled-NOT (CNOT) gate [18]. Later, Nemoto and Munro proposed a deterministic entangler based on XPM [19]. The impractical minus XPM phase shift $-\theta$ in the scheme of Ref. [19] can be avoided with a double XPM proposal by using two qubus beams [20]. Based on the double XPM method, we developed the entangler in Ref. [21], which was adopted as the basic gate for creating 2D cluster states in this paper.

In what follows, we present a brief review on the operation of the entangler in Ref. [21]. Its schematic is shown in Fig. 1. Here, we encode a qubit with the polarization modes of a single photon (i.e., $|0\rangle \equiv |H\rangle$ and $|1\rangle \equiv |V\rangle$). The input state of two single-photon qubits $|\psi\rangle = a|H\rangle + b|V\rangle$ and $|+\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ together with two ancilla beams in the state $|\alpha\rangle$ are transformed by the system in Fig. 1 to

$$\frac{1}{\sqrt{2}}(a|HH\rangle + b|VV\rangle)|0\rangle|\sqrt{2}\alpha\rangle + \frac{1}{\sqrt{2}}(a|HV\rangle|-\beta\rangle + b|VH\rangle|\beta\rangle)|\sqrt{2}\alpha\cos\theta\rangle, \tag{1}$$

where $|\beta\rangle = |i\sqrt{2}\alpha\sin\theta\rangle$. Then, the projection $|n\rangle\langle n|$ on the first output qubus beam will yield the proper output. If $n = 0$, the target output state $a|HH\rangle + b|VV\rangle$ will be projected out; if $n \neq 0$, on the other hand, one will obtain the state $|\psi\rangle_{\rm out} = e^{-in(\pi/2)}a|HV\rangle + e^{in(\pi/2)}b|VH\rangle$, which can be transformed to the target output state with the proper operations on the second photon according to the classically feedforwarded measurement results n. Therefore, the operation for the entangler is to deterministically entangle two qubits:

$$|\psi\rangle|+\rangle \stackrel{E}{\to} a|0\rangle|0\rangle + b|1\rangle|1\rangle, \tag{2}$$

where E denotes the entangler operation. A deterministic CNOT gate or CZ gate can be realized with two entanglers and one ancilla photon [18,19]. Alternatively, one can also use a pair of C-path and merging gates, together with a recyclable ancilla photon, to realize these logic gates [21,22].

An important step in the preceding entangler operation is the projection $|n\rangle\langle n|$ on the first output qubus beam, which should be highly efficient for realizing the deterministic entangler. It is implemented by the QND module shown inside the dashed-dotted line in Fig. 1. In this module, one of the two identical beams in the coherent state $|\gamma\rangle$ is coupled to the first output qubus beam through an XPM process, which outputs the following state:

$$|\pm\beta\rangle|\gamma\rangle|\gamma\rangle \to e^{-|\beta|^2/2}\sum_{n=0}^{\infty}\frac{(\pm\beta)^n}{\sqrt{n!}}|n\rangle\left|\frac{\gamma e^{in\theta}-\gamma}{\sqrt{2}}\right\rangle\left|\frac{\gamma e^{in\theta}+\gamma}{\sqrt{2}}\right\rangle = |\Phi_{\rm out}\rangle, \tag{3}$$

The photon number which resolves measurement on the qubus beam $|\pm\beta\rangle$ will be indirectly realized with a number nonresolving detection described by the positive-operator-value-measurement elements [23] $\Pi_0 = \sum_{m=0}^{\infty}(1-\eta)^m|m\rangle\langle m|$, $\Pi_1 = I - \Pi_0$ on the output states $|\frac{\gamma e^{in\theta}-\gamma}{\sqrt{2}}\rangle$ $(n = 0,1,\ldots,\infty)$. Here, Π_0 and Π_1 correspond to the detection of no photon and any number of photons, respectively, and $\eta < 1$ is the efficiency of the detector. Since each of the states $|\frac{\gamma e^{in\theta}-\gamma}{\sqrt{2}}\rangle$ has a certain distribution of photon numbers (Poisson peak), the action of Π_1 actually functions as the operators $\Pi_{1,k} = \sum_{m=n_k}^{n_k'}[1-(1-\eta)^m]|m\rangle\langle m|$ on each of them, if the dominant distribution for the kth Poisson peak is from n_k to n_k'. We can use a sufficiently large $|\gamma|$ so that the distributions of $|\frac{\gamma e^{in\theta}-\gamma}{\sqrt{2}}\rangle$ will be mutually separated. The number nonresolving detector for the detection of the beam in these coherent states can be a sensor, which is unable to resolve an exact Fock state $|n\rangle$ but could output the distinct signals proportional to the total photon detection probability $\langle\frac{\gamma e^{in\theta}-\gamma}{\sqrt{2}}|\Pi_{1,k}|\frac{\gamma e^{in\theta}-\gamma}{\sqrt{2}}\rangle$. Based on this indirect photon number resolving detection strategy, the error probability of our entangler is

$$P_E = \langle\Phi_{\rm out}|\Pi_0|\Phi_{\rm out}\rangle \sim \exp[-2(1-e^{-(1/2)\eta\gamma^2\theta^2})\alpha^2\sin^2\theta], \tag{4}$$

by considering $\theta \ll 1$ from a weak cross-Kerr nonlinearity. The possibly low efficiency of a realistic photon detector can be compensated by the large intensity $|\gamma|^2$ of the ancilla beams, and the entangler operation could be deterministic given that $2\alpha^2\sin^2\theta \gg 1$ and $\frac{1}{2}\eta\gamma^2\theta^2 \gg 1$ [21]. This greatly improves on the efficiency of the entanglers in Refs. [13,19].

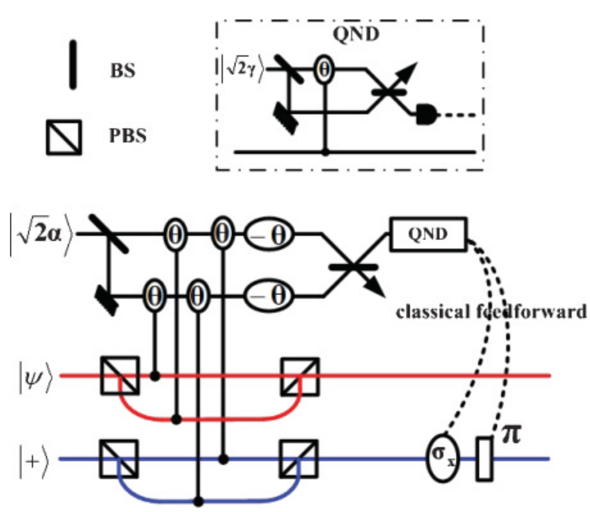

FIG. 1. (Color online) Schematic for the entangler. Two qubus beams are coupled to the two single photons as indicated. The XPM phases θ and two phase shifters $-\theta$ are applied to the qubus beams. The quantum nondemolition (QND) module and the classical feedforward are used to make this operation deterministic. This operation entangles the polarization modes of two single photons.

This type of entangler is suitable for performing very large numbers of repeated entangling operations without accumulating considerable error to the system. There is the following relation between the success probability P_{succ} of implementing N repeated entangling operations and the error probability P_E of a single operation ($P_E \ll 1$ with the proper parameters):

$$P_{\text{succ}} = (1 - P_E)^N \sim 1 - N P_E. \tag{5}$$

The allowed operation number N for a fixed P_{succ} scales exponentially with the parameter $\alpha^2 \sin^2\theta$:

$$N \sim (1 - P_{\text{succ}}) \exp[2(1 - e^{-(1/2)\eta\gamma^2\theta^2})\alpha^2 \sin^2\theta]. \tag{6}$$

For example, given the phase θ on the order of 10^{-2}, the detector with the efficiency $\eta = 0.6$ and the coherent beams of the moderate average photon numbers $|\gamma|^2 \sim 10^6$ and $|\alpha|^2 = 8 \times 10^4$, the entangler could perform at least 8.8×10^4 operations without the accumulated error probability going beyond 10^{-2} (i.e., $P_{\text{succ}} = 0.99$). The scaling shown in Eq. (6) is especially meaningful to the tasks such as entangling a large number of qubits to a cluster state.

III. GENERATION OF STRING CLUSTER STATE BY ENTANGLER

As we know, a cluster string or a chain cluster state can be generated by using CZ gate operations one after another [1,3,24]. Such a procedure can actually be simplified by using only one entangler in the operation. We begin with the initial product state $|+\rangle|+\rangle$. If a CZ gate is applied to this input, one will get $\frac{1}{\sqrt{2}}(|0\rangle|+\rangle + |1\rangle|-\rangle)$, a two-qubit cluster state. This process can be simplified by using one entangler operation described by Eq. (2), that is,

$$|+\rangle|+\rangle \xrightarrow{E} \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle). \tag{7}$$

Then, one single-qubit rotation called a Hadamard gate on the second photon will transform the preceding output to the target $\frac{1}{\sqrt{2}}(|0\rangle|+\rangle + |1\rangle|-\rangle)$. This method can be used to add one photon to an already generated cluster state, as shown in part (3) of Fig. 2. In general, an already created cluster state can be expressed in the form $|\Phi_1\rangle|0\rangle_p + |\Phi_2\rangle|1\rangle_p$, where $|\Phi_{1(2)}\rangle$ is a proper unnormalized pure state, which involves all qubits except the pth such that the whole state should be the eigenstates of the stabilizers [2]. Now, the process of adding one photon q in the state $|+\rangle$ to the already prepared cluster state is as follows:

$$(|\Phi_1\rangle|0\rangle_p + |\Phi_2\rangle|1\rangle_p)|+\rangle_q \xrightarrow{E_{pq}} |\Phi_1\rangle|0\rangle_p|0\rangle_q + |\Phi_2\rangle|1\rangle_p|1\rangle_q$$
$$\xrightarrow{H_q} |\Phi_1\rangle|0\rangle_p|+\rangle_q + |\Phi_2\rangle|1\rangle_p|-\rangle_q, \tag{8}$$

where E_{pq} denotes the entangler operation between the pth and qth qubits, and H_q is the Hadamard gate operation on the qth qubit.

By using this technique, one can easily generate any cluster state string as shown in part (1) of Fig. 2 and the star cluster state shown in part (2) of Fig. 2. In addition, it is feasible to use only entanglers to generate an alveolate graph shape deterministically (the projector of the PBS in Ref. [25] is actually an entangler) and a cluster state string simultaneously such as the scheme in [26]. Another advantage of the approach is that no ancilla single photon is necessary in the operation. By the way, it should be noted that, if one wants to connect two photons in two already different created cluster states, one CZ gate, or two entanglers plus one ancilla single photon equivalently, will be needed.

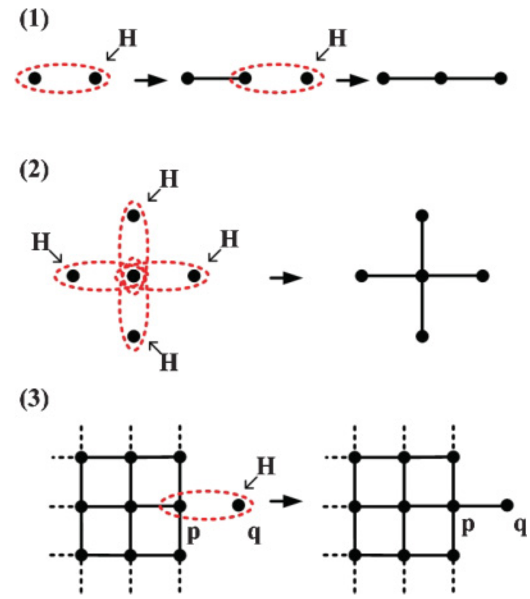

FIG. 2. (Color online) The generation of a cluster state string with entanglers. (1) Use of entanglers and Hadamard operations to generate a cluster state string. (2) Use of four entanglers and Hadamard operations to create a star cluster state. (3) Use of one entangler and a Hadamard operation to add one photon to the already created cluster state.

IV. GENERATION OF 2D BOX CLUSTER STATE BY ENTANGLER

The cluster states for practical MBQC are 2D ones. One could use CZ gates to connect cluster state strings to obtain a 2D cluster state. However, higher efficiency and fewer resources could be possible for this purpose if applying the improved strategies. In what follows, we will first show how to generate a box cluster state with entanglers, and then use the box cluster states as the basic elements to construct a general 2D cluster state in an efficient way.

A. Type-I box

The first scheme to generate a box cluster state is shown in part (1) of Fig. 3. At first, we use two entanglers to generate a cluster string of three photons in the following state:

$$\tfrac{1}{2}(|0\rangle|+\rangle|0\rangle + |0\rangle|-\rangle|1\rangle + |1\rangle|-\rangle|0\rangle + |1\rangle|+\rangle|1\rangle)_{123}. \tag{9}$$

Then, a Hadamard gate operation on the second photon will transform the previous state to

$$\tfrac{1}{2}(|0\rangle|0\rangle|0\rangle + |0\rangle|1\rangle|1\rangle + |1\rangle|1\rangle|0\rangle + |1\rangle|0\rangle|1\rangle)_{123}. \tag{10}$$

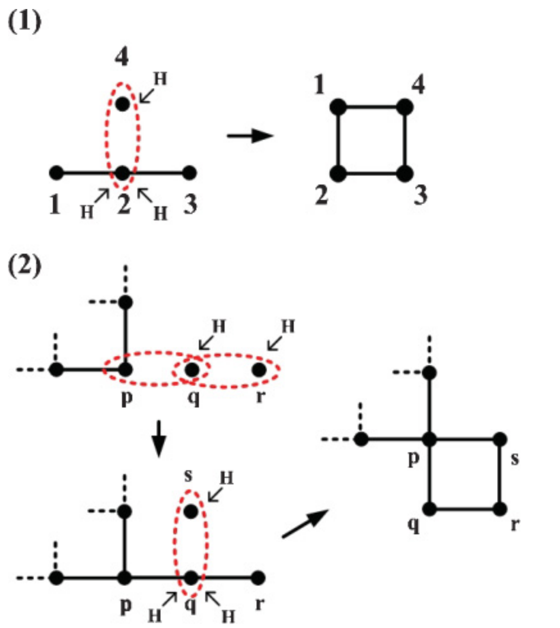

FIG. 3. (Color online) Schematic for generating a type-I box. (1) One entangler plus three Hadamard operations are performed on photons 2 and 4 in order to connect two links 1–4, 3–4. Then, a box cluster state could be generated by only three entangler operations without an ancilla photon. (2) The generalization of adding a box structure to an already generated cluster state.

Next, by applying an entangler operation on photons 2 and 4 (initially, in state $|+\rangle$) yields the state

$$\tfrac{1}{2}(|0\rangle|0\rangle|0\rangle|0\rangle + |0\rangle|1\rangle|1\rangle|1\rangle + |1\rangle|1\rangle|0\rangle|1\rangle + |1\rangle|0\rangle|1\rangle|0\rangle)_{1234}. \tag{11}$$

Finally, a Hadamard operation is performed on the second and fourth photons, respectively, so that one will obtain the state

$$\begin{aligned}&\tfrac{1}{2}(|0\rangle|+\rangle|0\rangle|+\rangle + |0\rangle|-\rangle|1\rangle|-\rangle\\ &\quad+|1\rangle|-\rangle|0\rangle|-\rangle + |1\rangle|+\rangle|1\rangle|+\rangle)_{1234},\end{aligned} \tag{12}$$

which is a box cluster state [15]. In this process, we generate two bonds ($4 \rightarrow 1,4 \rightarrow 3$) simply by one entangler operation and three Hadamard operations. Thus, the reason why the operation could be simplified with the entangler operation is that the box cluster state has a perfect symmetry. Seen from photon 1 or 3, photons 2 and 4 are symmetric, so the states of them are equivalent, and one entangler operation will be sufficient for connecting both bonds. In total, three entangler operations, not four CZ gates, will be necessary to generate a box cluster state. No ancilla photon is needed for the entangler operations here.

The generalization of the scheme to add a box cluster state to an already generated cluster state is straightforward. The schematic is shown in part (2) of Fig. 3. Generally, an already created cluster state is in the form $|\Phi_1\rangle|0\rangle_p + |\Phi_2\rangle|1\rangle_p$. First, we add one photon ($|+\rangle_q$) to an already created cluster state to get state $|\Phi_1\rangle|0\rangle_p|+\rangle_q + |\Phi_2\rangle|1\rangle_p|-\rangle_q$. Second, by continuing to add one more photon in state $|+\rangle_r$, we obtain

$$\begin{aligned}&\frac{1}{\sqrt{2}}[|\Phi_1\rangle|0\rangle_p(|0\rangle_q|+\rangle_r + |1\rangle_q|-\rangle_r)\\ &\quad+|\Phi_2\rangle|1\rangle_p(|0\rangle_q|+\rangle_r - |1\rangle_q|-\rangle_r)]\\ &= \frac{1}{\sqrt{2}}[|\Phi_1\rangle|0\rangle_p(|+\rangle_q|0\rangle_r + |-\rangle_q|1\rangle_r)\\ &\quad+|\Phi_2\rangle|1\rangle_p(|-\rangle_q|0\rangle_r + |+\rangle_q|1\rangle_r)].\end{aligned} \tag{13}$$

Finally, by a similar process shown from Eqs. (9) to (12), we can achieve the state

$$\begin{aligned}&\frac{1}{\sqrt{2}}|\Phi_1\rangle(|0\rangle|+\rangle|0\rangle|+\rangle + |0\rangle|-\rangle|1\rangle|-\rangle)_{pqrs}\\ &\quad+\frac{1}{\sqrt{2}}|\Phi_2\rangle(|1\rangle|-\rangle|0\rangle|-\rangle + |1\rangle|+\rangle|1\rangle|+\rangle)_{pqrs},\end{aligned} \tag{14}$$

which is our target cluster state. By using only three entanglers, we can add a box structure to an already created cluster state. Here, one photon in the added box belongs to the already created cluster state (i.e., the added box must include three photons, which are not in the already generated cluster states). We call this type of box a cluster state type-I box.

B. Type-II box

Since the type-I box can only be used to add three photons to an already generated cluster state, its application in generating an arbitrary 2D cluster state is limited. Here, we introduce another type of box cluster state called the type-II box (Fig. 4). This box could be used to connect two photons in an already created cluster state (or two different cluster states) to two independent photons. Suppose the already generated cluster state is initially prepared as (unnormalized):

$$|\Psi_1\rangle|0\rangle_p|0\rangle_s + |\Psi_2\rangle|0\rangle_p|1\rangle_s + |\Psi_3\rangle|1\rangle_p|0\rangle_s + |\Psi_4\rangle|1\rangle_p|1\rangle_s. \tag{15}$$

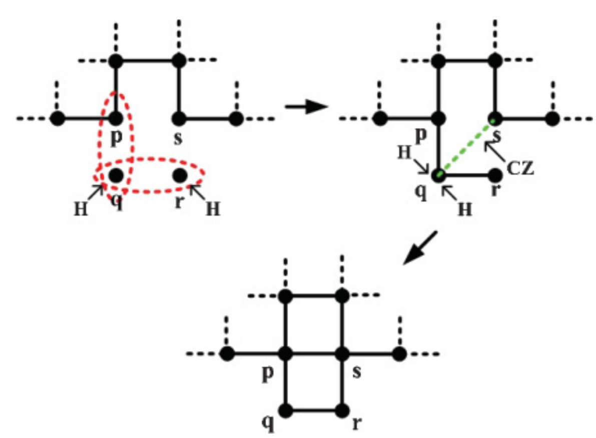

FIG. 4. (Color online) Schematic for generating a type-II box. Two entanglers are used to add two single photons to the already generated cluster state. Then, one CZ gate is performed on photons q and s for creating two links s–r, s–p. A total of four entanglers with one ancilla photon are required to add a type-II box to an already created cluster state or between two already created cluster states.

At first, we use two entanglers and some Hadamard operations to add two photons, respectively, in states $|+\rangle_q, |+\rangle_r$ to the preceding cluster state to get the following:

$$\begin{aligned}\frac{1}{\sqrt{2}}[&|\Psi_1\rangle|0\rangle_p|0\rangle_s(|+\rangle_q|0\rangle_r + |-\rangle_q|1\rangle_r)\\ &+|\Psi_2\rangle|0\rangle_p|1\rangle_s(|+\rangle_q|0\rangle_r + |-\rangle_q|1\rangle_r)\\ &+|\Psi_3\rangle|1\rangle_p|0\rangle_s(|-\rangle_q|0\rangle_r + |+\rangle_q|1\rangle_r)\\ &+|\Psi_4\rangle|1\rangle_p|1\rangle_s(|-\rangle_q|0\rangle_r + |+\rangle_q|1\rangle_r)]. \qquad (16)\end{aligned}$$

Next, after a Hadamard operation is performed on photon q, we perform a CZ operation on photons q and s, respectively. Finally, a Hadamard operation on photon q will realize the state

$$\begin{aligned}\frac{1}{\sqrt{2}}[&|\Psi_1\rangle|0\rangle_p|0\rangle_s(|+\rangle_q|0\rangle_r + |-\rangle_q|1\rangle_r)\\ &+|\Psi_2\rangle|0\rangle_p|1\rangle_s(|+\rangle_q|0\rangle_r - |-\rangle_q|1\rangle_r)\\ &+|\Psi_3\rangle|1\rangle_p|0\rangle_s(|-\rangle_q|0\rangle_r + |+\rangle_q|1\rangle_r)\\ &+|\Psi_4\rangle|1\rangle_p|1\rangle_s(-|-\rangle_q|0\rangle_r + |+\rangle_q|1\rangle_r)], \qquad (17)\end{aligned}$$

which is the target cluster state with the box structure of photons p, q, r, and s. Two entangler operations and one CZ gate are necessary for generating this type-II box. Since a CZ gate could be implemented by two entanglers, four entanglers will totally be necessary to create this type of box. On average, one entanglement bond requires one entangler operation by this method.

V. CREATING GENERAL 2D CLUSTER STATE WITH ENTANGLERS AND CZ GATES

In a classical computer, a simple computation task could involve thousands of bits. Although numerous experiments in MBQC have shown the power of quantum computation, all of them are proof of principle in nature [15,16]. The numbers of the qubits in these experiments are very limited, and only simple operations could be demonstrated. Highly efficient schemes for generating cluster states must be developed before large-scale computation in MBQC could materialize. As the main topic of this paper, we will show, in the following, how to generate an arbitrary 2D cluster state by using the previously discussed string box cluster states as the basic elements.

We illustrate the procedure with an example of a 5×5 cluster state, which is shown in Fig. 5. Six steps will complete the generation of this cluster state:

(1) Generating a cluster state string of nine qubits with eight entangler operations;

(2) creating a cluster state of four boxes by repeating the procedure of creating a type-I box for four times;

(3) adding two cluster state strings of four qubits to the second box and then, with two entanglers, two type-I boxes to the four-box cluster state;

(4) continuing to add two type-II boxes to the six-box cluster state with two cluster state strings and two CZ gates;

(5) adding two independent photons to the eight-box cluster state with two entanglers;

(6) connecting the indicated bonds to the target 5×5 cluster state by six CZ gates.

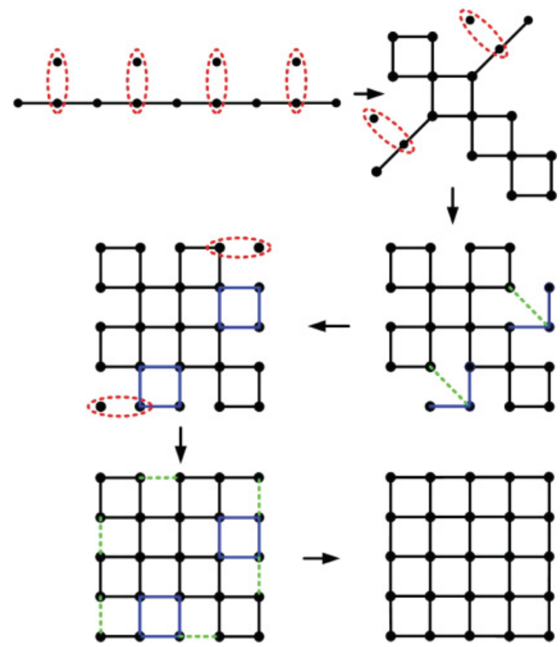

FIG. 5. (Color online) The generation of a 5×5 cluster state from a string and two types of box structures. A nine-photon cluster string and four entanglers are used to create four type-I box cluster states, and then two type-I boxes are added by six entanglers. Next, two type-II boxes are generated by four entanglers and two CZ gates. After that, with two entanglers, two independent photons will be added to the eight-box cluster state. At the last step, six CZ gates are applied to complete the connection of entanglement bonds. There is a total of 40 necessary entanglers. The number of entanglers is equal to that of the bonds of the cluster state.

Here, we neglect the use of Hadamard operations for a simpler illustration. Now, we calculate the resources required in this scheme. In addition to some single-qubit operations, 24 entanglers are required in the generation of a string, two types of box structures; eight CZ gates are required in the generation of a type-II box and in the final step. By considering the fact that one CZ gate could be realized by two entanglers, a total of 40 entanglers should be used in this scheme. The number of entanglers is exactly equal to the bonds of the 5×5 cluster state.

It is straightforward to generalize this method to create an $n \times n$ cluster state. If n is odd, $n^2 - 1$ entanglers and $(n-1)^2/2$ CZ gates, or a total of $2n(n-1)$ entanglers will be required in the generation of an $n \times n$ cluster state. If n is even, $n^2 - 1$ entanglers and $n(n-2)/2$ CZ gates, or a total of $2n(n-1) - 1$ entanglers will be necessary to generate the 2D cluster state. Evidently, the number of the entanglers is less than or equal to the number of the bonds [a total of $2n(n-1)$ bonds]. In other words, we could generate a universal 2D cluster state with one entangler operation per bond, so the scheme is highly efficient.

VI. DISCUSSION AND CONCLUSION

In this paper, we propose a deterministic scheme to generate 2D cluster states with entanglers plus CZ gates. The entangler operations need no ancilla single photon, and a CZ gate operation demands one ancilla photon, which could be recycled [21]. In principle, only one ancilla single photon is necessary for creating a 2D cluster state, which involves a large number of qubits. Compared with the previous works (e.g., Refs. [4,5,13]), the overhead for single photons could be greatly reduced.

With regard to the number of basic gates for creating a cluster state, the scheme in Ref. [13] could also generate the cluster strings or chain cluster states by one entangler per bond. The main advantage of our scheme is in generating the universal 2D cluster states, where we manage to reduce the number of CZ gates and effectively generate a 2D cluster state by one entangler per bond. In practice, like the operations in a classical computer, only the simultaneous operations require the different circuit resources, so the number of the necessary entanglers could be much smaller than the number of bonds in a cluster state. We indicate the operation time order with the arrows in the figures, where the different steps could be performed by the same entangler, CZ gate, etc. With fewer resources and greatly increased efficiency from the deterministic entangler operation, the scheme is more suitable for large-scale quantum computation than the previously proposed ones.

As mentioned in Sec. I, quantum memory is required in the schemes by using probabilistic gates, which repeat the operation until success. The already generated parts should be stored in the memory. However, the realization of high-quality quantum memory is still technically challenging. In our scheme, the entangler operation based on XPM is deterministic and very fast (the operation time is on the order of that for the signals, which go through the nonlinear medium). So, the storage time for the already generated parts need not be long, and one could use some temporary storage, such as delay lines, for the already created parts. In this sense, the scheme could be feasible with the current experimental technology.

The scheme improves on the previous ones by replacing the one-by-one fashion of generating the entanglement links with the strategies of string by string and box by box, thus, greatly increasing the efficiency. In particular, this approach to MBQC is deterministic, uses fewer resources, and uses no quantum memory. It could be a promising candidate for large-scale quantum computation.

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