Impedance-matched cavity quantum memory

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(Received 14 April 2010; published 11 August 2010)

We consider an atomic frequency comb based quantum memory inside an asymmetric optical cavity. In this configuration it is possible to absorb the input light completely in a system with an effective optical depth of one, provided that the absorption per cavity round trip exactly matches the transmission of the coupling mirror ("impedance matching"). We show that the impedance matching results in a readout efficiency only limited by irreversible atomic dephasing, whose effect can be made very small in systems with large inhomogeneous broadening. Our proposal opens up an attractive route toward quantum memories with close to unit efficiency.

DOI: 10.1103/PhysRevA.82.022310 PACS number(s): 03.67.Hk, 42.50.Pq, 03.67.Lx, 42.50.Ex

I. INTRODUCTION

Quantum memories for photons [1–4] are essential elements for many applications in quantum information processing, including quantum repeaters [5] and linear-optics quantum computing [6]. Most conceivable applications require memories with storage and readout efficiencies that are at or above the 90% level (and likely far above that level for quantum computing). While quantum memory experiments have progressed impressively over the last few years, efficiencies typically range from a few percent to a few tens of percent [7–16]. Only a few experiments have reached efficiencies above the 50% level [17–19], most notably a storage and readout efficiency close to 70% has been achieved [18] in a highly absorbing solid-state atomic ensemble using the gradient echo memory protocol [20].

It is usually thought that implementing memories in atomic ensembles [3] with efficiencies close to unity will require optical depths much greater than one [18,21,22]. However, reaching high optical depth is difficult in practice, in particular for the most attractive solid-state systems, such as rare-earth ion doped crystals [4]. Individual crystals with realistic dimensions and doping levels often have very limited optical depth. One exception is praseodymium-doped Y_2SiO_5 crystals [18]. But in order to fully exploit the potential of other materials, having considerably lower optical depth but otherwise interesting coherence properties, it would be of great interest to find a general method to overcome this crucial limitation.

Here we show that memories with unit efficiency can be realized in a cavity-memory system with an optical depth of one, by using the impedance matching condition. This condition is attained [23] when the absorption, per cavity round trip, is exactly matched to the transmission of the coupling mirror of the (asymmetric) cavity. The result is a complete absorption of the incoming light and we show that the resulting memory readout efficiency reaches 100% for optical depths around 1. The use of impedance matching had previously been suggested for quantum memories in homogeneously broadened systems [24], however, the results of Ref. [21] later showed that in such systems high effective optical depth is always required for high efficiency, because the effect of spontaneous decay cannot be ignored. In homogeneous systems the efficiency roughly scales with $1-1/d$ [21], where d is the optical depth.

Here we show that the situation is different in systems with inhomogeneous broadening, for instance in solid-state approaches [4,22]. In such systems there is an additional timescale given by the inverse of the inhomogeneously broadened bandwidth. This can be much shorter than the spontaneous decay time. As a consequence, the effects of spontaneous decay can be negligible during absorption and re-emission even for moderate optical depth. Long storage times are nevertheless possible because the inhomogeneous component of the dephasing can be made to be reversible, e.g., by tailoring the spectral density in the form of a frequency comb (AFC) [22] or by using an externally controlled reversible inhomogeneous broadening (CRIB) [4,20]. As a consequence, the principle of impedance matching can develop its full potential in inhomogeneously broadened systems, as we will now show in more detail.

II. PERFECT ABSORPTION THROUGH IMPEDANCE MATCHING

Let us start by considering the absorption of light by an inhomogeneously broadened atomic ensemble in a one-sided cavity, see Fig. 1. The readout step will be treated in Sec. III, for the case of an AFC-based control of the inhomogeneous dephasing. The dynamical equation for the cavity field $\mathcal{E}$ is

$$\dot{\mathcal{E}} = -\kappa\mathcal{E} + \sqrt{2\kappa}\mathcal{E}_{\text{in}} + i\tilde{\mathcal{P}}\int d\omega n(\omega)\sigma_\omega, \tag{1}$$

where κ is the cavity decay rate, $\tilde{\mathcal{P}}$ is proportional to the dipole moment [22], ω is the detuning, $n(\omega)$ is the inhomogeneous atomic spectral distribution, and σ_ω is the atomic polarization at detuning ω. The equation for the atomic polarization is

$$\dot{\sigma_\omega} = -i\omega\sigma_\omega - \gamma_h\sigma_\omega + i\mathcal{P}\mathcal{E}, \tag{2}$$

where γ_h is the homogeneous linewidth and $\mathcal{P}$ is the dipole moment. Finally the input-output relation for the cavity is

$$\mathcal{E}_{\text{out}} = -\mathcal{E}_{\text{in}} + \sqrt{2\kappa}\mathcal{E}, \tag{3}$$

which is valid for relatively high cavity finesse. (We will drop this simplifying assumption later on.)

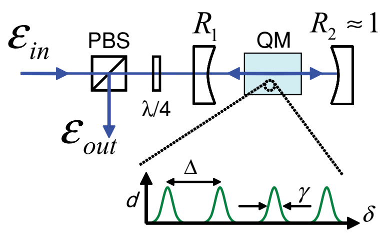

FIG. 1. (Color online) We consider a quantum memory (QM) based on an atomic frequency comb which is placed in an asymmetric optical cavity with reflectivity $R_1 < R_2 \approx 1$. The input and output fields $\mathcal{E}_{\rm in}$ and $\mathcal{E}_{\rm out}$ are separated by a quarter-wave plate ($\lambda/4$) and a polarization beam splitter (PBS). If the QM strongly absorbs only a particular linear polarization mode, one can also use a Faraday rotator and a half-wave plate as in Ref. [16]. The atomic comb memory is based on an inhomogeneously broadened transition, where the absorption depth d is shaped into a comb structure as function of detuning δ with periodicity Δ and tooth width γ. The interaction between the atomic comb structure and a incoming light pulse leads to a coherent re-emission at $t = 2\pi/\Delta$. Longer storage times can be achieved by using additional ground state levels [22,25].

Putting the solution of Eq. (2) into Eq. (1) gives

$$\dot{\mathcal{E}}(t) = -\kappa\mathcal{E}(t) + \sqrt{2\kappa}\mathcal{E}_{\rm in}(t) - \mathcal{P}\tilde{\mathcal{P}}\int_{-\infty}^{t} dt'\tilde{n}(t-t')e^{-\gamma_h(t-t')}\mathcal{E}(t'), \quad (4)$$

where $\tilde{n}(t)$ is the Fourier transform of $n(\omega)$. If $\gamma_i \gg \gamma_h$, where γ_i is the width of the inhomogeneous distribution $n(\omega)$, then the exponential containing γ_h can be ignored over the relevant timescales. If moreover γ_i is significantly larger than the bandwidth of the input light, then $\tilde{n}(t-t')$ can be approximated as $\frac{N}{\gamma_i}\delta(t-t')$ (for times around $t=0$, i.e., when the absorption happens, cf. below for much later times), where $N = \int d\omega n(\omega)$ is the total number of atoms, yielding

$$\dot{\mathcal{E}} = -\kappa\mathcal{E} + \sqrt{2\kappa}\mathcal{E}_{\rm in} - \Gamma\mathcal{E}, \quad (5)$$

where $\Gamma = \frac{N\mathcal{P}\tilde{\mathcal{P}}}{\gamma_i}$ emerges as the absorption rate of the cavity field by the atomic ensemble.

Under conditions where the input field varies much more slowly than the cavity lifetime, i.e., when the input spectrum is in resonance with the cavity, one can now adiabatically eliminate the cavity mode (i.e., set $\dot{\mathcal{E}} = 0$), which gives

$$\mathcal{E} = \frac{\sqrt{2\kappa}}{\kappa+\Gamma}\mathcal{E}_{\rm in}. \quad (6)$$

Plugging this into Eq. (3) results in

$$\mathcal{E}_{\rm out} = \frac{\kappa-\Gamma}{\kappa+\Gamma}\mathcal{E}_{\rm in}. \quad (7)$$

Total absorption, corresponding to $\mathcal{E}_{\rm out} = 0$, can thus be achieved for $\kappa = \Gamma$, which is the impedance matching condition in our case. The intuitive explanation is that in this situation the absorption losses have exactly the same effect as a second identical mirror would. To the input field the cavity-memory system therefore looks exactly like a symmetric Fabry-Perot cavity, leading to zero reflection on resonance [23]. The ratio Γ/κ is exactly the effective optical depth, or the cooperativity C in the notation of Ref. [21]. Perfect absorption is thus achieved for an optical depth of one, a very moderate value. Our results are nevertheless consistent with those of Ref. [21] in the sense that if all N atoms were concentrated into the homogeneous linewidth γ_h rather than the inhomogeneous linewidth γ_i, the resulting cooperativity would be very large, given our assumption that $\gamma_i \gg \gamma_h$. However, fortunately there is no need for all the N atoms to actually have the same frequency in the quantum memory schemes based on control of inhomogeneous dephasing.

III. HIGH-EFFICIENCY READOUT FOR AN AFC MEMORY

In the context of quantum memories it is crucial to also obtain an efficient readout of the stored excitation. Here we will limit our analysis to the case of an AFC-based [22] quantum memory. We only briefly remind the reader of the essential features, for details we refer to Ref. [22]. The inhomogeneous absorption is shaped into a comb structure, by optical pumping techniques, having periodicity Δ and peak width γ (see Fig. 1). The interaction between an incoming light pulse in resonance with the comb results in a coherent re-emission after a time $t = 2\pi/\Delta$, due to a periodic rephasing of the atomic coherence (we assume that the input spectrum is larger than Δ). Note that freely controllable storage times far beyond $2\pi/\Delta$ can be achieved by using an additional ground state level [22], as recently also shown experimentally [25].

In the case of a high AFC comb finesse $F_A = \Delta/\gamma$, the efficiency of this echo-type emission can be very large for large optical depths [22]. For a forward readout configuration it is limited to 54% due to re-absorption in the sample, while for a backward readout it can reach 100% due to an interference effect that is well understood [22]. We will show below that in our proposed cavity arrangement, the efficiency can reach 100% for a much lower optical depth, also without having to resort to the backward recall procedure [22].

We thus assume that $n(\Delta)$ has the shape of a comb, as in Fig. 1. As a consequence $\tilde{n}(t)$ has peaks not only at $t=0$ (as we used before), but also at integer multiples of $2\pi/\Delta$. Following Ref. [22] one can derive the following equation for the cavity field around the first rephasing at $t = 2\pi/\Delta$:

$$\dot{\mathcal{E}}(t) = -\kappa\mathcal{E}(t) - \frac{\mathcal{P}\tilde{\mathcal{P}}}{\sqrt{2\kappa}}\int_{-\infty}^{0} dt'\tilde{n}(t-t')\mathcal{E}_{\rm in}(t') - \mathcal{P}\tilde{\mathcal{P}}\int_{0}^{t} dt'\tilde{n}(t-t')\mathcal{E}(t'), \quad (8)$$

similar to Eq. (A15) in Ref. [22]. Using similar arguments as for the absorption, this reduces to

$$\dot{\mathcal{E}}(t) = -\kappa\mathcal{E}(t) - 2\frac{\Gamma\sqrt{\eta_F}}{\sqrt{2\kappa}}\mathcal{E}_{\rm in}\left(t - \frac{2\pi}{\Delta}\right) - \Gamma\mathcal{E}(t), \quad (9)$$

which is analogous to Eq. (A16) in Ref. [22]. Here η_F describes the reduction in efficiency due to the fact that the individual teeth in the frequency comb have finite width, which leads to irreversible atomic dephasing. In the case of Gaussian peaks [22] one finds $\eta_F \approx e^{-7/F_A^2}$. This should not be confused with the cavity finesse $\mathcal{F}_C = \frac{\pi(R_1R_2)^{\frac{1}{4}}}{1-\sqrt{R_1R_2}}$.

Adiabatically eliminating the cavity mode as before and using the fact that there is no input field at $t = 2\pi/\Delta$ we find

$$\mathcal{E}_{\text{out}}(t) = -\frac{2\Gamma\sqrt{\eta_F}}{\kappa+\Gamma}\mathcal{E}_{\text{in}}\left(t - \frac{2\pi}{\Delta}\right) = -\sqrt{\eta_F}\mathcal{E}_{\text{in}}\left(t - \frac{2\pi}{\Delta}\right), \tag{10}$$

where the last equality holds under the impedance matching condition ($\kappa = \Gamma$). One sees that the readout efficiency is only limited by the finesse of the atomic frequency comb, which without the cavity would correspond to an infinitely high optical depth d [22].

The above treatment applies to the regime where $R_1 \approx 1$ and $R_2 = 1$. More precise results for a general asymmetric cavity can be obtained in the following way. For the absorption it is sufficient to include absorption factors into the usual "sum over all roundtrips" treatment of a Fabry-Perot cavity. This yields

$$\mathcal{E}_{\text{out}} = \mathcal{E}_{\text{in}}\frac{-\sqrt{R_1}+\sqrt{R_2}e^{-\tilde{d}}}{1-\sqrt{R_1R_2}e^{-\tilde{d}}} \tag{11}$$

on resonance, where $\tilde{d}$ is the optical depth of the crystal inside the cavity (averaged over the frequency comb, cf. Ref. [22]). One sees that perfect absorption is still achievable provided that $\sqrt{R_1} = \sqrt{R_2}e^{-\tilde{d}}$, which is the impedance condition [23].

A similar treatment is possible for the memory readout. From Ref. [22] it is known that the readout efficiency can be obtained via a "sum over all amplitudes" approach. For example, for forward readout the relevant efficiency factor is given by Eq. (A19) of Ref. [22],

$$\int_0^L dz e^{-\tilde{\alpha}z/2}\tilde{\alpha}e^{-\tilde{\alpha}(L-z)/2} = \tilde{\alpha}Le^{-\tilde{\alpha}L/2}, \tag{12}$$

where L is the length of the crystal, $\tilde{\alpha}$ is the absorption coefficient ($\tilde{\alpha}L = \tilde{d}$), and one integrates over all possible points of absorption z. The first factor under the integral corresponds to the amplitude for the photon to be transmitted to the point z, the second factor can be interpreted as the amplitude for absorption and re-emission (in z), and the third factor is the amplitude to be transmitted from z to the end of the crystal after re-emission. This can be generalized for a Fabry-Perot cavity, taking into account the fact that the photon can do an arbitrary number of round trips in the cavity before absorption and after re-emission. The result is

$$2\int dz \frac{\sqrt{T_1}e^{-\tilde{\alpha}z/2}}{1-\sqrt{R_1R_2}e^{-\tilde{d}}}\tilde{\alpha}\frac{e^{-\tilde{\alpha}(L-z)/2}e^{-\tilde{\alpha}L/2}\sqrt{T_1R_2}}{1-\sqrt{R_1R_2}e^{-\tilde{d}}}, \tag{13}$$

where $T_1 = 1 - R_1$ is the transmission of the first mirror. Again the first factor under the integral corresponds to propagation before absorption, the second factor is the absorption and re-emission amplitude, and the third factor is for propagation after re-emission. The factor of 2 in front of the integral stems from the fact that the photon can be absorbed while propagating either in forward or in backward direction. Note that inside the cavity there is no change of direction upon re-emission. (Of course, the output field of the asymmetric cavity propagates predominantly in the opposite direction to the input field, but this is an automatic consequence of the interference between all the possible paths.) Simplifying the above expression, and multiplying by $\sqrt{\eta_F}$ to take into account the irreversible component of the atomic dephasing, one obtains the following expression for the square root of the total memory efficiency η (as is customary for quantum memories, we define efficiencies with respect to intensities, not amplitudes):

$$\sqrt{\eta} = \frac{2\tilde{d}e^{-\tilde{d}}T_1\sqrt{R_2}\sqrt{\eta_F}}{(1-\sqrt{R_1R_2}e^{-\tilde{d}})^2}. \tag{14}$$

Our previous results correspond to the limit $\sqrt{R_1} = 1 - \epsilon$ with $\epsilon \ll 1$, $R_2 = 1$, $\tilde{d} \ll 1$. In this case Eq. (14) becomes

$$\sqrt{\eta} = \frac{2\tilde{d}\sqrt{\eta_F}}{\epsilon+\tilde{d}}, \tag{15}$$

so that we recover our previous result ($\eta = \eta_F$) under the impedance matching condition, which is now expressed as $\epsilon = \tilde{d}$.

IV. IMPLEMENTATION ISSUES AND CONCLUSIONS

The total memory efficiency η (which includes absorption and re-emission) is shown in Fig. 2 as a function of input mirror reflectivity R_1. Clearly one can achieve very high efficiency for low reflectivities (in the context of optical cavities) and for very reasonable optical depths. For example, a memory with a peak optical depth $d = 1$ and AFC finesse $F_A = 10$, such that $\tilde{d} = 0.1$, has an efficiency of only 1% without cavity, but can be boosted to 92% efficiency by an impedance-matched cavity of finesse $\mathcal{F}_C \approx 31$.

An impedance-matched cavity memory can be operated for a large variety of conditions. Equation (14) allows us to find the best working conditions for a particular situation. There are some assumptions, however, that must be fulfilled. The

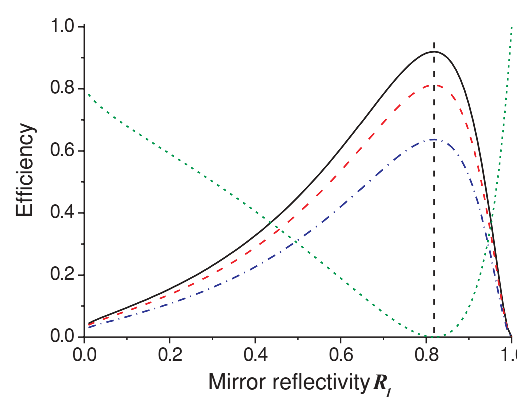

FIG. 2. (Color online) We here show the efficiency of an AFC-cavity quantum memory in an asymmetric cavity ($R_2 = 0.999$) as a function of the input mirror reflectivity R_1, based on Eq. (14). We show the result for different comb finesse $F_A = 10$ (solid line), $F_A = 6$ (dashed line), and $F_A = 4$ (dashed-dotted line). The single-pass effective absorption depth was set to $\tilde{d} = 0.1$, which in a memory without cavity would bound the efficiency to ∼1% [by use of Eq. (12)]. In the figure one clearly observes the great enhancement of memory efficiency using an impedance-matched cavity, i.e., at $R_1 = \exp(-2\tilde{d}) \approx 0.82$, reaching $\eta \sim 92\%$ for $F_A = 10$. At this point the efficiency is only limited by irreversible atomic dephasing due to the finite comb finesse F_A. We also plot the reflectivity of the combined AFC-cavity system (dotted line), showing the complete absorption of light at the optimal point.

quantum memory bandwidth must be significantly smaller than the width of the optical cavity in order to fulfill the resonance condition used above. As an example, if we assume a cavity length $L = 1\,\mathrm{cm}$ (reasonable for typical crystal dimensions) the cavity width would be $\approx$480 MHz for the example above. We have also assumed that the cavity has no losses. In general the losses must clearly be significantly lower than the memory absorption probability (per single pass). The effect of losses can be evaluated, however, by changing the reflectivity of the second mirror R_2, thus introducing a loss to the environment. For the example above, $R_2 = 0.99$ instead of $R_2 = 0.999$ would reduce the efficiency to 84%. Practically a good AR-coating on the crystal should keep the losses low enough.

In conclusion, we have shown that impedance matching to an optical cavity allows the implementation of highly efficient quantum memories for an effective optical depth of only one. Our proposal should make it much easier for experiments to reach the truly high efficiency regime.

Note added. Recently, we became aware of a recent related proposal [26].

ACKNOWLEDGMENTS

The authors acknowledge useful discussions with H. de Riedmatten, N. Sangouard, N. Gisin, and J.-L. Le Gouët. This work was supported by the Swiss NCCR Quantum Photonics, the EU projects Q-essence and QuRep.

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